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SOME QUESTIONS CONCERNING THE NONCONSTRUCTIBILITY AND COMPUTABILITY IN HOMOGENEOUS STRUCTURES

V. ALADJEV. MITTEKONSTRUEERITAVUSEST JA ARVUTATAVUSEST HOMOGEENSETES STRUKTUURIDES

В. АЛАДЬЕВ. НЕКОТОРЫЕ ВОПРОСЫ, КАСАЮЩИЕСЯ НЕКОНСТРУИРУЕМОСТИ И ВЫЧИСЛИМОСТИ В ОДНОРОДНЫХ СТРУКТУРАХ

In this paper we shall attempt to define some relations between different concepts of nonconstructibility in Homogeneous Structures. A considerable part of the results contained in this paper was taken from our book [1].

Key words and definitions. Homogeneous Structure (HS), Configuration (CF), Nonconstructible CF (NCF and NCF-1), Erasable CF (ECF), Computation Universality (CU) [1]; HS $\ni\in A$ — HS has property A; we denote the sets of all finite and any CF of HS by $\overline{C}$ and C, respectively.

An automaton is backwards-deterministic (BD) if and only if for each $i\in I$ and $s'\in S$ there is at most one $s\in S$ such that $T(i, s) = s'$, where S, I, $Q(S = Q)$ and T are the sets of internal, input, output states and transition function, respectively. An automaton is said to be backwards-indeterministic ($\overline{\text{BD}}$) if it is not BD. Roughly speaking, a BD automaton can never forget its state, for the internal state at t can be recovered from the input history and the present internal state.

HS with no set of quiescent states will be called an unstable HS. HS is self-erasable (SE) if and only if for every ordering of the cell states with O as the initial (quiescent) element the state of a cell may both increase and decrease.

Some results. E. Moore [2] and J. Myhill [3] have shown some interesting relations between the erasability in HS and the existence of NCF.

Theorem 1 (Moore-Myhill). *The existence of ECF is necessary and sufficient for the existence of NCF in HS (for C or $\overline{C}$).*

It can be shown that theorem 1 holds for an unstable HS. In work [4] we introduced the concept of NCF-1 which is not equivalent to the concept of NCF, generally.

Theorem 2 (Aladyev). *There exists an HS which has NCF-1 without NCF (for $\overline{C}$).*

Obviously, any CF containing NCF is NCF-1, whereas in the case of NCF-1 this is wrong. Thus CF $c_0 = x_1x_2\ldots x_n$, $\bar{c}_0 = x_n\ldots x_2x_1$ and even CF $c_0\bar{c}_0c_0$ may be NCF-1, but already CF c_0c_0, $\bar{c}_0\bar{c}_0$, $c_0\bar{c}_0$, $\bar{c}_0c_0$ are not. S. Amoroso and G. Cooper [5] have shown that the converse of Myhill's Theorem fails. But since there exist two concepts of non-constructibility (NCF and NCF-1), their work entails some confusions. This question is discussed in our book [1]. In an other case, if set $\overline{C}$ is replaced by a set C, there follows result [1].

T h e o r e m 3 (Aladyev). *The existence of ECF (NCF) is necessary and sufficient for the existence of NCF-1 in HS (for C).*

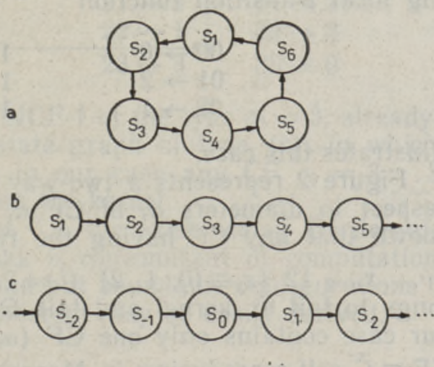

Fig. 1. State subgraphs of the graphs of homogeneous structure (HS). The work of HS may be defined as a state graph (the state of HS is a configuration of HS) which consists of some types of subgraphs. *a* — pure cycle; *b* — one-way infinite subgraph; *c* — two-way infinite subgraph.

From theorems 1—3 some other relations between NCF, NCF-1 and ECF follow easily. Theorem 3 gives a positive answer to Burks's question [6] (in our terms, of course): if HS possesses NCF-1, then it possesses NCF (for C)? We note that Burks ([6], 17, 25 pp.) repeats Amoroso-Cooper's confusions [5]. Furthermore, theorem 3 is used as a positive answer to Burks's following question: is there HS which is $\overline{BD}$ and without NCF-1?

A simple example will illustrate this fact. Let HS be governed by a local transition function

$$00 \to 0 \qquad 01 \to 1 \qquad 10 \to 1 \qquad 11 \to 0.$$

This HS is $\overrightarrow{BD}$, as the following transition shows:

$$\begin{array}{ll} t \quad : \ldots 11110111 \ldots & \ldots 00001000 \ldots \\ t+1 : \ldots 000110000 \ldots & \ldots 000110000 \ldots \end{array}$$

But this HS does not possess ECF. Thus, we have $\overline{BD}$ HS without NCF-1 and NCF (for C). Example 3 from [6] suggests $\overline{BD}$ HS for any complement of partial local transition function because in such HS there exists ECF already of the type

$$t: \begin{array}{l} 12|001212|00 \\ 12|212100|00 \end{array} \quad t+1: \quad 0|012210|0$$

for that partial local transition function. It gives an answer to one particular Burks's question [6].

T h e o r e m 4 (Aladyev). *HS (for C) is BD if and only if its state graphs consist only of subgraphs of types (a) or (a) and (c).*

T h e o r e m 4. 1 (Burks). *HS (for $\overline{C}$) is $\overline{BD}$ if and only if it has ECF.*

From theorem 4 a negative answer follows to Burks's question: is there a BD HS (for C) with one-way infinite subgraphs?

Let HS (for $\overline{C}$) be BD without NCF-1. Then there exist the following cases:

1. In HS there exist state graphs of the type (*a*) only. The first example of HS from ([1], p. 163) illustrates this case.

2. In HS there exist state graphs of the types (a) and (c), only. An example of HS from ([1], p. 122) illustrates this case and gives a negative answer to one particular Burks's question ([6], p. A3).

3. In HS there exist state graphs of the type (c), only, ignoring the periodical null-configuration. The example of HS defined by the following local transition function

$00 \to 0$	$10 \to 1$	$20 \to 0$
$01 \to 2$	$11 \to 2$	$21 \to 2$
$02 \to 1$	$12 \to 0$	$22 \to 1$

illustrates this case.

Figure 2 represents a two-way infinite subgraph $(\dots \alpha_{-1}\alpha_0\alpha_1 \dots)$ with respect to diameters d_i of CF α_i (state α_i) contained in it. It can be shown that any CF having the types $x_1 \dots x_{n-1}$ 1; $x_1 \dots x_{n-2}$ $\left(\begin{smallmatrix}02\\22\end{smallmatrix}\right)$ and $x_1 \dots x_{n-2}$ 12 ($x_i \in \{0, 1, 2\}$ ($i = \overline{2, n-1}$ ($i = \overline{2, n-1}$)), $x_1 = 1, 2$) belongs to tail $\oplus$, kernel and tail $\ominus$, respectively. Moreover, the kernel in our case contains only one CF (α_0 or α_1). Can this HS have almost all CF $\in \overline{C}$ self-reproducing in Moore's sense? If it is true, then we have an example to a certain extent distinct from Amoroso-Cooper and Ostrand's

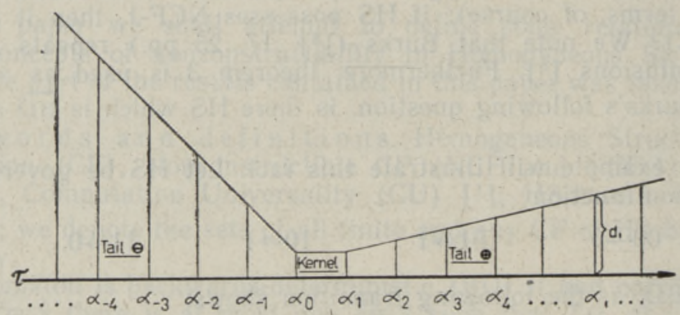

Fig. 2. d_i — diameter of CF α_i and τ — global transition functions of HS.

example [8–9]. In case of BD HS (for C) there exist only two possibilities; 1 and 2, because C contains a periodical in Amoroso-Yamada's sense, CF.

Let HS (for $\overline{C}$) be BD having NCF-1. Then there exist following cases:

1. In HS there exist state graphs of the type (b) only, ignoring the periodical null-configuration. The second example of HS from ([1], p. 119) illustrates this case.

2. In HS there exist state graphs of the types (a) and (b), only. An example of HS from ([1], p. 165) illustrates this case.

3. In HS there exist state graphs of the types (a), (b) and (c), only. The example of HS defined by following local transition function

$00 \to 0$	$10 \to 0$	$20 \to 2$	$30 \to 1$
$01 \to 1$	$11 \to 1$	$21 \to 0$	$31 \to 3$
$02 \to 2$	$12 \to 2$	$22 \to 1$	$32 \to 2$
$03 \to 3$	$13 \to 3$	$23 \to 3$	$33 \to 0$

illustrates this case.

4. In HS there exist state graphs of the types (*b*) and (*c*), only, ignoring the periodical null-configuration. The following example of HS defined by the local transition function

$00 \to 0$	$10 \to 1$	$20 \to 0$	$30 \to 1$
$01 \to 2$	$11 \to 2$	$21 \to 2$	$31 \to 2$
$02 \to 1$	$12 \to 0$	$22 \to 1$	$32 \to 3$
$03 \to 3$	$13 \to 3$	$23 \to 3$	$33 \to 0$

illustrates this case. Such HS possesses NCF-1 of the type $c_0 = 3$, already, whereas CF $c_0 = 3333$ belongs to the state graph of type (*c*), in which $\alpha_0 = 3232$ and $\alpha_1 = 3333$. Furthermore, in our case any CF $c_0 = \underbrace{3\ldots3}_{l}$ ($l = 2k + 1$; $k = 0, 1, \ldots$) is CF α_1 from the kernel.

In work [7] A. R. Smith writes that "...the existence of Garden-of-Eden configurations is probably as weak a concomitant of computation universality as is the existence of unbound but boundable configurations". But from our results [1] the following theorem may be easily derived.

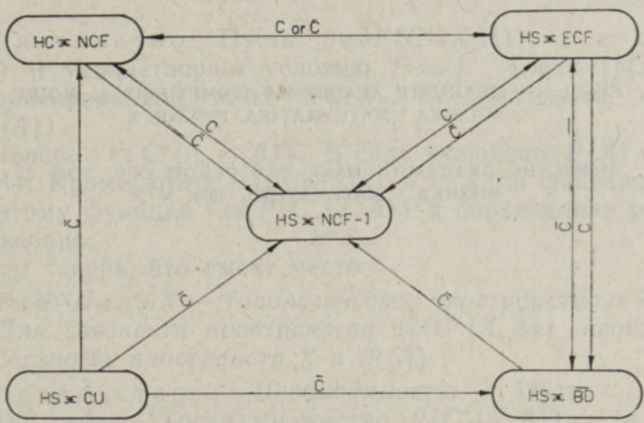

Fig. 3.

T h e o r e m 5 (Aladyev). *The existence of ECF (NCF and NCF-1) is necessary for the computation universality of HS.*

This theorem corroborates Burks's conjectures about the weakness of non-erasable cellular systems. Furthermore [1], no interesting form of self-reproduction can take place in HS which is non-erasable.

It can be shown that there exists HS which is not SE and possesses NCF (ECF) and NCF-1 and vice versa. Our conjecture is that the answer to Burks's question (Can a universal Turing machine (UMT) be embedded in HS which is not EC?) is negative. This opinion is based on the fact that cell states of HS should contain information both about the state of the tape of UMT as well as about the internal state of its finite automaton, which are generally erasable.

Graphical summaries of the results of theorems 1—5 are given in figure 3.

In our opinion, two concepts of nonconstructibility can be useful for studying formal languages generated by HS, for example.

We hope that this paper might help to clear up some of the questions which may arise after works [5–7] and give an answer to some of Burks's questions[6].

REFERENCES

1. Aladyev V., On the Theory of the Homogeneous Structures, Estonian Academic Press, Tallinn, 1972 (in Russian).
2. Moore E., Proc. Symp. Appl. Math., **14**, 17 (1962).
3. Myhill J., Proc. Am. Math. Soc., **14**, No. 4 (1963).
4. Аладьев В., К теории однородных структур, ВИНИТИ, № 4204, М., 1971.
5. Amoroso S., Cooper G., Proc. Am. Math. Soc., **26**, No. 1 (1970).
6. Burks A. W., On Backwards-Deterministic, Erasable and Garden-of-Eden Automata, Techn. Rept, No. 012520-4-T, Univ. Michigan (September, 1971).
7 Smith A. R., Cellular Automata Theory, Techn. Rept, No. 2, Stanford Univ. (1970)
8. Amoroso S., Cooper G., Comput. System Sci., No. 5 (1971).
9. Ostrand T. J., Property Preservation by Tessellation Automata, DCS Techn. Rept, No. 17, Hill Center Math. Sci. Livingston College (May, 1972).

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М. АБЕЛЬ

СВЯЗНОСТЬ ПРОСТРАНСТВА МАКСИМАЛЬНЫХ ИДЕАЛОВ АЛГЕБРЫ ОГРАНИЧЕННЫХ НЕПРЕРЫВНЫХ *A*-ЗНАЧНЫХ ФУНКЦИЙ

M. ABEL. *A*-VÄÄRTUSTEGA TÕKESTATUD PIDEVATE FUNKTSIOONIDE ALGEBRA MAKSIMAALSETE IDEAALIDE RUUMI SIDUSUS

M ABEL. THE CONNECTIVITY OF THE SPACE OF MAXIMAL IDEALS OF THE ALGEBRA OF *A*-VALUED BOUNDED CONTINUOUS FUNCTIONS

1. Пусть $C^*(X,A)$ — множество всех ограниченных непрерывных A-значных функций, определенных на топологическом пространстве X. В случае, когда A — (комплексная) коммутативная банахова алгебра с единицей [1], то множество $C^*(X,A)$ образует коммутативную банахову алгебру с единицей относительно обычных алгебраических операций над функциями и нормой

$$\|f\| = \sup_{x \in X} \|f(x)\|_A,$$

где $f \in C^*(X,A)$ и $\|\ldots\|_A$ — норма алгебры A.

[1] Всюду в дальнейшем вместо коммутативной банаховой алгебры с единицей будем говорить коротко банахова алгебра или алгебра.