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ALTERNATIVE ASYMPTOTIC WAVE SOLUTIONS OF THE FIELD EQUATIONS

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Abstract. In wave zone, new types of selfinteracting gravitational waves and their simple counterparts in the extended Maxwell theory have been determined. As a curious fact, the solutions are determined also by the nonhomogeneous Lorentz condition and the nonlinear harmonic coordinate conditions, respectively. Our solutions remove some difficulties inherent in standard approaches. New solutions describe photons and gravitons to which no specific value of spin can be ascribed and lead us to a class of unexplored theories.

Key words: wave solutions, selfinteracting gravitational radiation.

1. INTRODUCTION

It is well known that by imposing on the Maxwell equations

$$-F^{\mu\nu}{}_{,\nu} = \Box A^{\mu} - A^{\nu}{}_{,\nu}{}^{\mu} = 4\pi j^{\mu} \tag{1}$$

the Lorentz gauge condition

$$A^{\mu}{}_{,\mu} = 0, \tag{2}$$

one can write Eqs. (1) in the form

$$\Box A^{\mu} = 4\pi\ j^{\mu}, \tag{3}$$

where A^{μ} is the potential 4-vector and j^{μ} is the 4-vector of charge-current density (see, e.g., [1]). An analogous treatment is usually given to the weak linear approximation of the Einstein gravitational field [2], Section 11.2. A weak field is defined as the one for which the metric tensor $g^{\mu\nu}$, with a suitable choice of the coordinate system, can be separated into two parts

$$g^{\mu\nu} = \eta^{\mu\nu} + h^{\mu\nu},$$

where the components $h^{\mu\nu}$, corrections to the Minkowski metric $\eta^{\mu\nu}$, are small quantities. Subscript and superscript indices are raised and lowered with the metric tensor η. (We could define as well $g_{\mu\nu} = \eta_{\mu\nu} + H_{\mu\nu}$, then $H^{\mu\nu} = -h^{\mu\nu}$.) If we take $\chi^{\mu\nu} = \sqrt{-g}\, g^{\mu\nu} - \eta^{\mu\nu}$, or in the first approximation,

$$\chi^{\mu\nu} = h^{\mu\nu} - \frac{1}{2}\eta^{\mu\nu} h^{\alpha}_{\alpha}, \tag{4}$$

then the Einstein equations

$$R^{\mu\nu} = -8\pi\left(T^{\mu\nu} - \frac{1}{2} g^{\mu\nu} T\right) \tag{5}$$

can be put, in linear approximation, into the form [2]

$$\Box h^{\mu\nu} = 16\pi\left(T^{\mu\nu} - \frac{1}{2}\eta^{\mu\nu} T\right), \tag{6}$$

$$\chi^{\mu\nu}{}_{,\nu} = 0. \tag{7}$$

Here $R^{\mu\nu}$ is the Ricci tensor, $T^{\mu\nu}$ is the energy momentum tensor and T its trace, $g = \det g_{\mu\nu}$. The forms of Eqs. (6) and (7) are similar to those of (3) and (2), and linear gravitational radiation is treated following the example of well understood electromagnetic radiation.

Usually analogous treatment has been given also to higher-order wave solutions of the Einstein equations. Denoting nonlinear terms on the left-hand side of the Einstein equations (5) by $\frac{1}{8\pi} t^{\mu\nu}$, we can write the full nonlinear form of the equations as follows:

$$\Box h^{\mu\nu} - \chi^{\mu\alpha}{}_{,\alpha}{}^{\nu} - \chi^{\nu\alpha}{}_{,\alpha}{}^{\mu} = 16\pi\left(T^{\mu\nu} - \frac{1}{2} g^{\mu\nu} T + t^{\mu\nu}\right), \tag{8}$$

and add supplementary conditions (7). The nonlinear terms describe selfenergy of the gravitational field. The choice of the supplementary conditions (7) in the full theory “simply means that we are putting the interaction of the gravitational field with its own energy-momentum pseudotensor on the same footing as its interaction with the energy-momentum tensor of the matter field” (Gupta [3]). This is the basis of the standard approach to the selfinteracting gravitational radiation field and to the quantization of the nonlinear gravitational field. Equations (8), (4), and (7) are usually solved by some approximation method.

However, the standard approach leads to some difficulties, e.g., the energy loss in asymptotic solution is described by the angle dependent quantity $\Delta\varepsilon(u,\vartheta,\varphi)$ and in the wave zone terms of the type $\frac{\ln r}{r}\Delta\varepsilon$ occur ([4], Section 87). The logarithmic terms disappear in the Bondi coordinates [5] or in the Papapetrou radiation coordinates [6], but the angle dependence of $\Delta\varepsilon$ survives. More careful analysis reveals that solutions with angle dependent $\Delta\varepsilon$ diverge at time-like infinity. In this paper we propose new types of solutions of nonlinear gravitational wave equations in which the energy loss is described by angle independent quantity $M(u)$. In our solutions gravitational fields accompanying radiation energy behave differently as compared to fields emitted by massive sources.

To make clear the physical and mathematical difference between our new wave solutions with sources in the form of radiation energy flow and the standard wave solutions with massive sources, we shall first analyse in detail a much simpler model of the extended Maxwell theory. We shall extend the Maxwell equations by introducing into them fictitious light-like 4-currents $J^{\alpha}_{(rad)}$ moving at the velocity of light and having vanishing rest mass. The sources $J^{\alpha}_{(rad)}$ form a part of the current J^{μ} constructed from the massless complex scalar field. In solutions proposed below, light-like sources do not emit waves, but “drag the field along”, analogously to the *changes* of charge density in wave guides and electric transmission lines, propagating also at the velocity of light and “dragging” transverse electromagnetic waves along [7].

As a curious fact, it turns out that our new wave zone solutions are determined by the nonhomogeneous Lorentz condition

$$-A^{\mu}{}_{,\mu}=4\pi\int J^{0}_{(rad)}dt \tag{9}$$

and by the nonlinear harmonic conditions

$$\chi^{\mu\nu}{}_{,\nu}=-8\pi\int t^{\mu 0}_{(rad)}dt, \tag{10}$$

respectively. Here $t^{\mu\nu}_{(rad)}$ is the radiation part of $t^{\mu\nu}$, defined in Section 3.

Conditions (9) and (10) mean that no definite value of spin can be ascribed to our specific photons and to gravitons in nonlinear approximations. Theories of this type have been called, by Ogievetskij and Polubarinov [8], theories of class B (to discriminate them from ordinary theories of class A). As far as we know, no detailed study of theories of class B has been performed. The aim of the present paper is to initiate the research into the theories of class B, in the first place, to determine some solutions of class B theories.

In Section 2 we give a detailed gauge invariant derivation of the new types of wave solutions in the extended Maxwell theory. In Section 3 analogous solutions in the Einstein and Einstein–Maxwell theories will be proposed and discussed.

Notation. Latin indices i, k (except r) take the values 1, 2, 3 and denote the components of vectors in the orthogonal Cartesian coordinates. Greek indices (except ϑ, φ) take the values 0, 1, 2, 3; $x^{\mu} = \left(t, x^{i}\right)$, $A^{\mu} = \left(A^{0}, A^{i}\right)$, A is a three-dimensional vector. Indices are lowered and raised with the Minkowski metric $\eta_{\mu\nu} = \eta^{\mu\nu} =$ diag (1, –1, –1, –1), over repeated indices summation is assumed. The retarded time coordinate $u = t - r$ is also used; the wave vector $k^{\mu} \equiv u^{,\mu}$. The dot above a symbol denotes differentiation with respect to u. Occasionally polar coordinates r, ϑ, φ are used, $A^{r}, A^{\vartheta}, A^{\varphi}$ denote components of a vector in polar coordinates. As stated above, $A^{\mu}(u, r, \vartheta, \varphi)$, $A^{i}(u, r, \vartheta, \varphi)$ are components of a vector in the orthogonal Cartesian coordinates.

Units will be used in which the velocity of light $c = 1$ and the Newton gravitational constant $\gamma = 1$.

2. NEW SOLUTIONS OF THE EXTENDED MAXWELL EQUATIONS

In this section we shall determine solutions of a new type of the Maxwell equations. The solutions are generated by a (fictitious) spherical pulse of a light-like 4-current density

$$J^{\mu}_{(rad)} = \begin{cases} \dfrac{\sigma(u, \vartheta, \varphi)}{r^2} k^{\mu} & \text{if } u_1 < u < u_2, \\ 0 & \text{if } u < u_1 \text{ or } u > u_2, \end{cases} \tag{11}$$

where u_1 and u_2 are constants. Contributions to the linear electromagnetic field generated by j^{μ} and the remaining part of J^{μ} can be evaluated in a conventional way and added to our solution.

At first we propose a new solution of the Cauchy problem for the Maxwell equations

$$\operatorname{rot} \boldsymbol{E} = -\frac{\partial \boldsymbol{H}}{\partial t}, \tag{12}$$

$$\operatorname{div} \boldsymbol{H} = 0, \tag{13}$$

$$\operatorname{rot} \boldsymbol{H} = \frac{\partial \boldsymbol{E}}{\partial t} + 4\pi \boldsymbol{J}_{(rad)}, \tag{14}$$

$$\operatorname{div} \boldsymbol{E} = 4\pi J^{0}_{(rad)}, \tag{15}$$

then we shall demonstrate that our new solution can be obtained from the nonhomogeneous gauge condition (9) for potentials A^{μ}.

A solution to Eqs. (12)–(15) is uniquely determined by the values of $\boldsymbol{E}$ and $\boldsymbol{H}$ at some arbitrarily chosen initial moment of time $t = t_1$, but the initial values themselves for the fields $\boldsymbol{E}$ and $\boldsymbol{H}$ cannot be chosen arbitrarily; they must satisfy constraint equations (15) and (13). The novelty of our solution can be traced back to the choice of a new class of initial data, to nonstandard solution of Eq. (15).

The standard solution of Eq. (15) with the usual charge density j^0 can be found by assuming that [9–11]

$$\boldsymbol{E} = \operatorname{grad}\phi + \boldsymbol{E}', \quad \operatorname{div}\boldsymbol{E}' \equiv 0, \tag{16}$$

and inserting decomposition (16) into Eq. (15). We have

$$\Delta\phi = 4\pi j^0, \tag{17}$$

where $\Delta \equiv \operatorname{div}\operatorname{grad}$ is the Laplace operator. We see that ϕ is the constrained part of the field which is determined by the charge density j^0, while the initial value of $\boldsymbol{E}'$ can be chosen freely. Next, for the case of light-like sources (11), we shall determine an alternative solution of the constraint. Equation (15) is clearly underdetermined: three components of the electric field $\boldsymbol{E}$ at $t = t_1$ are described by one equation, and decompositions of field vectors, different from (16), lead to alternative solutions of Eq. (15) at $t = t_1$, hence to alternative solutions of the Maxwell equations in the whole space-time.

Let

$$\boldsymbol{E} = \boldsymbol{E}_{\Uparrow} + \boldsymbol{E}_{\perp}, \tag{18}$$

where the vector $\boldsymbol{E}_{\Uparrow}$ is parallel to radial vector and $\boldsymbol{E}_{\perp}$ is perpendicular to it. Suppose that at $t = t_1$

$$\operatorname{div}\boldsymbol{E}_{\Uparrow} = \frac{4\pi\sigma_0(u)}{r^2}, \tag{19}$$

$$\operatorname{div}\boldsymbol{E}_{\perp} = \frac{4\pi}{r^2}\left[\sigma(u,\vartheta,\varphi) - \sigma_0(u)\right], \tag{20}$$

$$\sigma_0(u) \equiv \frac{1}{4\pi}\int \sigma(u,\vartheta,\varphi)\sin\vartheta \,\mathrm{d}\vartheta\mathrm{d}\varphi. \tag{21}$$

In Eqs. (19) to (21) u has been taken on the initial surface $t = t_1$, i.e. $u = t_1 - r$.

Equation (19) gives us the Coulomb field with a variable charge density

$$\boldsymbol{E} = \frac{Q - q(u)}{r^2}\frac{\boldsymbol{r}}{r}, \tag{22}$$

where Q is an integration constant (the "total" charge) and

$$q(u) \equiv 4\pi \int_{u_1}^{u} \sigma_0(u)\,\mathrm{d}u. \tag{23}$$

Here integration variable r has been replaced by $u = t_1 - r$.

The treatment we shall give to Eq. (20) on the *surface of a two-sphere* $r = \text{const}$ is analogous to the standard treatment of the constrained field in three-space. Two-dimensional operators div, grad on a sphere $r = \text{const}$ we shall write with capital letters; they are appropriately chosen components of the corresponding three-dimensional operators in polar coordinates. Assume

$$\boldsymbol{E}_{\perp} = \operatorname{Grad} \Psi. \tag{24}$$

The corresponding physical components are $\left(0, \dfrac{1}{r}\dfrac{\partial \Psi}{\partial \vartheta}, \dfrac{1}{r\sin\vartheta}\dfrac{\partial \Psi}{\partial \varphi}\right)$. We have

$$\operatorname{div} \boldsymbol{E}_{\perp} = \operatorname{Div} \boldsymbol{E}_{\perp} = \frac{1}{r^2}\Delta^{*}\Psi = \frac{4\pi(\sigma - \sigma_0)}{r^2}, \tag{25}$$

where Δ^{*} is the Laplace operator on a unit sphere. To integrate Eq. (25), we expand $\sigma(u, \vartheta, \varphi)$ in spherical harmonics

$$\sigma(u, \vartheta, \varphi) - \sigma_0(u) = \sum_{l=1}^{\infty} \sum_{m=-l}^{l} \sigma_{lm}(u) Y_{lm}(\vartheta, \varphi) \tag{26}$$

and obtain from Eqs. (25), (24), and (19)

$$\Psi = -\sum_{l,m} \frac{4\pi\sigma_{lm}(u)}{l(l+1)} Y_{lm}(\vartheta, \varphi), \tag{27}$$

$$\boldsymbol{E} = \frac{Q - q(u)}{r^2}\frac{\boldsymbol{r}}{r} - \frac{4\pi}{r}\sum_{l,m}\frac{\sigma_{lm}(u)}{\sqrt{l(l+1)}} \boldsymbol{Y}^{(e)}_{lm}(\vartheta, \varphi), \tag{28}$$

$$\sqrt{l(l+1)}\,\boldsymbol{Y}^{(e)}_{lm}(\vartheta, \varphi) \equiv \left(0, \frac{\partial}{\partial\vartheta}, \frac{1}{\sin\vartheta}\frac{\partial}{\partial\varphi}\right) Y_{lm}(\vartheta, \varphi). \tag{29}$$

It is easy to verify that the value (28) of $\boldsymbol{E}$ satisfies the constraint equation (15) not only at $t = t_1$, but also at any later moment of time $t > t_1$. The second constraint equation $\operatorname{div} \boldsymbol{H} = 0$ is satisfied by $\boldsymbol{H} = \boldsymbol{n} \times \boldsymbol{E}$, $\boldsymbol{n} = \dfrac{\boldsymbol{r}}{r}$. We define $\boldsymbol{Y}^{(m)}_{lm}(\vartheta, \varphi) = \boldsymbol{n} \times \boldsymbol{Y}^{(e)}_{lm}(\vartheta, \varphi)$ and obtain

$$\boldsymbol{H} = -\frac{4\pi}{r}\sum_{l,m}\frac{\sigma_{lm}(u)}{\sqrt{l(l+1)}} \boldsymbol{Y}^{(m)}_{lm}(\vartheta, \varphi). \tag{30}$$

In solutions (28) and (30) the vector spherical harmonics of electric and magnetic types have been used [12]. The transverse part of electric field (28) and magnetic field (30) describe a TEM (transverse electromagnetic) wave which is "dragged" by light-like current, its inhomogeneities being described by $\sigma_{lm}(u)$.

The new solution has the following properties:

$$H_\vartheta = -\frac{1}{\sin\vartheta} E_\varphi, \quad \frac{1}{\sin\vartheta} H_\varphi = E_\vartheta. \tag{31}$$

We shall show that it satisfies the whole system of the Maxwell equations. By inserting (28), (30), and (24) into Eqs. (12) and (14), and taking into account that Rot Grad $\Psi \equiv 0$, we have

$$\frac{1}{\sin\vartheta}\dot{H}_\varphi = \dot{E}_\vartheta, \quad \dot{H}_\vartheta = -\frac{1}{\sin\vartheta}\dot{E}_\varphi, \tag{12a}$$

$$\frac{1}{r^2\sin\vartheta}\left(\frac{\partial}{\partial\vartheta}H_\varphi - \frac{\partial}{\partial\varphi}H_\vartheta\right) = \frac{\partial E_r}{\partial t} + \frac{4\pi\sigma(u,\vartheta,\varphi)}{r^2}, \tag{14a}$$

$$\dot{H}_\vartheta = -\frac{1}{\sin\vartheta}\dot{E}_\varphi, \quad \frac{1}{\sin\vartheta}\dot{H}_\varphi = \dot{E}_\vartheta. \tag{14b}$$

Due to the properties (31) of the new solutions, Eqs. (12a) and (14b) are satisfied, while Eq. (14a) is reduced to Eqs. (19) and (20) which are also satisfied by our solution. Four-potentials A^μ corresponding to fields (28) and (30) are

$$A^0 = A^r = \frac{Q - q(u)}{r}, \tag{32}$$

$$\mathbf{A}_\perp = \frac{4\pi}{r}\sum_{l,m}\frac{\varepsilon_{lm}(u)}{\sqrt{l(l+1)}}\mathbf{Y}_{lm}^{(e)}, \tag{33}$$

$$\varepsilon_{lm}(u) = \int_{u_1}^{u}\sigma_{lm}(u)du. \tag{34}$$

The solutions (32) and (33) can be derived from the nonhomogeneous Lorentz condition

$$-A^\mu{}_{,\mu} = 4\pi\int J^0_{(rad)}du \tag{35}$$

if we write [cf. Eq. (20)] $-\frac{d}{du}\operatorname{div}\mathbf{A}_\perp = \frac{4\pi(\sigma-\sigma_0)}{r^2}$. It is easy to see that by definition

$$E_{\perp}=-\frac{\partial}{\partial t}A_{\perp}=-\frac{\partial}{\partial u}A_{\perp},$$

A_0 being independent of the polar angles.

Nonhomogeneous gauge conditions are suitable for generalization to the case of the Einstein gravitation theory. Light-like sources in general relativity occur in a natural way in the form of the energy-momentum of electromagnetic and gravitational radiation.

3. NEW WAVE ZONE SOLUTIONS FOR SELFINTERACTING GRAVITATIONAL RADIATION

Let us determine wave zone solutions of the Einstein equations

$$\Box h^{\mu\nu}-\chi^{\mu\alpha}{}_{,\alpha}{}^{\nu}-\chi^{\nu\alpha}{}_{,\alpha}{}^{\mu}=16\pi\, t^{\mu\nu}_{(rad)}, \tag{36}$$

$$t^{\mu\nu}_{(rad)}=\frac{\sigma(u,\vartheta,\varphi)}{r^2}k^{\mu}k^{\nu}. \tag{37}$$

Assume that the Sommerfeld–Trautman [13] radiation conditions are satisfied. Hence

$$\chi^{\mu\nu}=\frac{a^{\mu\nu}(u,\vartheta,\varphi)}{r}+O(r^{-2}). \tag{38}$$

Functions $h^{\mu\nu}$ are of the same form. By inserting expressions (38) into Eqs. (36), we have for terms proportional to r^{-2} the following equations:

$$-\dot{\chi}^{\mu\alpha}{}_{,\alpha}k^{\nu}-\dot{\chi}^{\nu\alpha}{}_{,\alpha}k^{\mu}=\frac{16\pi\sigma k^{\mu}k^{\nu}}{r^2}. \tag{39}$$

There is a far-reaching analogy between extended electrodynamics and general relativity. We can impose nonhomogeneous gauge conditions, consistent with Eqs. (39)

$$\dot{\chi}^{\mu\alpha}{}_{,\alpha}=-\frac{8\pi\sigma(u,\vartheta,\varphi)k^{\mu}}{r^2}. \tag{40}$$

In the case of no φ-dependence, Eqs. (40) have the following solution (detailed calculations show that in the case $W_1=0$ it simultaneously satisfies all Eqs. (36)!):

$$\chi^{\mu\nu} = h^{\mu\nu},$$

$$h^{00} = h^{0r} = h^{rr} = \frac{2[m - M(u)]}{r},$$

$$h^{0\vartheta} = h^{r\vartheta} = \frac{1}{r^2}\sum_l \frac{2W_l(u)}{l(l+1)} P_l^{(1)}(\cos\vartheta), \qquad (41)$$

$$h^{\vartheta\vartheta} = -\frac{1}{\sin^2\vartheta} h^{\varphi\varphi} = \frac{1}{r^3}\sum_l \frac{4W_l(u)(l-2)!}{(l+2)!} P_l^{(2)}(\cos\vartheta),$$

$$\dot{M}(u) = \sigma_0(u), \; \dot{W}_l = \sigma_l(u),$$

$$\sigma(u,\vartheta) = \sigma_0(u) + \sum_{l=1}^{\infty} \sigma_l(u) P_l(\cos\vartheta).$$

Here $P_l(\cos\vartheta)$ are the Legendre polynomials and $P_l^{(m)}(\cos\vartheta)$ the associate Legendre polynomials. In the general case, instead of $P_l^{(m)}$ the spin m spherical harmonics should be used.

Equations (40) are, in fact, nonlinear equations; they can be solved explicitly by a successive approximation method, assuming in definition (38)

$$a^{\mu\nu} = a_1^{\mu\nu} + a_2^{\mu\nu} + \ldots$$

and taking account of the following form of σ:

$$\sigma(u,\vartheta,\varphi) = \frac{1}{8\pi}(\dot{a}^{\vartheta\vartheta}\dot{a}_{\vartheta\vartheta} + 2\dot{a}^{\vartheta\varphi}\dot{a}_{\vartheta\varphi} + \dot{a}^{\varphi\varphi}\dot{a}_{\varphi\varphi}).$$

The first-order values of $a^{\mu\nu}$ are determined by linear approximation equations (6) and (7). In higher approximations contributions from solutions of nonhomogeneous gauge conditions (40) must be added; they describe the self-interaction.

If we replace $t^{\mu\nu}_{(rad)}$ with the flat space-time electromagnetic radiation energy-momentum tensor $T^{\mu\nu}_{(rad)}$ which has the same form as (37), we get new solutions (41) in the linearized Einstein–Maxwell theory which should be added to the flat space-time electromagnetic waves.

In our solutions the gravitational field which accompanies ("is dragged by") photons and gravitons makes these particles heavier and spoils their standard spin structure. New solutions require modified quantization schemes. Up to now numerous attempts to construct a selfconsistent theory of quantum gravity within the class A of theories have failed. This justifies the investigation of the theories of class B, defined in [8].

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VÄLJAVÕRRANDITE ALTERNATIIVSED ASÜMPTOOTILISED LAINELAHENDID

Väino UNT

Lainetsoonis on leitud uut tüüpi iseenesega interakteeruvad gravitatsioonilained ja nende lihtne analoog Maxwelli teoorias, mille puhul on täiendavalt arvestatud valgussarnaseid allikaid. Rõhutamist väärib asjaolu, et uued lahendid on täielikult määratud mittehomogeense Lorentzi tingimusega (35) või mittelineaarsete harmoonilisuse tingimustega (40). Uued lahendid kõrvaldavad mõningad standardlahendite puudused ning kirjeldavad footoneid ja gravitone, millele ei saa omistada kindlat spinni väärtust. Seega kuuluvad nad seni uurimata teooriate klassi.