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MODEL MATCHING OF NONLINEAR DISCRETE TIME SYSTEMS VIA DYNAMIC STATE FEEDBACK

(Presented by O. Jaaksoo)

This paper considers the problem of designing the compensator for a nonlinear discrete time system under which the input-output behaviour of the compensated system becomes the same as the one of the prespecified nonlinear model. A local solution around a fixed (equilibrium) point of the system is given for the case when the so-called decoupling matrix of the system is nonsingular at the fixed point. For this subclass of right invertible systems the considered problem is solvable by dynamic state feedback locally around the fixed point of the system and the corresponding fixed point of the model if and only if the delay (relative) orders of the model are equal or greater than those of the system. The internal stability of the reduced-order right inverse system dynamics is shown to be the key factor in the model matching problem with internal stability.

1. Introduction

The model matching problem (MMP), which consists in designing a compensator for a certain plant such that the input-output map of the compensated system would match that of the prespecified model, has attracted a great deal of interest during the last two decades. Most papers consider linear systems, and a number of results have been obtained for the continuous time nonlinear systems either by tools of differential geometry [1, 2] differential algebra [3], linear algebra [4], by the so-called structure algorithm [5] and related zero-dynamics algorithm [6, 7], or by considering the MMP as the disturbance decoupling problem [8, 9]. The nonlinear MMP with internal stability of the closed-loop system has been considered by Byrnes, Castro and Isidori [10] for the single-input single-output systems, and by Huiberts [7] for the multi-input multi-output systems. Except for the papers by Conte, Moog and Perdon [3] and by Moog [4], all the results have been obtained for the systems which are linear in control and are local in the sense that they are valid in some neighbourhood of the initial point in the state space. To the author's knowledge there are no papers written on the topic of model matching of nonlinear discrete-time systems.

This paper deals with the MMP for discrete-time nonlinear analytic systems. In the case of nonlinear systems, the input-output map is usually described by Volterra series, and the purpose of the MMP is to achieve the coincidence of the corresponding Volterra kernels for the compensated system and the model. In the discrete-time case the Volterra kernels can be computed by the operator exponent technique [11, 12].

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Unfortunately, it is technically extremely complicated to obtain the expressions of Volterra kernels for a multi-input multi-output compensated system, nonlinear in control. In order to reduce the technical apparatus and to avoid lengthy proofs, this paper considers the MMP as the problem of finding such a compensator which would, beginning from some fixed time instant make the outputs of the compensated system to be equal to the outputs of the model, provided their inputs coincide. Similar approach was also followed by Okutani and Furuta [8], Huiberts [7] and Di Benedetto [6].

We shall adopt a local viewpoint throughout this paper. However, contrary to the continuous time case, in the discrete time case the local study is impossible around the arbitrary initial state, since even in one step the state evolution can move far from the initial point. For this reason we shall consider the MMP locally around the fixed (equilibrium) points. Such an approach was also used by Nijmeijer [13] in studying the input-output decoupling problem. Our paper considers a subclass of right invertible systems — the systems with nonsingular decoupling matrix. The necessary and sufficient condition for the solvability of the local finite time MMP for this class of systems is given, and the equations of the compensator in the form of the dynamic state feedback are found. The internal stability of the reduced-order right inverse system dynamics is shown to be the key factor in the MMP with internal stability.

2. Problem statement

Consider a discrete-time nonlinear plant P described by the equations of the form

$$\begin{aligned} x(t+1) &= f(x(t), u(t)), \\ y(t) &= h(x(t)), \end{aligned} \tag{1}$$

where the states $x(\cdot)$ belong to an open part X of R^n , the inputs $u(\cdot)$ belong to an open part U of R^m , and the outputs $y(\cdot)$ belong to an open part Y of R^m . The mappings f and h are supposed to be real analytic.

Furthermore, let a discrete-time nonlinear model M be given, which is described by the equations

$$\begin{aligned} x^M(t+1) &= f^M(x^M(t), u^M(t)), \\ y^M(t) &= h^M(x^M(t)), \end{aligned} \tag{2}$$

where the states $x^M(\cdot)$ belong to an open part X^M of R^{n_m} , the inputs $u^M(\cdot)$ belong to an open part U^M of R^m , and the outputs $y^M(\cdot)$ belong to an open part Y^M of R^m . The mappings f^M and h^M are supposed to be real analytic.

The compensator C used to control the plant P is a discrete-time nonlinear system described by the equations of the form

$$\begin{aligned} x^c(t+1) &= f^c(x^c(t), x(t), u^M(t)), \\ u(t) &= h^c(x^c(t), x(t), u^M(t)) \end{aligned} \tag{3}$$

with the state $x^c(\cdot) \in X^c$, an open part of R^v , and real analytic f^c and h^c .

The composition of (1) and (3), i. e. the system

$$x(t+1) = f(x(t), h^c(x^c(t), x(t), u^M(t))),$$

$$x^c(t+1) = f^c(x^c(t), x(t), u^M(t)),$$

$$y(t) = h(x(t))$$

is denoted by $P \circ C$: We are assumed to work in a neighbourhood of an equilibrium point of the system (1), that is around $(x^0, u^0) \in X \times U$ such that $f(x^0, u^0) = x^0$. For the input sequence $u(t) = u^0, t \geq 0$ there exists the constant output sequence $y(t) = y^0 = h(x^0), t \geq 0$. Let the corresponding equilibrium point (x^{M0}, u^{M0}) of the model be such that $y^0 = y^{M0}$.

Definition 1. Nonlinear discrete-time local finite time model matching problem. Given the system (1) around the equilibrium point (x^0, u^0) , the model (2) around the corresponding equilibrium point (x^{M0}, u^{M0}) and a point $(x(0), x^M(0)) \in X_0 \times X^{M0} \subset X \times X^M$, find the neighbourhoods V_1 of (x^0, x^{M0}, u^{M0}) in $X \times X^c \times U^M$ and V_2 of u^0 in U , the compensator $C: V_1 \rightarrow V_2$ with the initial state $x^c(0)$ defined by (3), with the property that

$$y^{P \circ C}(t, x(0), x^c(0), u^M(0), \dots, u^M(t-1)) = \\ = y^M(t, x^M(0), u^M(0), \dots, u^M(t-1)), \quad t_0 \leq t \leq t_F$$

for all $(x(0), x^M(0)) \in X_0 \times X^{M0}$ and for some t_F , where t_0 is some fixed adjustment time depending only on the system.

3. Preliminaries

In this Section, some background material is reviewed.

When necessary, we denote the components of a vector by using the lower indices, e. g. $y(t) = (y_1(t), \dots, y_m(t))^T$, $h(x) = (h_1(x), \dots, h_m(x))^T$.

With each component of the output y_i , we can associate a delay order q_i (referred to in the literature also as a characteristic number or relative order) in the following manner; see also [14].

Given an arbitrary initial state $x \in X$, we can compute for $i = \{1, \dots, m\}$ the derivative

$$\frac{\partial}{\partial u} h_i^1(f(x, u)) = \frac{\partial}{\partial u} h_i(f(x, u)). \quad (4)$$

It follows from the analyticity of the system that either the vector in (4) is nonzero for all (x, u) belonging to an open and dense subset 0_i of $X \times U$, or this vector vanishes for all $(x, u) \in X \times U$. In the first case we define $q_i = 1$, whereas in the latter case we continue by observing that the function $h_i^1(f(x, u))$ does not depend on u and so we may write $h_i^1(f(x, u)) = h_i^2(x)$ for some analytic function h_i^2 on X . Next we compute in an analogous fashion

$$\frac{\partial}{\partial u} h_i^2(f(x, u)).$$

If this vector is nonzero on an open and dense subset 0_i of $X \times U$, we set $q_i = 2$, otherwise we continue with the function $h_i^3(x) = h_i^2(f(x, u))$.

In this way the number q_i — if it exists — determines the inherent delay between the inputs and i th output. Namely, the input $u(0) = u$ affects the i th output only after q_i steps, that is at the time instant $t = q_i$. In case none of the iterated functions $h_i^k(f(x, u))$ depend on u , we define

$q_i = \infty$. When $q_i = \infty$, the i th output evolves in time independently of the input sequence applied to the system (1). Notice that a finite delay order satisfies [14] the inequality

$$q_i \leq n. \quad (5)$$

Assuming that each delay order q_i is finite, one can introduce the so-called decoupling matrix

$$A(x, u) = \begin{bmatrix} \frac{\partial}{\partial u} h_1^{p_1}(f(x, u)) \\ \vdots \\ \frac{\partial}{\partial u} h_m^{p_m}(f(x, u)) \end{bmatrix}.$$

From the definition of the q_i 's the rows of the matrix $A(x, u)$ are non-vanishing functions on an open and dense subset $0 = O_1 \cap O_2 \cap \dots \cap O_m$ of $X \times U$.

Lemma. [15] If the decoupling matrix $A(x, u)$ is nonsingular on an open and dense subset contained in $X \times U$, then

$$\text{rank } \frac{\partial}{\partial x} \begin{bmatrix} h_1^1(x) \\ h_1^{p_1}(x) \\ \vdots \\ h_m^1(x) \\ h_m^{p_m}(x) \end{bmatrix} = \sum_{i=1}^m q_i = \mu$$

on an open and dense subset of X .

4. Local right invertibility

Let us define by Y^0 the space of sequences $y^*(t)$, $t \geq 0$ which are sufficiently close to y^0 , i.e. $\|y^*(t) - y^0\| < \epsilon$ for some $\epsilon > 0$ and all $t \geq 0$.

Analogously, define by U^0 the space of sequences $u^*(t)$, $t \geq 0$ which are sufficiently close to u^0 , i.e. $\|u^*(t) - u^0\| < \delta$ for some $\delta > 0$ and all $t \geq 0$.

Definition 2. [13] The system (1) is said to be locally finite time right invertible at the equilibrium point (x^0, u^0) , if there exists a fixed adjustment time t_0 — depending only on the system — such that for all possible time sequences $y^*(t)$, $t \geq 0$ in the space Y^0 there exists a control sequence $u^*(t)$, $t \geq 0$ in the space U^0 which yields $y(t, x^0, u^*(0), \dots, u^*(t-1) = y^*(t)$, $t_0 \leq t \leq t_F$ for some t_F .

The following theorem will give a sufficient condition for local finite time right invertibility.

Theorem 1. The system (1) is locally finite time right invertible at the equilibrium point (x^0, u^0) if $\text{rank } A(x^0, u^0) = m$.

Proof. Consider the system of equations

$$\begin{aligned} y_1^*(t+q_1) - h_1^{p_1}(f(x(t), u(t))) &= 0, \\ y_m^*(t+q_m) - h_m^{p_m}(f(x(t), u(t))) &= 0 \end{aligned} \quad (6)$$

with $y^*(t) \in Y^0$, $t \geq 0$. Observe that the Jacobian matrix of the left hand side of (6) with respect to $u(t)$ equals $A(x(t), u(t))$. As $A(x^0, u^0)$ is nonsingular and $h_i^{p_i}(f(x^0, u^0)) = y_i^0$, $i = 1, \dots, m$, we may apply Implicit

Function Theorem around the point (x^0, u^0, y^0) yielding locally $u(t)$ as an analytic function of $x(t)$ and $y_1^*(t+\varrho_1), \dots, y_m^*(t+\varrho_m)$ that is

$$u(t) = \varphi(x(t), y_1^*(t+\varrho_1), \dots, y_m^*(t+\varrho_m))$$

and which is such that

$$y_i^*(t+\varrho_i) - h_i^{0t}(f(x(t), \varphi(x(t), y_1^*(t+\varrho_1), \dots, y_m^*(t+\varrho_m)))) = 0, \\ i=1, \dots, m.$$

Notice that $\varphi : V_3 \rightarrow V_2$ is analytic for some (possibly small) neighbourhoods V_3 and V_2 of (x^0, y^0) in $X^0 \times Y^0$ and of u^0 in U^0 .

This implies that when we apply a control sequence

$$u^*(t) = \varphi(x(t), y_1^*(t+\varrho_1), \dots, y_m^*(t+\varrho_m)), \quad t \geq 0 \quad (7)$$

to the system (1), then the output of the system is given as

$$y_i(t+\varrho_i) = y_i^*(t+\varrho_i), \quad i \in \{1, \dots, m\} \quad (8)$$

as long as $(x(t), y_1^*(t+\varrho_1), \dots, y_m^*(t+\varrho_m)) \in V_3$ and $u^*(t) \in V_2$.

Of course, the reproducibility property is lost if we leave the neighbourhoods V_3 resp. V_2 , which may happen for some t_F . So $y_i(t) = y_i^*(t)$ for $t_{0i} \leq t \leq t_F$, which proves the Theorem.

The control that produces the reference signal $y^*(t)$ can be found as the output of another dynamic system, the so-called right inverse system, operating on $y^*(t)$ [16]:

$$x(t+1) = f(x(t), \varphi(x(t), y_1^*(t+\varrho_1), \dots, y_m^*(t+\varrho_m))),$$

$$u(t) = \varphi(x(t), y_1^*(t+\varrho_1), \dots, y_m^*(t+\varrho_m)).$$

5. Problem solution

Next, we formulate our main result on the MMP.

Theorem 2. Consider the system (1) around the equilibrium point (x^0, u^0) and the model (2) around the corresponding equilibrium point (x^{M0}, u^{M0}) . Suppose that the decoupling matrix $A(x, u)$ of the system (1) is nonsingular at the equilibrium point (x^0, u^0) . Then the model matching problem is locally finite time solvable on some neighbourhoods V_1 of (x^0, x^{c0}, u^{M0}) and V_2 of u^0 if and only if the delay orders of the model (2) are equal to or greater than those of the original system (1).

Proof of Theorem 2. The proof relies on Theorem 1. Let us consider the model around the equilibrium point (x^{M0}, u^{M0}) which corresponds to the equilibrium point (x^0, u^0) of the system (1). Then, from (7) and (8) it follows that if we apply a compensator C given by equation

$$u(t) = \varphi(x(t), y_1^M(t+\varrho_1), \dots, y_m^M(t+\varrho_m))$$

to the system (1), then the outputs of the model and compensated system coincide:

$$y_i^{poc}(t+\varrho_i) = y_i^M(t+\varrho_i), \quad i \in \{1, \dots, m\},$$

as long as $(x(t), y_1^M(t+\varrho_1), \dots, y_m^M(t+\varrho_m)) \in V_3$ and $u(t) \in V_2$. The compensator C can be given in the form (3) if and only if the delay orders ϱ_i^M of the model are equal to or greater than the corresponding delay orders ϱ_i of the system (1), that is

$$\varrho_i^M \geq \varrho_i, \quad i = 1, \dots, m,$$

In that case, defining the functions $h_i^{M\rho_t}(x^M)$ analogously to functions $h_{\rho_t}(x)$ (see Section 3), we obtain

$$y_i^M(t+q_i) = h_i^{M\rho_t}(f^M(x^M(t), u^M(t))), \quad i=1, \dots, m \quad (9)$$

and the compensator C solving the model matching problem is the following:

$$x^M(t+1) = f^M(x^M(t), u^M(t)), \quad (10)$$

$$\begin{aligned} u(t) &= \varphi(x(t), h_i^{M\rho_t}(f^M(x^M(t), u^M(t)))), \quad i=1, \dots, m = \\ &= \bar{\varphi}(x(t), x^M(t), u^M(t)). \end{aligned}$$

From (9) we can see that $(x(t), y_1^M(t+q_1), \dots, y_m^M(t+q_m))$ belong to V_3 as long as $(x(t), x^M(t), u^M(t))$ belong to some neighbourhood V_1 of (x^0, x^{M0}, u^{M0}) .

6. Internal behaviour of the closed-loop system

As in the continuous-time case [7, 10], the notion of the normal form is helpful in understanding the internal behaviour of the closed-loop system (1), (10). In a discrete-time system with delay orders q_1, \dots, q_m , because of the Lemma, one can choose $h_i^k(x)$, $i=1, \dots, m$, $k=1, \dots, q_i$ as the new (partial) coordinates in the state space

$$\zeta(x) = [h_1^1(x), \dots, h_1^{q_1}(x), \dots, h_m^1(x), \dots, h_m^{q_m}(x)]^\top$$

locally around x^0 . Moreover, this set can be completed with the functions $\eta_1(x), \dots, \eta_{n-\mu}(x)$ such that

$$\text{rank } \frac{\partial}{\partial x} \begin{bmatrix} \zeta \\ \eta \end{bmatrix} = n.$$

The system (1) in the new coordinates (ζ, η) becomes

$$\zeta_1(t+1) = \zeta_2(t),$$

$$\zeta_{p_1-1}(t+1) = \zeta_{p_1}(t),$$

$$\zeta_{p_1}(t+1) = h_1^{\rho_1}(f(x(t), u(t)))|_{x=x(\zeta, \eta)},$$

$$\zeta_{\sum_{i=1}^m p_i+1}(t+1) = \zeta_{\sum_{i=1}^m p_i+2}(t),$$

$$\zeta_\mu(t+1) = h_m^{\rho_m}(f(x(t), u(t)))|_{x=x(\zeta, \eta)},$$

$$\eta(t+1) = \psi(\eta(t), \zeta(t), u(t)),$$

$$y(t) = \left[\zeta_1(t), \zeta_{p_1+1}(t), \dots, \zeta_{\sum_{i=1}^m p_i+1}(t) \right]^\top,$$

or in the compact form

$$\zeta(t+1) = \pi(\zeta(t), \eta(t), u(t)), \quad (11)$$

$$\eta(t+1) = \psi(\eta(t), \zeta(t), u(t)),$$

$$y(t) = \left[\zeta_1(t), \zeta_{p_1+1}(t), \dots, \zeta_{\sum_{i=1}^m p_i+1}(t) \right]^\top.$$

Note that the original equilibrium point (x^0, u^0) of the system (1) transforms into the equilibrium (ξ^0, η^0, u^0) of the system (11), where $\xi^0 = \xi(x^0)$ and $\eta^0 = \eta(x^0)$.

Remark. Differently from continuous time linear analytic systems [7] for the discrete-time case we cannot fulfil under reasonable assumptions such constraints for $\eta(x)$ which imply the independence of the dynamics of η from a control u .

Let the function $\varphi(x(t), y_1^*(t+\varrho_1), \dots, y_m^*(t+\varrho_m))$ (defined in Section 4) be denoted in the new coordinates by $\varphi^*(\eta(t), \xi(t), y_1^*(t+\varrho_1), \dots, y_m^*(t+\varrho_m))$. From the structure of the equations (11), which are said to be in the normal form, it can be immediately understood that the system (1) has a reduced order right inverse (that is the right inverse with minimal order dynamics) described by

$$\begin{aligned}\eta(t+1) &= \psi(\eta(t), \xi(t), \varphi^*(\eta(t), \xi(t), y_1^*(t+\varrho_1), \dots, y_m^*(t+\varrho_m))), \\ u(t) &= \varphi^*(\eta(t), \xi(t), y_1^*(t+\varrho_1), \dots, y_m^*(t+\varrho_m)).\end{aligned}\quad (12)$$

Let us note that the fixed point of the right inverse system is (η^0, ξ^0, y^0) .

Definition 3. The reduced order right inverse system dynamics (12) is said to be locally around the fixed point (η^0, ξ^0, y^0) bounded state stable if for any initial condition $\eta(0)$ from some neighbourhood H of η^0 and any output sequence $\{y^*(t) \in Y^0, t \geq 0\}$ there exists $K > 0$ such that $\|\eta(t)\| < K$ for all $t \geq 0$.

The stability properties of this inverse system are exactly the ones which characterize the internal behaviour of the closed-loop system (1), (10). In order to understand this, consider the composite system

$$\begin{aligned}x(t+1) &= f(x(t), u(t)), \\ x^M(t+1) &= f^M(x^M(t), u^M(t))\end{aligned}$$

with the output

$$y(t) = h(x(t)),$$

and choose (ξ, η, x^M) as the new coordinates around (x^0, x^{M0}) , with (ξ, η) as before. This is indeed an admissible choice, because the associated jacobian matrix is nonsingular. In the new coordinates the composite system becomes

$$\begin{aligned}\xi(t+1) &= \pi(\xi(t), \eta(t), u(t)), \\ \eta(t+1) &= \psi(\eta(t), \xi(t), u(t)), \\ x^M(t+1) &= f^M(x^M(t), u^M(t)).\end{aligned}$$

The choice of $u(t)$ as in (10), which in the new coordinates looks like

$$u(t) = \varphi^*(\eta(t), \xi(t), h_i^{M\rho_t}(f^M(x^M(t), u^M(t)))),$$

provides the description of the closed-loop system (1), (10) in the new coordinates, namely

$$\xi(t+1) = \pi(\xi(t), \eta(t), \varphi^*(\xi(t), \eta(t), h_i^{M\rho_t}(f^M(x^M(t), u^M(t))))),$$

$$\eta(t+1) = \psi(\eta(t), \xi(t), \varphi^*(\xi(t), \eta(t), h_i^{M\rho_t}(f^M(x^M(t), u^M(t))))),$$

$$x^M(t+1) = f^M(x^M(t), u^M(t)).$$

From the equations thus derived we see that the closed-loop system dynamics consists of three subsystems:

(i) the one described by the equation

$$\zeta(t+1) = \pi(\zeta(t), \eta(t), \varphi^*(\zeta(t), \eta(t), h_i^{M_{\theta_t}}(f^M(x^M(t), u^M(t))))),$$

whose dynamics (except the first $\max_i q_i$ steps) is totally defined by the outputs of the compensated system,

(ii) the one described by the equation

$$\eta(t+1) = \psi(\eta(t), \zeta(t), \varphi^*(\zeta(t), \eta(t), h_i^{M_p}(f^M(x^M(t), u^M(t)))))$$

which represents the dynamics of the reduced order right inverse system, driven by the outputs of the system and the model,

(iii) the one described by the equations

$$x^M(t+1) = f^M(x^M(t), u^M(t))$$

which represents the dynamics of the model.

Definition 4. The dynamics of the model (2) is said to be locally around the fixed point (x^{M_0}, u^{M_0}) bounded state stable, if for any initial condition $x^M(0)$ from some neighbourhood X^{M_0} of x^{M_0} and any input sequence $\{u^M(t) \in U^0, t \geq 0\}$ there exist $L > 0$ such that $\|x^M(t)\| < L$ for all $t \geq 0$.

Since the internal response of the closed-loop system can be brought to coincide with the second and third subsystems in at most $\max q_i$ steps,

and since the dynamics of the third subsystem can be made bounded by the assumption, it is clear that the internal response of the closed-loop system essentially depends on the properties of the reduced order inverse system dynamics (12). In particular if the reduced order right inverse system dynamics as well as the model are locally bounded state stable, then the internal response of the closed-loop system (1), (10) is bounded.

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DISKREETSETE MITTELINEAARSETE SÜSTEEMIDE SOBITAMINE DÜNAAMILISE TAGASISIDEGA KASUTADES SÜSTEEMI OLEKUT

Mõnede mittelineaarsete diskreetsete süsteemide alamklassi puhul on uuritud dünaamilise tagasiside kujul antud kompensaatori konstrueerimise ülesannet eesmärgiga tagada suletud süsteemi ning etteantud mudelsüsteemi sisend—väljundkujutiste kokkulangevus. On leitud ülesande lokaalne lahend juhtmisobjekti tasakaalupunkti ümbruses lisaeeldusel, et süsteemi dekomponeeritavuse maatriks on tasakaalupunktis mittesingu-laarne. Nimetatud paremalt pööratavate süsteemide alamklassi tarvis on ülesande lahenduvuse tarvilikud ja piisavad tingimused formuleeritud lähtesüsteemi ja mudelsüsteemi täisarvuliste mittenegatiivsete struktuuriparametreite, niinimetatud hilistumisjärkude abil. Vaadeldav ülesanne on lahenduv siis ja ainult siis, kui mudelsüsteemi hilistumisjärgud d_i^M on võrdsed lähtesüsteemi vastavate hilistumisjärkudega d_i või neist suuremad: $d_i^M \geq d_i, i=1, \dots, p$. Ulaltoodud tingimuste täidetuse korral on teletatud

kompensaatori vörrandid dünaamilise tagasiside kujul kasutades olekut. On näidatud, et parempoolse pöördsüsteemi sisemine stabiilsus mängib olulist rolli mittelineaarsete süsteemide sobitamise ülesande lahendamisel koos suletud süsteemi sisemise stabiilsuse tagamisega.

Юлле КОТТА

СОГЛАСОВАНИЕ НЕЛИНЕЙНЫХ СИСТЕМ ДИСКРЕТНОГО ВРЕМЕНИ С ПОМОЩЬЮ ОБРАТНОЙ СВЯЗИ, ДИНАМИЧЕСКОЙ ПО СОСТОЯНИЮ

Изучается задача построения компенсатора в виде динамической обратной связи по состоянию системы, обеспечивающего совпадение вход—выход отображений замкнутой и заданной систем для одного подкласса нелинейных систем дискретного времени. Предполагается локальное решение задачи в окрестности точки равновесия объекта управления, имеющего в точке равновесия несингулярную матрицу расщепимости. Для этого подкласса обратимых справа систем необходимые и достаточные условия разрешимости задачи сформулированы в терминах неких целочисленных неподнадежательных структурных параметров, т. н. порядков запаздывания, исходной и заданной систем. А именно, рассматриваемая задача имеет решение тогда и только тогда, когда порядки запаздывания d_i^M заданной системы равны или больше соответствующих порядков запаздывания d_i исходной системы: $d_i^M \geq d_i, i=1, \dots, p$. При выполнении вышеуказанных условий найдены уравнения компенсатора в виде динамической по состоянию обратной связи. Показывается, что внутренняя устойчивость правой обратной системы играет ключевую роль при решении задачи согласования нелинейных систем с внутренней устойчивостью.