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R. MANKIN, I. PIIR

SCALAR WAVE EQUATION IN A WEAK STATIC GRAVITATIONAL FIELD

1. Introduction

A simple method for analysing wave propagation in a weak static gravitational field with the metric

$$ds^2 = (1+2\Phi(\vec{r}))\,dt^2 - [1-2\Phi(\vec{r})]\,d\vec{r}^2 \tag{1}$$

was outlined in [1,2]. Here, $d\vec{r}^2$ is the metric of the Euclidean 3-space and $\Phi(\vec{r})$ is the Newtonian potential satisfying the Poisson equation *

$$\Delta_0\Phi(\vec{r}) = 4\pi\varkappa\varrho(\vec{r})$$

($\varkappa$ — gravitational constant, ϱ — mass density).

The method was based on an approximate solution of the scalar wave equation

$$\Box\Psi(t,\vec{r}) \equiv (-g)^{-1/2}[(-g)^{1/2}g^{ab}\Psi_{,a}]_{,b} = 0. \tag{2a}$$

Using metric (1), the last equation can be written as

$$\Box_0\Psi(t,\vec{r}) = 4\Phi(\vec{r})\Psi_{tt}(t,\vec{r}). \tag{2b}$$

Here and in what follows, only linear terms in Φ will be taken into account. The solution under discussion describing a point source which is located at $P'(=\vec{r}')$ and which radiates in a finite time interval $0<t<t_0$, was expressed as

$$\Psi = \Psi_0 + \Psi_1, \tag{3}$$

where Ψ_0 is the usual outgoing spherical wave emitted from the point P'

$$\Psi_0 = (1/q)f[t - q(1+I)] \tag{4}$$

with $q = |\vec{q}| = |\vec{r} - \vec{r}'|$
and

$$I(\vec{r},\vec{r}') = -2\int_0^1 \Phi(\vec{r}'+\lambda\vec{q})\,d\lambda. \tag{5}$$

The argument of the function f in (4) takes into consideration the dependence of the signal propagation velocity on the gravitational potential.

The correction term Ψ_1 determined by the inhomogeneous wave equation

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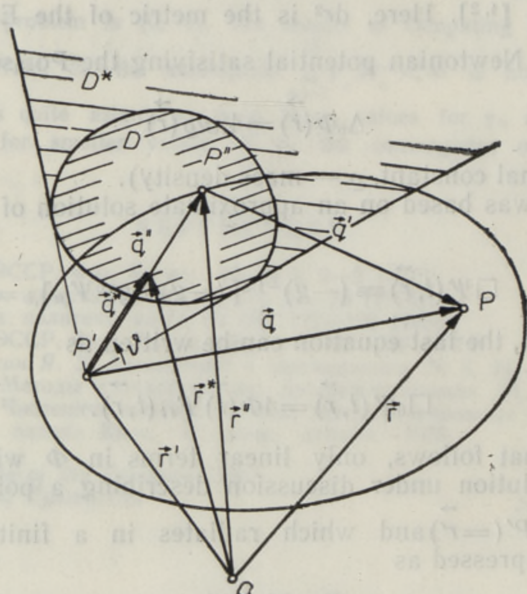
* The subscript zero indicates that the Laplacian Δ_0, d'Alembertian $\Box_0$, the null-cone $C_0(x')$ etc., belong to flat space-time.

$$\Box_0 \Psi_1(t,\vec{r}) = 8\pi\varkappa f'(t-q)\int_0^1 \varrho(\vec{r'}+\lambda\vec{q})\lambda^2\,d\lambda \equiv F(t,\vec{r}), \tag{6}$$

was expressed as the retarded potential

$$\Psi_1(t,\vec{r}) = (1/4\pi)\iiint_{q'\leqslant t}(1/q')F(t-q',\vec{r''})\,dV(\vec{r''}) \tag{7}$$

$(\vec{q'}=\vec{r''}-\vec{r})$. Ψ_1 describes the backreflection of the primary wave Ψ_0 on the curvature of the space-time. One can see that equation (6) turns inhomogeneous at the moment when the primary wave reaches the region D, where $\varrho \neq 0$ (see Figure). Consequently, the backreflection also begins at the same moment. As the inhomogeneous term in (6) can vanish neither in the region D nor in the shadow-region D^*, the backreflection will last forever, although it fades out after the primary wave has passed the region D. This result is in full agreement with the results by P. Günther [3], stating that the general covariant wave equation in a weak background gravitational field turns out to be a Huygens' equation with accuracy to the first order terms everywhere in the empty space-time.



In this paper subsequent improvements to the described approach are proposed. First, a method for calculating the backreflected wave Ψ_1 is elaborated and then this method is associated with the fundamental solution (or Green's function) formalism.

2. Backreflected wave Ψ_1

Formulae (6) and (7) give

$$\Psi_1(t,\vec{r};\vec{r'}) = 2\varkappa\iiint_{q'\leqslant 1}(1/q')f'(t-q'-q'')\int_0^1 \varrho(\vec{r'}+\lambda\vec{q''})\lambda^2\,d\lambda\,dV(\vec{r''}) \tag{8}$$

$(\vec{q''}=\vec{r''}-\vec{r'})$ (see Figure). The family of confocal ellipsoids of revolution with the foci at P' and $P(=\vec{r})$ determines the surfaces of the constant phase $\tau=t-q^*=t-(q'+q'')=\text{const}$. Therefore it is useful to integrate in (8) at first over the surface of the ellipsoid and then over the ellipsoids available in P at the observation moment t. Introducing polar co-ordinates with the origin at P' and with the polar axis in the direction $\vec{q}$, we get:

$$\mathrm{d}V(\vec{r}'')=\mathrm{d}V(q'',\vartheta,\varphi)=2q'q''^3(q^{*2}-q^2)^{-1}\sin\vartheta\,\mathrm{d}q^*\sin\vartheta\,\mathrm{d}\vartheta\,\mathrm{d}\varphi. \quad (9)$$

Then

$$\Psi_1(t,\vec{r};\vec{r}')=4\varkappa\int_q^t f'(t-q^*)M(q^*,\vec{q})\,(q^{*2}-q^2)^{-1}\,\mathrm{d}q^*, \quad (10)$$

where the mass-term M is determined by the gravitational mass distribution

$$M(q^*,\vec{q})=\int_0^{2\pi}\mathrm{d}\varphi\int_0^{\pi}\sin\vartheta\,\mathrm{d}\vartheta\int_0^{q''}\varrho(r^*,\vartheta,\varphi)\,r^{*2}\,\mathrm{d}r^* \quad (11)$$

with $q''=(1/2)\,(q^{*2}-q^2)\,(q^*-\vec{q}\vec{n})^{-1}$ and $\vec{n}=\vec{q}''/q''$.

For the following analysis it is useful to introduce new independent variables

$$u=t-q,\quad v=t+q,\quad \tau=t-q^*. \quad (12)$$

Then in (11) we have

$$q''=(u-\tau)\,(v-\tau)\,[2(u-\tau)+(v-u)\,(1-\cos\vartheta)\,]^{-1}$$

and

$$\Psi_1=\Psi_1(u,v;\vec{q})=\int_0^u f'(\tau)M(\tau,u,v;\vec{q})\,[\,(v-\tau)\,(u-\tau)\,]^{-1}\,\mathrm{d}\tau. \quad (13)$$

If the distances of the points P and P' from the region D are much larger than the dimensions of the region D, then the point-mass approximation is applicable, i.e. we can assume that the mass is concentrated at a point $P_0=(q''_0,\vartheta_0,0)\in D$ and formulae (11)—(13) are essentially simplified:

$$M=\begin{cases}0, & \text{if } q''<q''_0,\\ m, & \text{if } q''>q''_0;\end{cases} \quad (14)$$

$$\Psi_1=(2\varkappa m/q)\int_{q_0^*}^t f'(t-q^*)\,[\,(q^*-q)^{-1}-(q^*+q)^{-1}]\,\mathrm{d}q^* \quad (15a)$$

or

$$\Psi_1=(2\varkappa m/q)\int_0^{u_1=t-q_0^*} f'(\tau)\,[\,(u-\tau)^{-1}-(v-\tau)^{-1}]\,\mathrm{d}\tau, \quad (15b)$$

with $q^*_0=q''_0+(q''^2_0+q^2-2qq''_0\cos\vartheta_0)^{1/2}$.

At the moment $t\geqslant q''_0$ the function Ψ_1 does not vanish inside the sphere $S^{P_0}_{t-q_0''}$ (with the centre at P_0 and radius $t-q''_0$). Inside the sphere $S^{P_0}_{t-q_0''-t_0}$, it is necessary to integrate only from 0 to t_0 in (15b) (note that the function $f(t)$ does not vanish if $0<t<t_0$) and then

$$\Psi_1=(2\varkappa m/q)\int_0^{t_0}[\,(u-\tau)^{-1}-(v-\tau)^{-1}]f'(\tau)\,\mathrm{d}\tau=$$

$$= (2\varkappa m/q) \int_0^{t_0} [(v-\tau)^{-2} - (u-\tau)^{-2}] f(\tau)\, d\tau. \quad (16)$$

This leads to an interesting conclusion: inside the sphere $S^{P_0}_{t-t_0-q_0''}$, the function Ψ_1 depends only on t and q, but not on the angle ϑ_0 and it is presented as a sum of (direct) waves going out from the centre P' and of incoming (reverse) waves.

If the mass distribution is spherically symmetric (i. e. $\varrho = \varrho(r) \neq 0$ with $r < r_0$) and the scalar wave source lies in the centre of symmetry O, formula (11) can be written as follows

$$M = (\pi/r)(q^{*2} - r^2) \int_{1/2(q^*-r)}^{1/2(q^*+r)} (dq''/q''^2) \int_0^{q''} \varrho(r^*) r^{*2}\, dr^*.$$

Taking into account the formal expression of spherically symmetric Newtonian potential

$$\Phi(r) = \varkappa \int_\infty^r m(r')(r')^{-2}\, dr',$$

where $m(r) = 4\pi \int_0^r \varrho(r') r'^2\, dr'$ is the mass inside the sphere S^O_r, we finally get

$$M = (1/4\varkappa r)(q^{*2} - r^2)\{\Phi[(q^* + r)/2] - \Phi[(q^* - r)/2]\}. \quad (17)$$

Now, integrating by parts and substituting $q^* = t - \tau$, formula (10) gives

$$\Psi_1 = (f(u)/r)[\Phi(r) - \Phi(0)] +$$
$$+ (2\varkappa/r) \int_0^u f(\tau)\{m[(v-\tau)/2](v-\tau)^{-2} - m[(u-\tau)/2](u-\tau)^{-2}\}\, d\tau. \quad (18a)$$

This result specifies and, in a certain sence, accomplishes, our general analysis concerning conditions of validity of Huygens' principle and of appearing of wave tails in a weak gravitational field ([4], section 4).

If the observation point P lies outside the spherical mass, formula (18a) takes the form

$$\Psi_1 = -(f(u)/r)[\varkappa m/r - \Phi(0)] + (2\varkappa m/r) \int_0^u f(\tau)(v-\tau)^{-2}\, d\tau -$$
$$- (2\varkappa/r) \int_0^u f(\tau) m[(u-\tau)/2](u-\tau)^{-2}\, d\tau. \quad (18b)$$

This formula, except for the last term, gives well-known solution of the scalar wave equation in a weak Schwarzschild field (see, e. g. [4], where in (4b) one must substitute $F(u) = f(u)[1 + \Phi(0)]$). The last term in (18b) describes the backreflected wave running through the centre, and it satisfies the homogeneous equation (6) in the region $r > r_0$. If the problem is formulated as in paper [4], this term may also be included into the primary wave $F(u)$.

It is necessary to note that there is a certain arbitrariness in the choice of the primary wave Ψ_0. For example, following B. S. de Witt and R. W. John [5, 6], the primary wave can be expressed as

$$\Psi_0^* = [1 + 4\varkappa q^2 \int_0^1 (1-\lambda)\lambda\varrho(\vec{r}' + \lambda\vec{q})\, d\lambda](1/q) f[t - q(1+I)], \quad (19)$$

but then the second term in (3) is determined by the equation

$$\Box_0 \Psi_1^*(t,\vec{r}) = 4\pi\varkappa f(\tau - q)\,[\,(2/q)\int_0^1 (2\lambda - 1)\lambda\varrho(\vec{r}' + \lambda\vec{q})\,d\lambda +$$

$$+ q\int_0^1 (1-\lambda)\lambda^3 \Delta_0 \varrho(\vec{r}' + \lambda\vec{q})\,d\lambda\,]. \qquad (20)$$

3. Fundamental solution in the weak gravitational field approximation

The limiting process $t \to t_0$ in formulae (3), (4), (10), and (11) with the condition $\int_0^{t_0} f(\tau)\,d\tau = \text{const}$ allows us to find an approximate expression for the retarded fundamental solution of equation (2b). Starting from the fundamental solution, we can construct solutions of various particular problems as it is demonstrated in [7]. A more rigorous formulation of these problems requires the formalism of the distribution theory to be used. In the present section based on [7, 8] a principal scheme of corresponding calculations is outlined.

The retarded fundamental solution $G^+(x,x')$, where $x = (t,\vec{r})$ and $x' = (t',\vec{r}')$, is defined as a solution of the equation

$$\Box_x G^+(x,x') = \delta^4(x,x') \qquad (21a)$$

satisfying the condition

$$G^+(x,x') = 0 \quad (t < t'), \qquad (21b)$$

and $\delta^4(x,x')$ is defined as

$$(\delta^4(x,x'), f(x')) \equiv \int d^4x' [-g(x')]^{1/2} \delta^4(x,x') f(x') = f(x), \quad \forall f(x) \in C^\infty. \qquad (22)$$

Bearing in mind the results presented in the preceding sections, $G^+(x,x')$ may be written as follows —

$$G^+(x,x') = (1/4\pi)\,\theta(x^0 - x^{0\prime})\,\delta[\sigma(x,x')] + (\partial/\partial x_0)\,G_1(x,x'), \qquad (23)$$

where θ is the Heaviside function and σ is, up to a sign, one half of the square of the geodesic distance from x' to x. The distribution $\theta\,\delta(\sigma)$ is defined as

$$(\theta(x^0 - x^{0\prime})\,\delta[\sigma(x,x')], f(x)) \equiv \int_{C^+(x')} |\sigma_{,x_0}|^{-1} [-g(x)]^{1/2} f(x)\,d\vec{r}^2, \qquad (24)$$

where $C^+(x')$ is the future null semi-cone at x' ($x \in C^+(x')$; $\sigma(x,x') = 0$, $x^0 - x^{0\prime} \geqslant 0$). In case of the metric (1),

$$\sigma(x,x') = (1/2)\,(q^{02} - q^2) + (q^{02} + q^2)\int_0^1 \Phi(\vec{r}' + \lambda\vec{q})\,d\lambda \qquad (25)$$

and the null semi-cone $C^+(x')$ is defined by the equation

$$q^0 = q\,[1 - 2\int_0^1 \Phi(\vec{r}' + \lambda\vec{q})\,d\lambda]; \quad q^0 \geqslant 0. \qquad (26)$$

One has

$$(\Box_x \theta(x^0 - x^{0\prime})\,\delta[\sigma(x,x')], \varphi(x)) = (\theta(x^0 - x^{0\prime})\,\delta[\sigma(x,x')], \Box\varphi(x)) =$$

$$= 4\pi\{\varphi(x) + 2\varkappa \iiint_{R^3} d^3\vec{r} \int_0^1 \lambda^2 \varrho(\vec{r}' + \lambda\vec{q})\,d\lambda\,[\,(\partial/\partial x^0)\,\varphi(x)\,]_{x^0 = x^{0\prime} + q}\},$$

from where it follows that

$$\Box_x \theta(x^0 - x^{0'}) \delta[\sigma(x, x')] = 4\pi(\delta(x, x') -$$
$$-2\varkappa(\partial/\partial x^0)\{\theta(x^0 - x^{0'})\delta[\sigma(x, x')]\} q \int_0^1 \lambda^2 \varrho(\vec{r}' + \lambda\vec{q})\, d\lambda)^{**}$$

and, therefore, the equation (21a) gives

$$\Box_{0x} G_1(x, x') = 2\varkappa q \int_0^1 \lambda^2 \varrho(\vec{r}' + \lambda\vec{q})\, d\lambda\, \theta(x^0 - x^{0'})\delta[\sigma_0(x, x')]. \tag{27}$$

Formulae (10) and (11) suggest that $G_1(x, x')$ may be written as

$$G_1(x, x') = \begin{cases} [2\pi\sigma_0(x, x')]^{-1}\varkappa \int_0^{2\pi} d\varphi \int_0^{\pi} \sin\vartheta\, d\vartheta \int_0^{q''} \varrho(\vec{r}' + \vec{q}^*)\, q^{*2}\, dq^* & (q_0 > q), \\ 0 \quad (q^0 < q) \end{cases} \tag{28}$$

with $q'' = \sigma_0(x, x')(q^0 - \vec{n}\vec{q})^{-1}$ and $\vec{n} = \vec{q}''/q'' = \vec{q}^*/q^*$.

It is easy to see that in the interior $D_0^+(x')$ of the future null semi-cone $C_0^+(x')$ $(x \in D_0^+(x')\colon \sigma_0(x, x') > 0,\ q^0 > 0)$ the function $G_1(x, x')$ belongs to the same class of differentiable functions as the mass density $\varrho(x)$ and $\Box_{0x} G_1 = 0$, if $q^0 > q$. However, on the null-cone the formula (28) does not define the function $G_1(x, x')$. In order to prove that $G_1(x, x')$ actually satisfies the equation (27), we must first continue $G_1(x, x')$ from the region $D_0^+(x')$ to its boundary surface $C_0^+(x')$. This can be done by replacing the variables of integration q^*, ϑ, φ in (28) by the cylindrical co-ordinates $\tilde{r}, \tilde{z}, \tilde{\varphi}$ with the origin at $O' = 1/2(\vec{r} + \vec{r}')$

$$\begin{aligned} q^{*\mu} &= \tilde{r}[\sigma_0(x, x')/2]^{1/2} l^\mu + (\tilde{z} q^0/2 + q/2)(q^\mu/q), \\ l^\mu &= \cos\tilde{\varphi}\,(q^\mu/q)_{,\theta} + \sin\tilde{\varphi}\,(q^\mu/q)_{,\Phi} \sin^{-1}\theta; \\ (q^{*\mu} &= (q^* \sin\vartheta \cos\varphi,\ q^* \sin\vartheta \sin\varphi,\ q^* \cos\vartheta), \\ q^\mu &= (q \sin\theta \cos\Phi,\ q \sin\theta \sin\Phi,\ q \cos\theta)). \end{aligned} \tag{29}$$

In the new co-ordinates the region of integration in (28) transforms into the interior of the unit sphere with the centre at O', and we have

$$G_1(x, x') = (\varkappa/8\pi)\, q^0 \int_{-1}^{1} d\tilde{z} \int_0^{(1-\tilde{z}^2)^{1/2}} \tilde{r}\, d\tilde{r} \int_0^{2\pi} d\tilde{\varphi}\, \varrho(\vec{r}' + \vec{q}^*) \quad (q^0 \geqslant q > 0). \tag{30}$$

The new expression (30) for the function $G_1(x, x')$ is well-defined on the null-cone $C_0^+(x')$ and in the region $D_0^+(x')$ it naturally coincides with the previous expression (28) (only on the straight line $q = 0$ expression (30) is not defined). By a straightforward differentation one can prove that $G_1 \in C^2(D_0^+(x'))$ and $G_1 \in C^1(C_0^+(x') \cup D_0^+(x'))$. Hence one can apply Green's integral formula, in analogy with [8] (see p. 44—45), that gives

$$\Box_{0x} G_1 = (\Box_{0x})_{cl} G_1 + (\partial/\partial N)\, \bar{G}_1 \cdot \delta_S + (\partial/\partial N)\, (\bar{G}_1 \delta_S). \tag{31}$$

Here $S \equiv C_0^+(x')$ and $\partial/\partial N = \partial/\partial x^0 + (q^\mu/q)\,\partial/\partial x^\mu$ is derivative in the

** It should be taken into consideration that

$$(\theta(x^0 - x^{0'})\delta[\sigma(x, x')],\ \varphi(x)) = \int_{C_0^+(x')} [\sigma_{0,x^0}]^{-1} \varphi(x)\, d^3\vec{r}$$

and

$$\sigma_0 = 1/2(q^{02} - q)^2.$$

direction of the external normal to the surface, $\bar{G}^1$ and $(\partial/\partial N)\bar{G}_1$ being respectively the values of G_1 and $(\partial/\partial N)G_1$ on S. The distributions $\mu(x)\delta_S$ and $(\partial/\partial N)(\mu\delta_S)$ are determined as follows:

$$(\mu(x)\delta_S(x), \varphi(x)) = \int_S \mu(x)\varphi(x)\,dS,$$

$$((\partial/\partial N)(\mu(x)\delta_S(x)), \varphi(x)) = -\int \mu(x)(\partial/\partial N)\varphi(x)\,dS. \quad (32a)$$

Consequently,

$$\mu(x)\delta_S(x) = q\mu(x)\vartheta(x^0 - x^{0'})\delta[\sigma_0(x, x')]. \quad (32b)$$

The classical value $(\Box_{0x})_{cl}G_1$ of the distribution $\Box_{0x}G_1$ is found, calculating the usual partial derivatives of $G_1(x, x')$ given either in (28) or in (30), to be

$$(\Box_{0x})_{cl}G_1(x, x') = 0. \quad (33)$$

From (30) one gets

$$\bar{G}_1 = \varkappa q \int_0^1 \lambda(1-\lambda)\varrho(\vec{r}' + \lambda\vec{q})\,d\lambda,$$

$$(\partial/\partial N)\bar{G}_1 = \varkappa \int_0^1 \lambda(2\lambda - 1)\varrho(\vec{r}' + \lambda\vec{q})\,d\lambda. \quad (34)$$

Using these results, it is easy to demonstrate that $G_1(x, x')$ satisfies equation (27).

Taking into account the relation

$$(\partial/\partial x^0)G_1 = (\partial/\partial x^0)_{cl}G_1 + \varkappa q^2 \int_0^1 \lambda(1-\lambda)\varrho(\vec{r}' + \lambda\vec{q})\,d\lambda\,\vartheta(x^0 - x^{0'})\delta[\sigma_0(x, x')], \quad (35)$$

the fundamental solution may also be presented in the following form:

$$G(x, x') = (1/4\pi)[1 + 4\pi\varkappa q^2 \int_0^1 \lambda(1-\lambda)\varrho(\vec{r}' + \lambda\vec{q})\,d\lambda]\vartheta(x^0 - x^{0'})\delta[\sigma_0(x, x')] + G_1^*(x, x'), \quad (36)$$

where

$$G_1^*(x, x') = (\partial/\partial x^0)_{cl}G_1(x, x') \quad (37a)$$

or, using (28),

$$G_1^*(x, x') = \begin{cases} 0 \quad (q^0 < q), \\ (\varkappa/4\pi)\sigma_0^{-2}(x, x')\{-2q^0 \int_0^{2\pi} d\varphi \int_0^{\pi} \sin\vartheta\, d\vartheta \int_0^{q''} \varrho(\vec{r}' + \vec{q}^*)q^{*2}\,dq^* + \\ + \sigma_0^{-1}(x, x') \int_0^{2\pi} d\varphi \int_0^{\pi} \sin\vartheta\, d\vartheta\, q''^4 [q^{02} + q^2 - 2q^0(\vec{q}\vec{n})]\varrho(\vec{r}' + \vec{q}'')\} \\ \qquad (q^0 > q); \end{cases} \quad (37b)$$

An equivalent representation of $G_1^*(x, x')$ may be found, starting from (30).

The new expression of the fundamental solution given in (36) is in some respects preferable to that given in (23) since here the coefficient by $\vartheta\,\delta(\sigma)$ has the same structure as in [5, 6].

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Tallinn Polytechnic Institute
Tartu State University

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R. MANKIN, I. PIIR

SKALAARNE LAINEVÕRRAND NÕRGA STAATILISE GRAVITATSIOONIVÄLJA KORRAL

On leitud nõrga staatilise gravitatsioonivälja meetrika korral skalaarse lainevõrrandi lahend, mis kirjeldab kiirguse levikut lõpliku ajavahemiku jooksul tegutsevast punktallikast. On näidatud, et kui piirduda gravitatsioonivälja potentsiaali Φ suhtes lineaarsete liikmetega, on tühjas aegruumis (s. t. kui $R_{ab}=0$) vaadeldavaks lahendiks keralaine, mille laineargument arvestab lainete levimiskiiruse sõltuvust potentsiaalist Φ. Parandusliige, mis on seotud Huygensi printsiibi rikkumisega, saab nullist erinevaks alles siis, kui keralaine on jõudnud gravitatsioonivälja allikate piirkonda, kus $R_{ab}\neq 0$. Tulemusi on rakendatud punktmassi gravitatsioonivälja ja tsentraalsümmeetrilise ülesande erijuhtudel. Saadud tulemustele toetudes on leitud nõrga gravitatsioonivälja lähenduses ka skalaarse lainevõrrandi retardeeritud põhilahend (Greeni funktsioon).

Р. МАНКИН, И. ПИЙР

СКАЛЯРНОЕ ВОЛНОВОЕ УРАВНЕНИЕ В СЛАБОМ СТАТИСТИЧЕСКОМ ГРАВИТАЦИОННОМ ПОЛЕ

На фоне метрики слабого гравитационного поля найдено приближенное решение скалярного волнового уравнения, описывающее распространение излучения от точечного источника, действующего в течение конечного интервала времени. Показано, что если ограничиваться лишь линейными относительно гравитационного потенциала Ф членами, рассматриваемое решение имеет в пустом пространстве-времени (т. е. при $R_{ab}=0$) вид сферической волны, волновой аргумент которой учитывает зависимость скорости распространения сигнала от потенциала Ф. Поправочный член, связанный с нарушением принципа Гюйгенса, возникает лишь в тот момент, когда сферическая волна достигает области, где $R_{ab}\neq 0$. В качестве частных примеров рассмотрены случаи гравитационного поля точечной массы и центральносимметричная задача. На основе найденного решения получено в приближении слабого гравитационного поля явное выражение для запаздывающего фундаментального решения (функции Грина) скалярного волнового уравнения.