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Arkady BEREZOVSKY\* and Felix KAPLANSKY\*\*

## DYNAMICS OF THIN VORTEX RINGS IN A VISCOUS FLUID

(Presented by J. Engelbrecht)

### 1. Introduction

The idea of the description of fluid motion on the basis of vorticity dynamics has attracted researchers for a long time. This idea has formed following Aref et al. [1], «a basis for a variety of numerical procedures for flow simulation, now commonly referred to under the rubric of Vortex Methods». Sarpkaya [2] explains that the error arising by the approximation of real vorticity is the main source of difficulties by the realization of the vortex methods. As a rule, the distribution of vorticity in compact domains of a flow field is assumed to be uniform or Gaussian. In both cases such distribution is stationary.

The well-known and easily reproducible example of a vortex flow is the motion of a vortex ring. There are two familiar descriptions of this phenomenon in the classical theory of inviscid fluid. One is constructed in the form of the stream function for an axially symmetric flow induced by an isolated circular vortex filament. The other is the Hill's spherical vortex. In Maxworthy's opinion, «there is a basic inconsistency between the classical models and even the most casual observations of the motion of real vortex rings» [3].

It can be seen from the observation of vortex rings in the real fluid that two types of vortex shapes are realized at the initial and final stages of their evolution. At the initial stage the Reynolds numbers is large, while the inner radius of a vortex ring is much smaller than the outer one. At the final stage, vice versa, the Reynolds number is small, and the inner radius of a vortex ring is much larger than the outer one. The self-similar distribution of vorticity was shown by Phillips [4] for the final stage of decay of a vortex ring in viscous fluid. This direction was developed by Kambe and Oshima [5] analytically and numerically for an asymptotic state at  $t \rightarrow \infty$ . As they have written, «the physical situation might be termed a motion of «a vortex sphere» rather than a vortex ring». The radius of this vortex sphere grows as  $t^{1/2}$ , and its circulation decreases like  $t^{-1}$ . The experimental investigation by Maxworthy [3] did not give the same result. It was found that the radius of a viscous vortex ring increased like  $t^{1/3}$ , and decaying of its circulation was proportional to  $t^{-2/3}$ .

\* Eesti Teaduste Akadeemia Küberneetika Instituut (Institute of Cybernetics, Estonian Academy of Sciences). EE0108 Tallinn, Akadeemia tee 21. Estonia.

\*\* Eesti Teaduste Akadeemia Termofüüsika ja Elektrofüüsika Instituut (Institute of Thermophysics and Electrophysics, Estonian Academy of Sciences). EE0001 Tallinn, Paldiski mnt. 1. Estonia.

The asymptotic analysis of the motion and the decay of a vortex ring submerged in a background potential flow, was presented by Tung and Ting [6]. They assumed that the circulation of the vortex ring was conserved. Such an assumption is incompatible with the conservation of the vortex impulse, because in this case the outer radius of the vortex ring is invariable in time. Fraenkel [7] established the expressions for integral characteristics of thin steady vortex rings depending on the distribution of vorticity inside the vortex core. The form of the vorticity distribution was unspecified.

Thus, there is no theoretical description of the vorticity distribution inside the core of a vortex ring conformed with the outer flow and the variation of the integral parameters of a ring. In this paper we present a new vorticity distribution according to the thin vortex ring in a viscous incompressible fluid. This distribution is the first approximation of the equation of vorticity dynamics by an asymptotic solution. The self-similar solution obtained in this paper recovers and enlarges the previous results, and furnishes an explanation of the experimental relations.

## 2. Statement of the problem

We consider the vortex ring as a motion in an incompressible unbounded fluid induced by the vorticity occupying the part of space like the torus with the outer radius  $R_0$  and the inner radius  $r_0$  at the initial moment  $t_0$  (See Fig. 1). The viscous fluid is assumed to be at rest at infinity. We suppose the flow field to be axisymmetric. In this case the vorticity possesses only the azimuthal component in the frame of cylindrical polar coordinates  $(r, z)$ . We represent the vorticity dynamics and continuity equations governing the flow in the form [8]

$$\frac{\partial \zeta}{\partial t} + \frac{\partial}{\partial r} (U\zeta) + \frac{\partial}{\partial z} (V\zeta) = \nu \left( \frac{\partial^2 \zeta}{\partial z^2} + \frac{\partial^2 \zeta}{\partial r^2} + \frac{1}{r} \frac{\partial \zeta}{\partial r} - \frac{\zeta}{r^2} \right) \quad (2.1)$$

$$\frac{\partial}{\partial r} (rU) + \frac{\partial}{\partial z} (rV) = 0. \quad (2.2)$$

Here,  $U$  and  $V$  are the  $r$  and  $z$  components of the velocity vector, respectively,  $t$  is time,  $\zeta$  is the vorticity,  $\nu$  is the kinematic viscosity.

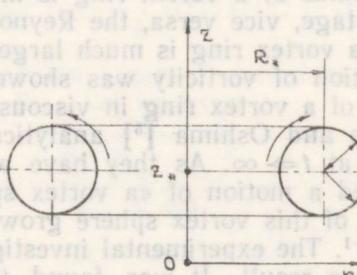


Fig. 1. Definition sketch.

Due to the continuity equation, a stream function can be introduced

$$U = -\frac{1}{r} \frac{\partial \varphi}{\partial z}, \quad V = \frac{1}{r} \frac{\partial \varphi}{\partial r}, \quad (2.3)$$

The stream function is related to the vorticity as follows:

$$-r\zeta = \frac{\partial^2 \varphi}{\partial z^2} + \frac{\partial^2 \varphi}{\partial r^2} - \frac{1}{r} \frac{\partial \varphi}{\partial r}. \quad (2.4)$$

This relation permits to restore the distribution of the stream function while the vorticity is known.

The condition of regularity to be satisfied on the symmetry axis is

$$\zeta = 0, \quad \varphi = 0 \quad \text{at} \quad r = 0. \quad (2.5)$$

Integration of (2.1) under condition (2.5) gives the following condition for the conservation of the vortex impuls [9]:

$$\pi \int_{-\infty}^{\infty} \int_0^{\infty} r^2 \zeta dz dr = M_0. \quad (2.6)$$

The conservation of the vortex impulse causes a nontrivial solution of the problem (i.e. of the motion of a vortex ring). As initial conditions, we have initial inner and outer radii of the vortex ring, which is just sufficient for the self-similar solution.

### 3. The limit cases of motion

In order to find the governing parameters of the problem, it is necessary to transform the governing equations into a nondimensional form. We easily come to the existence of two characteristic length scales. The first is the diffusion scale [8]

$$r_* = (2vt)^{1/2}, \quad (3.1)$$

and it may be chosen as a core size, i.e. the inner radius.

The second one is the inertial scale. Usually it is identified by the current outer radius of the vortex ring  $R_*$ . Both of the introduced scales change with time.

There are two limit cases for the motion of a vortex ring. The first is determined by the relation

$$\frac{r_*}{R_*} \ll 1. \quad (3.2)$$

The vortex ring with an infinitesimal inner radius is called the circular vortex filament. In the inviscid fluid the circular vortex filament induces the potential flow with the distribution of a stream function in the following form [10]:

$$\varphi(r, z) = \frac{\Gamma}{2\pi} (rR_*)^{1/2} \left\{ \left( \frac{2}{k} - k \right) K(k) - \frac{2}{k} E(k) \right\}, \quad (3.3)$$

$$k^2 = 4\pi R_* r / (z^2 + (r + R_*)^2).$$

Here  $\Gamma$  is the given circulation of the vortex filament  $K(k)$  and  $E(k)$  are the complete elliptic integrals of the first and second kind, respectively.

The distribution of the stream function (3.3) is singular. That is why the infinite velocity of the circular vortex filament emerges in the inviscid fluid.

In another limit case the outer radius of the vortex ring is assumed to be much smaller than the inner one

$$\frac{r_*}{R_*} = \epsilon \gg 1, \quad (3.4)$$

This condition corresponds to the regime of viscous diffusion. For the final stage of the viscous decay of the vortex ring the self-similar solution of the problem is well-known, and can be presented in the form [4]

$$\zeta = t^{-1} r \exp\left(-\frac{r^2+z^2}{8vt}\right), \quad (3.5)$$

$$\begin{aligned} \varphi &= \left(\frac{r^2}{r^2+z^2}\right) \left( \left(\frac{1}{(r^2+z^2)^{1/2}}\right) \int_0^x \exp(-s^2) ds - \exp\left(-\frac{r^2+z^2}{8vt}\right) \right) \\ z &= (r^2+z^2)^{1/2}. \end{aligned} \quad (3.6)$$

Both first- and second-limit solutions did not describe the real vortex rings. In the case of the circular vortex filament the vortex ring simply does not exist yet. It is a pure outer flow without an inner one. At the final stage of the viscous decay the vortex ring can not be identified because it occupies all the space and it actually is the pure inner flow without the outer one. For that reason a more detailed description of the motion of the vortex ring can be constructed only on the basis of its finite sizes.

#### 4. Inertial regime

In this case viscous effects are confined to a small neighbourhood of the centre of the vortex core. More exactly, if the condition (3.2) is observed at the initial moment

$$\frac{r_0}{R_0} = \varepsilon_0 \ll 1 \quad (4.1)$$

it will still be satisfied for some time.

It is convenient to decompose the full flow field into two parts: (i) the interior of the viscous core and (ii) the outer region, where the influence of viscosity is much smaller. We look for a solution of Eqs. (2.1) and (2.4) under the conditions (2.5), (2.6) and (3.2). The problem has a small parameter which is the ratio of length scales. Such a problem following Van Dyke [11], is the problem of singular perturbation, and we will apply the method of multiple scales for solving it. For this purpose we shall introduce the dimensionless variables of two different kinds. The origin of coordinates is replaced in the centre of the vortex core

$$(4.2) \quad \bar{x} = \frac{r - R_*}{R_*}, \quad \bar{y} = \frac{z - Z_*}{R_*}, \quad \bar{\tau} = \frac{R_0}{R_*},$$

$$(4.3) \quad x = \frac{r - R_*}{r_*}, \quad y = \frac{z - Z_*}{r_*}, \quad \tau = \frac{r_0}{r_*},$$

where  $Z_*$  is the current distance travelled by the vortex ring. The barred quantities indicate those associated with the outer region. The vorticity and the stream function are assumed to be as a sum of two terms each of which depending on the variables of their own kind only

$$(4.4) \quad \frac{\zeta}{\omega_0}(r, z, t; \varepsilon) = \bar{W}(\bar{x}, \bar{y}, \bar{\tau}; \varepsilon) + \frac{W}{\varepsilon^2}(x, y, \tau; \varepsilon),$$

$$\frac{\varphi}{\omega_0 R_*^3} (r, z, t; \varepsilon) = \bar{\psi}(\bar{x}, \bar{y}, \bar{z}; \varepsilon) + \psi(x, y, z; \varepsilon). \quad (4.5)$$

Here  $\omega_0$  is the characteristic scale of vorticity. Its value will be determined later.

By rearranging (2.1)–(2.2), we transform the governing equations into their dimensionless form. This form can be again split into two parts. The first one depends on outer variables  $\bar{x}$ ,  $\bar{y}$ ,  $\bar{z}$ , and the second one on the inner ones  $x$ ,  $y$ ,  $z$ . The equality between these parts is possible only if each of them is equal to the same constant. However, the first part differs from the second one by two orders of  $\varepsilon$ . Hence, we can equate each part to zero individually. By the identification of the velocity components in the centre of the vortex core with the velocity components of the outer flow

$$u(t) = \frac{1}{\bar{x}+1} \frac{\partial \bar{\psi}}{\partial \bar{y}} \left( x\varepsilon, y\varepsilon, \tau \frac{\varepsilon}{\omega_0} \right) R_* \omega_0 = \frac{dR_*}{dt}, \quad (4.6)$$

$$v(t) = \frac{1}{\bar{x}+1} \frac{\partial \bar{\psi}}{\partial \bar{x}} \left( x\varepsilon, y\varepsilon, \tau \frac{\varepsilon}{\omega_0} \right) R_* \omega_0 = \frac{dZ_*}{dt}, \quad (4.7)$$

we will get the following equations for the inner problem:

$$\begin{aligned} \frac{W}{\omega_0^2} \frac{\partial \omega_0}{\partial t} - \frac{\dot{r}_*}{\omega_0 r_*} \left( x \frac{\partial W}{\partial x} + y \frac{\partial W}{\partial y} + \tau \frac{\partial W}{\partial \tau} \right) - \frac{1}{\varepsilon^2} \frac{1}{(ex+1)} \left[ \frac{\partial \psi}{\partial y} \frac{\partial W}{\partial x} - \right. \\ \left. - \frac{\partial \psi}{\partial x} \frac{\partial W}{\partial y} \right] + \frac{1}{(ex+1)} \left[ \frac{B(t)}{\omega_0} + \frac{1}{\varepsilon} \frac{1}{(ex+1)} \frac{\partial \psi}{\partial y} \right] W = \\ = \frac{1}{Re_*} \left[ \frac{\partial^2 W}{\partial x^2} + \frac{\partial^2 W}{\partial y^2} + \frac{\varepsilon}{(ex+1)} \frac{\partial W}{\partial x} - \frac{\varepsilon^2 W}{(ex+1)^2} \right], \quad (4.8) \end{aligned}$$

$$+ \left( \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} - \frac{\varepsilon}{ex+1} \frac{\partial \psi}{\partial x} \right) = -(ex+1) W, \quad (4.9)$$

where

$$B(t) = \frac{u(t)}{R_*(t)} = \frac{\dot{R}_*(t)}{R_*(t)}, \quad Re_* = \frac{\omega_0 r_*}{\nu}. \quad (4.10)$$

We suppose the outer flow to be potential. Therefore the boundary conditions for the vorticity and the stream functions become

$$W \Rightarrow 0, \quad \psi \Rightarrow 0 \quad \text{at} \quad x^2 + y^2 \Rightarrow \infty, \quad (4.11)$$

and the vortex impulse is preserved due to the inner flow only.

## 5. Inner flow. The zero-order approximation

The flow field generated near the centre of the cross section of the vortex ring will obviously be symmetrical. Therefore it is convenient to use the polar coordinates in this region

$$x = q \cos \theta, \quad y = q \sin \theta. \quad (5.1)$$

The equations of vorticity dynamics and stream function (4.8)–(4.9) in new coordinates become

$$\begin{aligned} & \frac{W}{\omega_0^2} \frac{\partial \omega_0}{\partial t} - \frac{\dot{r}_*}{\omega_0 r_*} \left( \varrho \frac{\partial W}{\partial \varrho} + \tau \frac{\partial W}{\partial \tau} \right) - \frac{1}{\varepsilon^2} \frac{1}{(\varepsilon \varrho \cos \theta + 1)} \left[ \frac{\partial \psi}{\partial \varrho} \frac{\partial W}{\partial \theta} - \right. \\ & \left. - \frac{\partial \psi}{\partial \theta} \frac{\partial W}{\partial \varrho} \right] + \frac{1}{(\varepsilon \varrho \cos \theta + 1)} \left[ \frac{B}{\omega_0} + \frac{1}{\varepsilon} \frac{1}{(\varepsilon \varrho \cos \theta + 1)} \left( \frac{\partial \psi}{\partial \varrho} \sin \theta - \right. \right. \\ & \left. \left. - \frac{\cos \theta}{\varrho} \frac{\partial \psi}{\partial \theta} \right) \right] W = \frac{1}{Re_*} \left[ \frac{\partial^2 W}{\partial \varrho^2} + \frac{1}{\varrho^2} \frac{\partial^2 W}{\partial \theta^2} + \frac{1}{\varrho} \frac{\partial W}{\partial \varrho} \right], \quad (5.2) \\ & \frac{\partial^2 \psi}{\partial \varrho^2} + \frac{1}{\varrho^2} \frac{\partial^2 \psi}{\partial \theta^2} + \frac{1}{\varrho} \frac{\partial \psi}{\partial \varrho} = -W. \quad (5.3) \end{aligned}$$

The form of the governing equations (5.2), (5.3) is obtained by neglecting the terms of the order  $\varepsilon$  and larger. We will seek the approximate solution of Eqs. (5.2), (5.3) in the form of asymptotic expansions

$$W(\varrho, \theta, \tau; \varepsilon) = W_0(\varrho, \tau) + \varepsilon \cos \theta W_1(\varrho, \tau) + \varepsilon^2 \cos 2\theta W_2(\varrho, \tau) + \dots \quad (5.4)$$

$$\psi(\varrho, \theta, \tau; \varepsilon) = \psi_0(\varrho, \tau) + \varepsilon \cos \theta \psi_1(\varrho, \tau) + \varepsilon^2 \cos 2\theta \psi_2(\varrho, \tau) + \dots \quad (5.5)$$

which are valid as  $\varepsilon_0 \rightarrow 0$  for fixed values of  $\varrho$  and  $\tau$ .

In addition, the following expansion is used

$$\frac{1}{(\varepsilon \varrho \cos \theta + 1)} = 1 - \varepsilon \varrho \cos \theta + \varepsilon^2 \varrho^2 \cos^2 \theta - \dots \quad (5.6)$$

Finally, in order to get a governing system for determining the unknown functions  $W_0$ ,  $W_1$ ,  $W_2$  and  $\psi_0$ ,  $\psi_1$ ,  $\psi_2$  we substitute expansions (5.4), (5.5) into system (5.2), (5.3), yielding

$$[\psi'_0 W_0 + \psi'_0 W_1 - \psi_1 W'_0] = 0, \quad (5.7)$$

$$\begin{aligned} & \left[ \psi'_0 W_0 \varrho + \alpha \left( \frac{\psi'_0 W_1 \varrho}{2} - \frac{\psi'_0 W_1}{2} - \frac{\psi'_1 W_0}{2} - \frac{\psi_1 W_0}{2} - \frac{\psi_1 W'_0 \varrho}{2} \right) + \right. \\ & \left. + \alpha^2 \left( \frac{\psi_1 W'_1}{2} - \frac{\psi_1 W_0}{2} - 2\psi'_0 W_2 + 2\psi_2 W'_0 \right) \right] \sin 2\theta + \frac{W_0 \omega_0}{\omega_0^2} - \\ & - \frac{\dot{r}_*}{\omega_0 r_*} \left( \varrho W'_0 + \tau W_0 \right) - \frac{B(t) W_0}{\omega_0^2} - \frac{1}{Re_*} \left[ W''_0 + \frac{1}{\varrho} W'_0 \right] = 0, \quad (5.8) \end{aligned}$$

$$\psi''_0 + \frac{1}{\varrho} \psi'_0 = -W_0, \quad (5.9)$$

$$\psi''_1 + \frac{1}{\varrho} \psi'_1 = -W_1, \quad (5.10)$$

$$\psi''_2 + \frac{1}{\varrho} \psi'_2 = -W_2 - \Delta \bar{\psi}_0, \quad (5.11)$$

$$\alpha = \frac{\varepsilon}{\varepsilon_0}.$$

Here «'» indicate the space derivation, «·» the time derivation and «\*» derivation with respect to  $\tau$ . The existence of trigonometric functions in (5.8) (i.e. the dependence on polar coordinates  $\theta$ ) gives the possibility to isolate the part that does not depend on  $\theta$ . The zero-order approximation of (5.8) yields

$$2kW_0 - \left( \varrho \frac{\partial W_0}{\partial \varrho} + \tau \frac{\partial W_0}{\partial \tau} \right) + CW_0 = \frac{\partial^2 W_0}{\partial \varrho^2} + \frac{1}{\varrho} \frac{\partial W_0}{\partial \varrho}, \quad (5.12)$$

where  $C = 2tB(t)$ ,  $\omega_0 = t^k$ .

From the physical viewpoint, the constant  $C$  describes the influence of ambient flow. Its value is fixed by the existence of a self-similar solution of Eq. (5.12)

$$C = -2k - 2. \quad (5.13)$$

Substituting this constant value into Eq. (5.12) we obtain the solution of the inner problem

$$W_0 = e^{-\frac{\varrho^2 + \tau^2}{2}} I_0(\varrho\tau), \quad (5.14)$$

where  $I_0$  is the modified Bessel function.

Thus, the zero-order approximation gives the self-similar distribution of the vorticity in the core of the vortex ring. Such a self-similarity is obtained due to the scaled system of coordinates using the compatibility of outer and inner velocities in the centre of vortex core.

The stream function is related to  $W_0$  by Eq. (5.9.). The zero-order solution will be completed after the determination of the degree  $k$ .

## 6. Integral properties of a vortex ring

As it was pointed out earlier, the outer flow is potential. Hence, all the integral characteristics of a vortex ring are determined by the inner flow only. Using the vorticity distribution obtained above, we can find the circulation. Integration of (5.14) gives

$$\Gamma = \int_{-\infty}^{\infty} \int_0^{\infty} \zeta dz dr = \pi \omega_0 R_*^2. \quad (6.1)$$

The value of the vortex impulse may be obtained analogically

$$M_0 = \int_{-\infty}^{\infty} \int_0^{\infty} r^2 \zeta dz dr = \Gamma R_*^2. \quad (6.2)$$

Assuming

$$R_* \sim t^p \quad (6.3)$$

we obtain, from the expressions (4.10), (5.13) and (6.2), a system of equations for determining the parameters  $k$  and  $p$

$$k + 4p = 0, \quad (6.4)$$

$$2p = -2k - 2. \quad (6.5)$$

The values resulting from this system are

$$p = 1/3, \quad k = -4/3. \quad (6.6)$$

Consequently, the outer radius of a vortex ring and its characteristic vorticity depend on time like

$$R_*(t) = R_0 t^{1/3} t_0^{-1/3}, \quad (6.7)$$

$$\omega_0(t) = A(M_0) t^{-4/3} t_0^{1/3}. \quad (6.8)$$

The value of the constant of  $A$  can be determined by the condition of the conservation of a vortex impulse (2.6). At the initial moment we have

$$A(M_0) = \frac{M_0 t_0}{\pi R_0^4}. \quad (6.9)$$

Hence, the dependence on time for circulation has the form

$$\Gamma = \frac{M_0}{R_0^2} \left( \frac{t}{t_0} \right)^{-2/3}. \quad (6.10)$$

These results agree with the measurements of  $R_*$  and  $\Gamma$  by Maxworthy [3].

## 7. Discussion of results

The comparison of the theoretical vorticity distribution inside the vortex core with the experimental data is shown in Fig. 2. The values of experimental parameters  $Re = 2 * 10^4$ ,  $R_0 = 2.5$  cm,  $v = 0.01$  cm/s,  $\zeta = -680$  s<sup>-1</sup> give the value  $M_0 = 13 * 10^2$  cm<sup>4</sup>/s. A good agreement between the theoretical and experimental results can be achieved by choosing  $r_0 = 0.217$  cm. The corresponding values of time are  $t_0 = 0.8$  s and  $t = 2.36$  s. As it can be seen (in Fig. 2 dots indicate experimental data), the conclusion on the symmetry of the motion inside the vortex core is confirmed. This conclusion was noted by the preliminary experimental investigation [12]. A unique feature of this motion is the leading role of the flow in the vortex core. The vorticity actually determines the appearance of circulation which causes the outer potential flow. This can be described by the well-known solution for the circular vortex filament in perfect fluid (3.3) in addition to which circulation vanishes in time. The suggested formulation of the problem gives the possibility to describe the interaction between two coaxial vortex rings in the viscous fluid. In this case it is convenient to use the modification of the familiar approach [13]. The phase portrait illustrating the «game» between vortex rings is presented in Fig. 3. The difference from the description of the same phenomenon in a perfect fluid is obvious. The influence of viscosity causes the spreading of an unclosed trajectory.

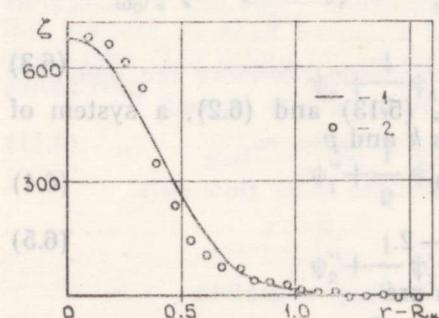


Fig. 2. Vorticity distribution inside the vortex core. 1 — theoretical curve according (5.15), 2 — experimental data [12].

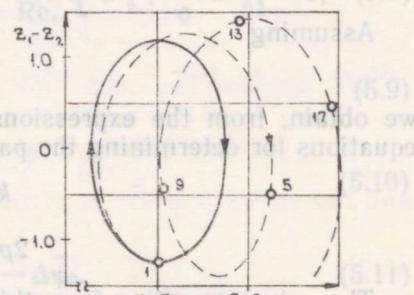


Fig. 3. Phase portrait explaining the interaction between two vortex rings;  $(z_1 - z_2)$  is the distance between vortex rings,  $r$  is the outer radius for one of the vortex rings, numbers show the time intervals. Continuous curve corresponds to the case of a perfect fluid (mapping back to 1), dashed curve — to the case of a viscous fluid.

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Arkadi BEREZOVSKI, Felix KAPLANSKI

## OHUKESTE KEERISRÖNGASTE DÜNAAMIKA Viskoossetes vedelikes

Töös on vaadeldud väikese viskoossusega vedelikku koosnevana välisest potentiaalst ja sisemistest keeriselisest osast ning uuritud ohukesti keerisrõngaste automodelset jaotust keerise südamikus. Teoreetilise kinnituse said eksperimentaalselt leitud keerisrõnga tsirkulatsiooni ja raadiuse ajalise muutumise seaduspärasused. Saadud tulemuste kasutamise näitena on tehtud kahe samatelgse keerisrõnga koostoime («mängu») arvutus.

Аркадий БЕРЕЗОВСКИЙ, Феликс КАПЛАНСКИЙ

## ДИНАМИКА ТОНКИХ ВИХРЕВЫХ КОЛЕЦ В МАЛОВЯЗКОЙ ЖИДКОСТИ

Изучено движение тонких вихревых колец в маловязкой жидкости путем разделения течения на внешнюю потенциальную и внутреннюю вихревую части. В нулевом приближении получено автомодельное распределение завихренности внутри ядра вихря. Теоретически подтверждены экспериментально обнаруженные законы изменения циркуляции и радиуса вихревого кольца во времени. В качестве примера использования полученных результатов произведен расчет взаимодействия («игры») двух коаксиальных вихревых колец.