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Ülle KOTTA

MODEL MATCHING OF NONLINEAR SINGLE-INPUT SINGLE-OUTPUT DISCRETE-TIME SYSTEMS: FORMAL AND LOCAL SOLUTIONS

(Presented by Ü. Jaaksoo)

1. Introduction

The model-matching problem (MMP) consists of designing a controller to compensate a given plant so that the resulting controlled system has a specified input-output behaviour. This problem has attracted a great deal of interest during the last two decades. Most papers consider linear systems, a number of results have also been obtained for continuous time nonlinear systems either by tools of differential geometry [1,2], differential algebra [3], by the so-called structure algorithm [4], or by zeroing the output of the extended system, i. e. considering the model-matching problem as the disturbance decoupling problem [5,6]. Except for the paper [3], all the results have been obtained for systems which are linear in control, and are local in the sense that they are valid in some neighbourhood of the initial point in the state space. The result by Conte and others [3] is global (though a global representation of a compensator may not exist) and, differently from other papers, the compensator is not assumed to be causal. To the author's knowledge, there are no papers written on the topic of model matching of nonlinear discrete-time systems.

This paper deals with the MMP for discrete-time nonlinear analytic systems with single input and single output. Moreover, the considered systems are assumed to be right invertible. The solution to MMP is obtained via the right inverse system. A similar approach is followed in [7,8] to related problems of input-output linearization and decoupling.

We shall adopt two — a formal and a local — viewpoints throughout this paper. At first, the necessary and sufficient conditions for formal solvability are given, and under these conditions the formal solution is presented. Then, it will be shown when the formal solution can be used for computing a local solution around a fixed (equilibrium) point.

2. Problem statement

Consider the nonlinear discrete-time system S described by equations

$$\begin{aligned} &x(t+1)=x(t)+f(x(t),\, u(t)), \quad x(0)=x^0, \\ &y(t)=h(x(t)), \end{aligned} \tag{1}$$

where the state $x(t)\in R^n$, the input $u(t)\in R$, the output $y(t)\in R$, $f\colon R^{n+1}\to R^n$ and $h\colon R^n\to R$ are analytic functions. We are assumed to

* Eesti Teaduste Akadeemia Küberneetika Instituut (Institute of Cybernetics, Estonian Academy of Sciences). 200 108 Tallinn, Akadeemia tee, 21. Estonia.

work in a neighbourhood $X^0 \times U^0$ of a fixed point, i. e. around such a (x^0, u^0), that $f(x^0, u^0)=0$. For the initial point x^0 and the input sequence $u(t)=u^0$, $t \geqslant 0$, there exists the constant output sequence $y(t)=y^0=h(x^0)$.

In addition, suppose a model M is given, described by the equations

$$
\begin{aligned}
& z(t+1)=z(t)+g(z(t), v(t)), z(0)=z^0, \\
& y^*(t)=h^*(z(t)),
\end{aligned} \tag{2}
$$

where $z(t) \in R^l$, $v(t) \in R$, $y^*(t) \in R$, $g: R^{l+1} \to R^l$ and $h^*: R^l \to R$ are analytic functions. Again, the model is considered around a fixed point (z^0, v^0). For z^0 and $v(t)=v^0$, $t \geqslant 0$, there exists the constant output sequence $y^*(t)=y^{*0}=h^*(z^0)$.

The problem is to find a compensator C for the system S so that the compensated system displays the same input-output behaviour as the model M. The compensator C is assumed to be described by equations

$$
\begin{aligned}
& \gamma(t+1)=\gamma(t)+a(x(t), \gamma(t), v(t)), \gamma(0)=\gamma^0, \\
& u(t)=c(x(t), \gamma(t), v(t))
\end{aligned} \tag{3}
$$

in which $\gamma(t) \in R^r$, $a: R^{r+n+1} \to R^r$ and $c: R^{r+n+1} \to R$ are analytic functions defined on a suitable open and dense subset of R^{r+n+1}. The composition $S \circ C$ of (1) and (3) is clearly a new dynamical system with the same structure as (1).

In the case of nonlinear systems, the input-output behaviour is usually described by the Volterra series expansion. To be precise, the input-output map of the system (1) associated with any initial state x^0, with $u(t) \in U^0$, a neighbourhood of a fixed reference input u^0, can be expanded in a discrete-time Volterra series. With $\varepsilon(\tau)=u(\tau)-u^0$, $\tau=0, 1, \ldots, t-1$, we have

$$
y(t)=w_0(t; x^0)+\sum_{\tau_1=0}^{t-1} w_1(t, \tau_1; x^0) \varepsilon(\tau_1)+\ldots
$$

$$
\ldots+\sum_{\tau_k \geqslant \tau_{k-1} \geqslant \ldots \geqslant \tau_1=0}^{t-1} w_k(t, \tau_1, \ldots, \tau_k; x^0) \varepsilon(\tau_1) \ldots \varepsilon(\tau_k)+\ldots
$$

which is the Taylor expansion around u^0 of $y(t)$ considered, for any given x^0, as a function of the variables $u(0), \ldots, u(t-1)$ [9].

Let $w_k^M(t, \tau_1, \ldots, \tau_k; z^0)$ denote the k-th kernel of the model M and $w_k^{S \circ C}(t, \tau_1, \ldots, \tau_k; x^0, \gamma^0)$ the k-th kernel of the compensated system. Now we are ready to present the precise problem statement.

Model-matching problem (MMP). Given the system S with its fixed point (x^0, u^0), the model M with its fixed point (z^0, v^0), find, if possible, the neighbourhoods X of x^0 and Z of z^0, an integer r, open subsets $V \subset R$, $\Gamma \subset R^r$, a *compensator* C defined on $X \times \Gamma \times V$, a map $F: X \times Z \to \Gamma$ with the property that

$$
w_k^{S \circ C}(t, \tau_1, \ldots, \tau_k; x, F(x, z))=w_k^M(t, \tau_1, \ldots, \tau_k; z)
$$

for all $k \geqslant 1$ and for all $(x, z) \in X \times Z$.

Note that following the linear MMP we do not require the 0-th order kernels $w_0^{S \circ C}$ and w_0^M to be the same.

3. Preliminaries

This section is devoted to introducing briefly the tools used in the sequel. The more detailed presentation of this material can be found in [9]. Given a family of analytic functions $\sigma_i: R^n \to R^n$, denote the j-th component of σ_i by σ_i^j, and identity function by I.

The iterated composition of an analytic function $h: R^n \to R$ with $I+\sigma_i$, $i=1, \ldots, k$, $h \circ (I+\sigma_k) \circ \ldots \circ (I+\sigma_1): R^n \to R$ can be expressed as

$$h \circ (I+\sigma_k) \circ \ldots \circ (I+\sigma_1)(x) = \Delta_{\sigma_1} \circ \ldots \circ \Delta_{\sigma_k} h|_x,$$

where

$$\Delta_{\sigma_i}: I_d + \sum_{r \geqslant 1} \frac{1}{r!} L_{\sigma_i}^{\otimes r} = I_d + \sum_{r \geqslant 1} \frac{1}{r!} \sum_{i_1, \ldots, i_r = 1}^{n} \sigma_i^{i_1} \ldots \sigma_i^{i_r} \frac{\partial^r}{\partial x_{i_1} \ldots \partial x_{i_r}}$$

is the formal differential series associated with σ_i which acts on analytic functions defined on R^n; I_d denotes the identity operator and $|_x$ evaluation at x.

Let us consider now the system (1), the expansion of the function $f(x, u)$ in powers of $u - u^0$ around $f(x, u^0) = f_0(x)$ gives

$$x(t+1) = x(t) + f_0(x(t)) + \sum_{k \geqslant 1} f_k(x(t)) [u(t) - u^0]^k,$$

where

$$f_k = \frac{1}{k!} \frac{\partial^k f(x, u)}{\partial u^k}\Big|_{u=u^0} \qquad k \geqslant 1,$$

are analytic functions.

For a fixed initial state x^0 and input sequence $\{u(0), \ldots, u(t-1)\}$, the output y at time instant t is given by the application to the output function h the formal differential operator

$$\Delta_{\sum\limits_{i \geqslant 0} f_i \varepsilon^i(0)} \circ \ldots \circ \Delta_{\sum\limits_{i \geqslant 0} f_i \varepsilon^i(t-1)}$$

evaluated at x^0.

Analogously, the model (2) can be presented in powers of $v - v^0$ around $g(z, v^0) = g_0(z)$:

$$z(t+1) = z(t) + g_0(z(t)) + \sum_{k \geqslant 1} g_k(z(t)) [v(t) - v^0]^k,$$

and the output of the model y^* at time t for fixed z^0 and $\{v(0), \ldots$ $\ldots, v(t-1)\}$ is given by the application to the function h^* the formal differential operator

$$\Delta_{\sum\limits_{i \geqslant 0} g_i \mu^i(0)} \circ \ldots \circ \Delta_{\sum\limits_{i \geqslant 0} g_i \mu^i(t-1)}$$

evaluated at z^0. Here $\mu(t) = v(t) - v^0$.

Let us introduce the following differential operators

$$\delta_0 = \Delta_{f_0}, \quad \delta_0^* = \Delta_{g_0},$$

$$\delta_p = \Delta_{f_0} \otimes \sum \frac{1}{n_1! \ldots n_k! \ldots} L_{f_1}^{\otimes n_1} \otimes \ldots \otimes L_{f_k}^{\otimes n_k} \otimes \ldots, \quad p \geqslant 1,$$

$$\delta_p^* = \Delta_{g_0} \otimes \sum \frac{1}{n_1! \ldots n_k! \ldots} L_{g_1}^{\otimes n_1} \otimes \ldots \otimes L_{g_k}^{\otimes n_k} \otimes \ldots, \quad p \geqslant 1, \tag{4}$$

where the summations are taken over all sets of nonnegative integers n_i such that the equation $n_1+2n_2+\ldots+kn_k+\ldots=p$ holds. Using these operators, we can write

$$\Delta_{\sum_{i\geq 0} f_i\varepsilon^i}=\delta_0+\delta_1\varepsilon+\delta_2\varepsilon^2+\ldots+\delta_k\varepsilon^k+\ldots,$$

$$\Delta_{\sum_{i\geq 0} g_i\mu^i}=\delta_0^*+\delta_1^*\mu+\delta_2^*\mu^2+\ldots+\delta_k^*\mu^k+\ldots.$$

4. Right inverse system

The right invertibility means, roughly speaking the possibility of realizing a given class of sequences as outputs of the system through the choice of appropriate inputs.

Let us define Y^0 as the space of sequences $\zeta(t)$, $t\geqslant 0$, which are sufficiently close to y^0, i. e. $\|\zeta(t)-y^0\|<\varepsilon$ for some $\varepsilon>0$ and all $t\geqslant 0$.

Definition 1. *The system* (1) *is said to be locally t_0-delay right invertible at the equilibrium point* (x^0, u^0) *if there exists a fixed adjustment time t_0 — depending only on the system — such that for all possible time sequences $\zeta(t)$ in the space Y^0 there exists a control sequence $u^*(t)$, $t\geqslant 0$ which yields*

$$y(t, x^0, u^*(0), \ldots, u^*(t))=\zeta(t), \quad t\geqslant t_0.$$

An input $u^*(t)$ can be found using the delay right inverse system operating on $\zeta(t)$.

Definition 2. *The order of delay d (referred to in the literature also as relative order or characteristic number) of the system* (1) *is defined as the largest integer k, such that* [10]

$$\delta_s\circ\delta_0^{k-1}h\equiv 0, \quad \forall s\geqslant 1.$$

Here $\delta_0^{k-1}=\delta_0\circ\ldots\circ\delta_0$ *($k-1$ times).*

Remark 1. $t=d+1$ is the first instant of time at which the output $y(t)$ is affected by the input applied at time $t=0$.

Let us introduce the following notations

$$a_i(x)=\delta_i\circ\delta_0^d h|_x, \quad i\geqslant 0;$$

$$\alpha_n(x)=\sum\frac{(n+c_1+c_2+\ldots+c_n-1)!}{n!\,c_2!\ldots c_n!\,a_1^n(x)}\left(-\frac{a_2(x)}{a_1(x)}\right)^{c_2}\ldots\left(-\frac{a_n(x)}{a_1(x)}\right)^{c_n},$$

where the summation is taken over all sets of nonnegative integers c_i such that $c_2+2c_3+\ldots+(n-1)c_n=n-1$;

$$\gamma_r=\sum\frac{(l_1+\ldots+l_r)!}{l_1!\ldots l_r!}f_{l_1+\cdots+l_r}\,\alpha_1^{l_1}(x)\ldots\alpha_r^{l_r}(x),$$

where the summation is taken over all sets of nonnegative integers l_i such that $l_1+2l_2+\ldots+rl_r=r$.

Theorem 1 [10]. *Consider the system* (1) *around the equilibrium point* (x^0, u^0) *for which the condition $\delta_1h|_x\neq 0$ holds in some open and dense subset X of X^0. Then, around u^0, the system has in X a $(d+1)$-delay right inverse, which is defined by equations*

$$x(t+1)=x(t)+f_0(x(t))+\sum_{r\geqslant 1}\gamma_r(x(t))[y(t+d+1)-a_0(x)]^r, \quad (5a)$$

$$\varepsilon(t)=\sum_{n\geqslant 1}\alpha_n(x)[y(t+d+1)-a_0(x)]^n. \quad (5b)$$

In the sequel we will assume that the assumption of the theorem holds.

5. MMP solution via right inverse system

We look for the compensator such that the input-output behaviour of the compensated system coincides with the input-output behaviour of the model system (2). Let the input-output maps of the original system, its inverse, the model and the compensator be denoted by $\hat{S}$, $\hat{S}_R^{-1}$, $\hat{M}$ and $\hat{C}$ respectively. Moreover, denote by z the one step forward shift operator: $zy(t)=y(t+1)$. For $(d+1)$-delay right invertible systems $\hat{S}\circ\hat{S}_R^{-1}=$ $=z^{-d-1}$ [10]. So, the choice

$$\hat{C}=\hat{S}_R^{-1}\circ z^{d+1}\circ\hat{M}$$

guarantees the desired goal. Thus, to obtain the required control law, we must feed into the output equation of the right inverse system the appropriate shift of the output of the model system $y^*(t+d+1)$, and then express $y^*(t+d+1)$ by $z(t)$ and $v(t)$.

Let us feed into the output equation of the right inverse system (5b) the $d+1$-step forward shift of the output of the model system (2):

$$\varepsilon(t)=\sum_{n\geqslant 1}\alpha_n(x)[y^*(t+d+1)-a_0(x)]^n. \quad (6)$$

Let us assume that the delay order d^* of the model system is equal or greater than that of the original system, i. e.

$$\delta_s^*\circ\delta_0^{*r}h^*|_z\equiv 0, \quad \forall r<d, \quad \forall s\geqslant 1.$$

Introducing the notations

$$a_i^*(z)=\delta_i^*\circ\delta_0^{*d}h^*|_z, \quad i\geqslant 0$$

and taking into account (6), we obtain from equations of model

$$y^*(t+d+1)=a_0^*(z(t))+\sum_{s\geqslant 1}a_s^*(z(t))\mu^s(t). \quad (7)$$

The equations (6) and (7) show that probably the dynamic feedback

$$z(t+1)=z(t)+g_0(z(t))+\sum_{s\geqslant 1}g_s(z(t))\mu^s(t), \quad (8a)$$

$$\varepsilon(t)=\sum_{n\geqslant 1}\alpha_n(x(t))[a_0^*(z(t))-a_0(x(t))+\sum_{s\geqslant 1}a_s^*(z(t))\mu^s(t)]^n \quad (8b)$$

gives the formal solution to the MMP. We shall formulate this result as a Theorem.

T h e o r e m 2. *Consider the system* (1) *around the equilibrium point* (x^0, u^0) *for which the condition* $\delta_1 h|_x\neq 0$ *holds in some open and dense subset* X *of* X^0. *The MMP can be formally solved by dynamic state feedback given by equations* (8) *if and only if the delay order of the model system* (2) *is equal or greated than that of the original system.*
The proof of Theorem 2 is given in the Appendix.

R e m a r k 2. In fact, the feedback control law (8) gives only a formal solution to the MMP. The formulas (8) can be used for computing a local solution around fixed point (x^0, u^0) to the posed problem if and only if $y^{*0} \in Y^0$. This implies that $\sum_{s\geqslant 0} a_s(z(t))\mu^s(t)$ belongs to a neighbourhood of $a_0(x(t)) \subset Y^0$. The reason lies in the following. The feedback control law (8b) is a formal solution of the equation

$$\sum_{n\geqslant 0} a_n(x(t))\varepsilon^i(t) = \sum_{s\geqslant 0} \overset{*}{a}_s(z(t))\mu^s(t). \tag{9}$$

In case $\varepsilon(t)=0$, $\sum_{n\geqslant 0} a_n(x(t))\varepsilon^i(t) = a_0(x(t))$ and therefore $a_0(x(t))$ belongs to the image of the function $\sum_{n\geqslant 0} a_n(x(t))\varepsilon^i(t)$. This implies [11] that the solution of equation (9) can be computed by (8b) locally around (u^0, y^0). Moreover, the series (8b) is absolutely and uniformly convergent in a neighbourhood of (u^0, y^0).

6. Conclusions

In this paper we have tackled the problem of compensating a nonlinear analytic discrete-time system around a fixed point in order to make its input-output behaviour the same as the one of the prespecified nonlinear analytic discrete-time model around prespecified fixed point. The considered system is assumed to be at the fixed point formally $(d+1)$-delay right invertible.

For this subclass of systems the posed problem is formally solvable by dynamic state feedback if and only if the order of delay of the model is equal or greater than that of the original system. The formal solution cannot always be used for computing a local solution around fixed point (x^0, u^0). For such a possibility one should, in addition, require that the constant outputs y^0 and y^{*0}, corresponding to the fixed points (x^0, u^0) and (z^0, v^0), are sufficiently close to each other. When particular examples are considered, it is often possible to solve the problem in a global sense.

It turns out that the concept of right inverse plays a central role in the solution of this problem. In fact, the required feedback law can be given by the output equation of delay right inverse system if we feed into it as inputs $(d+1)$-step forward shift of outputs of the prespecified model coupled with the equations of the model.

For the simplicity only single-input single-output systems are considered, but the results are easily generalizable to a multivariable case, provided we consider formally $(d_1+1, \ldots, d_p+1)$-delay right invertible systems.

The result obtained in this paper is weaker than the result for continuous time system [1, 2, 4–6] in several sense. Firstly, for continuous time system, each input-output behaviour of the model is reproduced from any initial state of the process (provided that the compensator is set in a suitable initial state). For discrete-time system, only such behaviour of the model can be reproduced from fixed initial state of the system for which the model outputs remain sufficiently close to the system output. The difference comes from the fact that in a discrete-time case we consider the systems nonlinear in control. The same restrictions will appear for continuous time systems, if we consider systems nonlinear in control. Secondly, in discrete-time case we are required to work around a fixed point. For continuous time system there is no need to consider systems only around the equilibrium point.

Appendix: Proof of Theorem 2.

The closed-loop system (1), (8) is also the analytic system

$$\tilde{x}(t+1)=\tilde{x}(t)+\tilde{f}_0(\tilde{x}(t))+\sum_{i\geqslant 1}\tilde{f}_i(\tilde{x}(t))\mu^i(t), \tag{10}$$

$$y(t)=\tilde{h}(\tilde{x}(t)),$$

where

$$\tilde{x}^T(t)=[x^T(t),\, z^T(t)],$$

$$\tilde{f}_0(\tilde{x})=\begin{bmatrix} f_0(x)+\sum_{i\geqslant 1} f_i(x)\alpha^i(x,z) \\ g_0(z)\end{bmatrix},$$

$$\tilde{f}_i(\tilde{x})=\begin{bmatrix}\beta_i(x,z)\\ g_i(z)\end{bmatrix}, \quad \tilde{h}(\tilde{x})=h(x),$$

$$\alpha(x,z)=\sum_{n\geqslant 1}\alpha_n(x)\,[a_0^*(z)-a_0(x)]^n,$$

$$\beta_i(x,z)=\sum_{j\geqslant 1} f_j(x)\zeta_j(i);$$

$\zeta_j(i)$ is computed recursively:

$$\zeta_j(i)=\sum_{k=0}^{i}\zeta_{j-1}(i-k)\zeta_1(k), \quad \zeta_1(r)=\pi_r, \quad r\geqslant 0,$$

$$\pi_t=\sum_{n\geqslant 1}\alpha_n(x)\sum_{k=0}^{n}C_n^k\gamma_{t,k}(z)\,[-a_0(x)]^{n-k};$$

$\gamma_{t,k}$ is computed recursively

$$\gamma_{t,r}=\frac{1}{ta_0^*(z)}\sum_{p=1}^{t}(rp-t+p)\,a_p^*(z)\gamma_{t-p,r}, \quad t\geqslant 1,$$

$$\gamma_{0,r}=[a_0^*(z)]^r, \quad C_n^k=\frac{n!}{k!\,(n-k)!}.$$

Let us introduce the differential operators $\tilde{\delta}_p$, $p\geqslant 0$ associated with the closed-loop system (10) analogously to operators (4). The Volterra kernels of the closed-loop system can be expressed in terms of these operators

$$w_k^{S\circ C}(t+1,\tau_1,\ldots,\tau_k;\tilde{x}^0)=\tilde{\delta}_0^{\tau_1}\circ\tilde{\delta}_1\circ\tilde{\delta}_0^{\tau_2-\tau_1-1}\circ\tilde{\delta}_1\circ\ldots\circ\tilde{\delta}_1\circ\tilde{\delta}_0^{t-\tau_k}\tilde{h}|_{\tilde{x}^0}, \quad k\geqslant 1.$$

The Volterra kernels of the model (2) can be expressed analogously via the operators δ_p^*, $p\geqslant 0$:

$$w_k^M(t+1,\tau_1,\ldots,\tau_k;z^0)=\delta_0^{*\tau_1}\circ\delta_1^*\circ\delta_0^{*\tau_2-\tau_1-1}\circ\delta_1^*\circ\ldots$$
$$\ldots\circ\delta_1^*\circ\delta_0^{*t-\tau_k}h^*|_{z^0}, \quad k\geqslant 1.$$

For compact expression of the kernels, the following notations have been used (here δ_0^{-1} has not been defined and carries no meaning)

$$\tilde{\delta}_{i_1}\circ\tilde{\delta}_0^{-1}\circ\tilde{\delta}_{i_2}\circ\tilde{\delta}_0^{-1}\circ\ldots\circ\tilde{\delta}_0^{-1}\circ\tilde{\delta}_{i_r}=\tilde{\delta}_r,$$

$$\delta_{i_1}^*\circ\delta_0^{*-1}\circ\delta_{i_2}^*\circ\delta_0^{*-1}\circ\ldots\circ\delta_0^{*-1}\circ\delta_{i_r}^*=\delta_r^*, \tag{11}$$

$$r\geqslant 2, \quad i_1=\ldots=i_r=1.$$

(Sufficiency). Let us first observe that

$$\tilde{\delta}_0\tilde{h}=\sum_{i\geqslant 0}\delta_i\alpha^i\circ h.$$

If $d>0$, then $\tilde{\delta}_0\tilde{h}=\delta_0 h$. Similarly

$$\tilde{\delta}_0^2\tilde{h}=\delta_0^2 h,\ \ldots,\ \tilde{\delta}_0^d\tilde{h}=\delta_0^d h,$$

but

$$\tilde{\delta}_0^{d+1}\tilde{h}=\tilde{\delta}_0\circ\delta_0^d h=\delta_0^{d+1}h+\sum_{i\geqslant 1}\alpha^i\delta_i\circ\delta_0^d h=$$
$$=\delta_0^{d+1}h+a_0^*(z)-a_0(x)=a_0^*(z)=\delta_0^{*d+1}h^*$$

because the formal series $\alpha=\sum_{k\geqslant 1}\alpha_k(x)[a_0^*(z)-a_0(x)]^k$ is the inverse of the formal series $\sum_{i\geqslant 1}\delta_i\circ\delta_0^d h\varepsilon^i$ with respect to substitution of one series into another, and $a_0(x)=\delta_0^{d+1}h$. Furthermore, as $\tilde{\delta}_0\circ\delta_0^{*d+k}h^*=\delta_0^*\circ\delta_0^{*d+k}h^*$ it is not difficult to show that the equality $\tilde{\delta}_0^{d+k}\tilde{h}=\delta_0^{*d+k}h^*$ yields $\tilde{\delta}_0^{d+k+1}\tilde{h}=$ $=\delta_0^{*d+k+1}h^*$.

Therefore

$$\tilde{\delta}_0^{t+1}\tilde{h}=\begin{cases}\delta_0^{t+1}h, & \text{if } t\leqslant d-1\\ \delta_0^{*t+1}h^*, & \text{if } t>d-1.\end{cases}$$

Next, we shall show that

$$\tilde{\delta}_k\circ\tilde{\delta}_0^t\tilde{h}=\delta_k^*\circ\delta_0^{*t}h^*. \tag{12}$$

Actually, if $t<d$, then $\tilde{\delta}_k\circ\tilde{\delta}_0^t\tilde{h}=\tilde{\delta}_k\circ\delta_0^t h=0$ because of the definition d. If $t>d$, then for every $k\geqslant 1$

$$\tilde{\delta}_k\circ\tilde{\delta}_0^t\tilde{h}=\tilde{\delta}_k\circ\delta_0^{*t}h^*=\delta_k^*\circ\delta_0^{*t}h^*.$$

If $t=d$, then

a) $\tilde{\delta}_1\circ\tilde{\delta}_0^d\tilde{h}=\pi_1\sum_{i\geqslant 0}\delta_i\alpha^i\otimes(L_{f_1}+2\alpha L_{f_2}+3\alpha^2 L_{f_3}+\ldots)\delta_0^d h=$
$=\pi_1(a_1+2a_2\alpha+3a_3\alpha^2+\ldots)=\pi_1 a'(\alpha)=$
$=a_1^*\alpha'(a_0^*-a_0)a'(\alpha)=a_1^*=\delta_1^*\circ\delta_0^{*d}h^*$

because of the equalities
$\pi_1=a_1^*\alpha'(a_0^*-a_0)$ and $\alpha'(a_0^*-a_0)a'(\alpha)=1$;

b) $\tilde{\delta}_2\circ\tilde{\delta}_0^d\tilde{h}=\tilde{\delta}_2\circ\delta_0^d h=[\pi_2\sum_{i\geqslant 0}\delta_i\alpha^i\otimes(L_{f_1}+2\alpha L_{f_2}+3\alpha^2 L_{f_3}+\ldots)+$
$+\frac{1}{2}\pi_1^2\sum_{i\geqslant 0}\delta_i\alpha^i\otimes(L_{f_1}^{\otimes 2}+2L_{f_2}+6\alpha L_{f_3}+4\alpha L_{f_1}\otimes L_{f_2}+\ldots]\circ\delta_0^d h=$
$=\pi_2 a'(\alpha)+\pi_1^2 a''(\alpha)=[a_2^*a'+(a_1^*)^2\alpha'']a'(\alpha)+$
$+\frac{1}{2}(a_1^*)^2[\alpha'(a_0^*-a_0)]^2 a''(\alpha)=(a_1^*)^2[\alpha''a'(\alpha)+$
$+(\alpha'(a_0^*-a_0))^2 a''(\alpha)]+a_2^*[\alpha'a'(\alpha)]=a_2^*=$
$=\delta_2^*\circ\delta_0^{*d}h^*,$

etc.

Now, consider the k-th order kernel (k arbitrary) $w_k^{S\circ C}(t+1, \tau_1, \ldots$ $\ldots, \tau_k; \tilde{x}^0)$ of the closed-loop system (10). If $\tau_k = \tau_{k-1} = \ldots = \tau_{k-r} >$ $> \tau_{k-r-1}$, $r \geqslant 0$, then by (11) and (12)

$$w_k^{S\circ C}(t+1, \tau_1, \ldots, \tau_k; \tilde{x}^0) = \tilde{\delta}_0^{\tau_1} \circ \tilde{\delta}_1 \circ \tilde{\delta}_0^{\tau_2-\tau_1-1} \circ \tilde{\delta}_1 \circ \ldots$$

$$\ldots \circ \tilde{\delta}_0^{\tau_{k-r}-\tau_{k-r-1}-1} \circ \delta_{r+1}^* \circ \delta_0^{*t-\tau_k} h^* =$$

$$= \delta_0^{*\tau_1} \circ \delta_1^* \circ \delta_0^{*\tau_2-\tau_1-1} \circ \delta_1^* \circ \ldots \circ \delta_0^{*\tau_{k-r}-\tau_{k-r-1}-1} \circ$$

$$\circ \delta_{r+1}^* \circ \delta_0^{*t-\tau_k} h^* = w_k^M(t+1, \tau_1, \ldots, \tau_k; z^0).$$

(Necessity.) Suppose that the feedback (8) guarantees formal solution of the MMP. In that case the kernels of the closed-loop system (10) and the model (2) must be equal, i. e. for every $k \geqslant 1$, $t \geqslant \tau_k \geqslant \ldots \geqslant \tau_1 \geqslant 0$

$$\delta_0^{*\tau_1} \circ \delta_1^* \circ \delta_0^{*\tau_2-\tau_1-1} \circ \delta_1^* \circ \ldots \circ \delta_1^* \circ \delta_0^{*t-\tau_k} h^* |_{z^0} =$$

$$= \tilde{\delta}_0^{\tau_1} \circ \tilde{\delta}_1 \circ \tilde{\delta}_0^{\tau_2-\tau_1-1} \circ \tilde{\delta}_1 \circ \ldots \circ \tilde{\delta}_1 \circ \tilde{\delta}_0^{t-\tau_k} \tilde{h} |_{\tilde{x}^0}.$$

This implies

$$\delta_s^* \circ \delta_0^{*t} h^* = \tilde{\delta}_s \circ \tilde{\delta}_0^t \tilde{h} \equiv 0, \quad s \geqslant 1, \quad 0 \leqslant t \leqslant d-1$$

which means that the delay order of the model is equal to or greater than the delay order of the system.

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Ülle KOTTA

MITTELINEAARSETE DISKREETSETE ÜHE SISENDI JA ÜHE VÄLJUNDIGA SÜSTEEMIDE KOOSKÕLASTAMISE ÜLESANDE FORMAALNE JA LOKAALNE LAHENDAMINE

Teatud mittelineaarsete diskreetsete süsteemide alamklassi puhul on uuritud dünaamilise tagasiside kujul antud kompensaatori konstrueerimise ülesannet eesmärgiga tagada suletud süsteemi ja etteantud mudelsüsteemi sisend-väljundkujutiste kokkulangemine. Vaadeldavatel süsteemidel on üks sisend ja üks väljund ning nad on paremalt pööratavad. Ülesande lahenduvuseks tarvilikud ja piisavad tingimused on formuleeritud lähtesüsteemi ja mudelsüsteemi teatud täisarvuliste struktuuriparameetrite, niinimetatud hilistumisjärkude abil. Vaadeldav ülesanne on lahenduv siis ja ainult siis, kui mudelsüsteemi hilistumisjärk d^M on võrdne lähtesüsteemi hilistumisjärguga d või sellest suurem: $d^M \geqslant d$. Kompensaatori formaalsed võrrandid on saadud parempoolse pöördsüsteemi väljundvõrranditest, kui süsteemi sisendiks on võetud mudelsüsteemi väljund $(d+1)$-sammulise nihkega ettepoole. On näidatud, millal on võimalik kasutada seda formaalset lahendit lokaalse lahendi arvutamiseks süsteemi tasakaaluoleku ümbruses.

Юлле КОТТА

ФОРМАЛЬНОЕ И ЛОКАЛЬНОЕ РЕШЕНИЯ ЗАДАЧИ СОГЛАСОВАНИЯ НЕЛИНЕЙНЫХ СИСТЕМ ДИСКРЕТНОГО ВРЕМЕНИ С ОДНИМ ВХОДОМ И ВЫХОДОМ

Изучается задача построения компенсатора в виде динамической обратной связи по состоянию системы, обеспечивающего совпадение вход—выход отображений замкнутой и заданной систем для одного подкласса нелинейных систем дискретного времени. Рассматриваемые системы имеют один вход и один выход и являются обратимыми справа. Необходимые и достаточные условия разрешимости задачи сформулированы в терминах неких целочисленных структурных параметров, т. н. порядков запаздывания исходной и заданной систем. А именно, рассматриваемая задача имеет решение тогда и только тогда, когда порядок запаздывания d^M заданной системы равен или больше порядка запаздывания d исходной системы: $d^M \geqslant d$. Формальные уравнения компенсатора определяются выходным уравнением правой обратной системы, если подставить в качестве входа в это уравнение выход заданной системы со сдвигом $d+1$ шагов вперед. Выясняется, когда можно пользоваться этим формальным решением для вычисления локального решения в окрестности точки равновесия системы.