Proc. Estonian Acad. Sci. Phys. Math., 1997, **46**, 1/2, 55–63
https://doi.org/10.3176/phys.math.1997.1/2.07

SIGNAL PROPAGATION AND INTERNAL MEASUREMENT IN CRYSTALLINE SOLIDS

Helmut GÜNTHER

Fachhochschule Bielefeld, Fachbereich Elektrotechnik, Wilhelm–Bertelsmann-Str. 10, D-33602 Bielefeld, Deutschland; e-mail: guenther@fhzinfo.fh-bielefeld.de

Received 2 October 1996, accepted 17 March 1997

Abstract. Natural standard lengths and clocks in a crystalline solid with the help of soliton solutions of the sine-Gordon equation are introduced. The idea of an internal measurement for mutually moving reference systems can be well-founded, including especially a synchronization for moving clocks. As a result, the coordinates of an event measured from two reference systems are related by the Lorentz transformation. Hence, an arbitrary moving internal observer measures one and the same isotropic propagation for limiting signal velocity of the sine-Gordon equation. Lorentz symmetries in solids become more transparent in this description. Moreover, we have found a mechanical model for the special theory of relativity without falling back into the mechanical ether concept.

Key words: sine-Gordon equation, lattice structure, Lorentz symmetry, special relativity.

PRELIMINARY REMARK

Even four years after the discovery of special relativity by A. Einstein, H. Poincaré in 1909 held the view that the contraction hypothesis for moving lengths was postulated independently of the special principle of relativity, the universal constancy for the speed of light. We know that this view was wrong. However, there is a grain of truth in it, which we will discuss below considering special relativity based on a lattice structure.

1. INTRODUCTION

The displacement q, perpendicular to a straight dislocation line along the x-axis, in a crystal is described by the sine-Gordon equation (cf. [1–3]):

$$\frac{\partial^2 q}{\partial x^2} - \frac{1}{c_o^2}\frac{\partial^2 q}{\partial t^2} = \lambda_o^{-2} \sin q. \qquad \text{sine-Gordon equation} \quad (1)$$

Here c_o is a limiting velocity for the propagation of plastic deformations through a crystal with numerical values near that of the transversal sound velocity, cf. [4]. The length λ_o results from the physical constants of the lattice and has numerical values of around 10^{-8} cm. We will consider the following solutions of (1), cf. [5], also [6, 7]:

$$q_o^I(x) = 4\arctan\exp\left(\frac{x}{\lambda_o}\right), \qquad \text{static kink} \qquad (2)$$

$$q^I(x,t) = 4\arctan\exp\frac{x-vt}{\lambda'}, \quad \lambda' = \lambda_o\gamma, \quad \text{moving kink} \qquad (3)$$

$$\gamma = \sqrt{1 - v^2/c_o^2}, \qquad (4)$$

$$q_o^{III}(x,t) = 4\arctan\frac{\sin(2\pi t/T_o)}{\cosh(x/\sqrt{2}\lambda_o)}, \quad T_o = \sqrt{2}2\pi\lambda_o/c_o, \text{localized breather} \qquad (5)$$

$$q^{III}(x,t) = 4\arctan\frac{\sin[2\pi(t - vx/c_o^2)/T']}{\cosh[\pi(x-vt)/\sqrt{2}\lambda']}, \quad T' = T_o\gamma. \text{ moving breather} \qquad (6)$$

Without going into detail we notice: These solutions behave as quasi-particles, i.e., particles with inertia in reference to the lattice. The energy-momentum tensor of these solutions allows an energy-momentum vector of a particle. The vacuum of these particles is the lattice, whereas the particles themselves, if we take an idea of Weizsäcker [8], are nothing but shapes, this means here structural defects of the ideal lattice, cf. [9].

2. NATURAL STANDARD LENGTHS AND CLOCKS

With the help of the kink solution (2) a natural standard length, e.g., $L_o = \pi\lambda_o$ for measuring distances in the crystal can be defined (cf. [9], see Fig. 1).

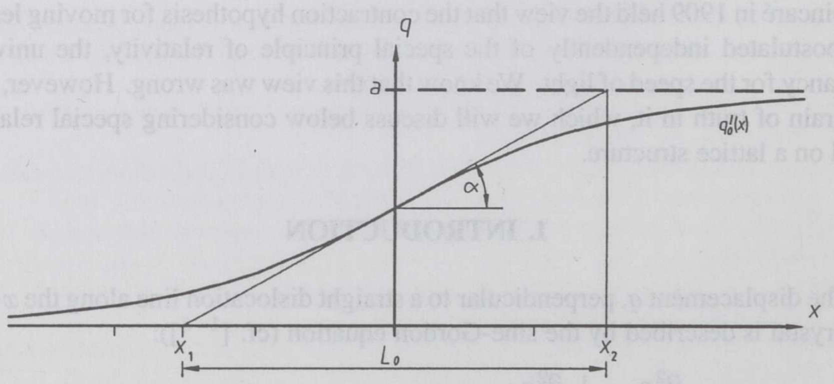

Fig. 1. Definition of the natural standard length L_o out of the kink solution.

On the other hand, the solution (5) supplies us with a period of oscillation T_o, which allows us to define a natural clock, cf. [6, 9], see Fig. 2. Then it is easy to see from the solution (3) that our natural standard length L_o, if it is moving with a constant velocity v, is contracted with the Lorentz factor (4), cf. Fig. 3. That is, we measure a length L' for the moving standard length according to the equation

$$L' = L_o\sqrt{1 - v^2/c_o^2}\,. \tag{7}$$

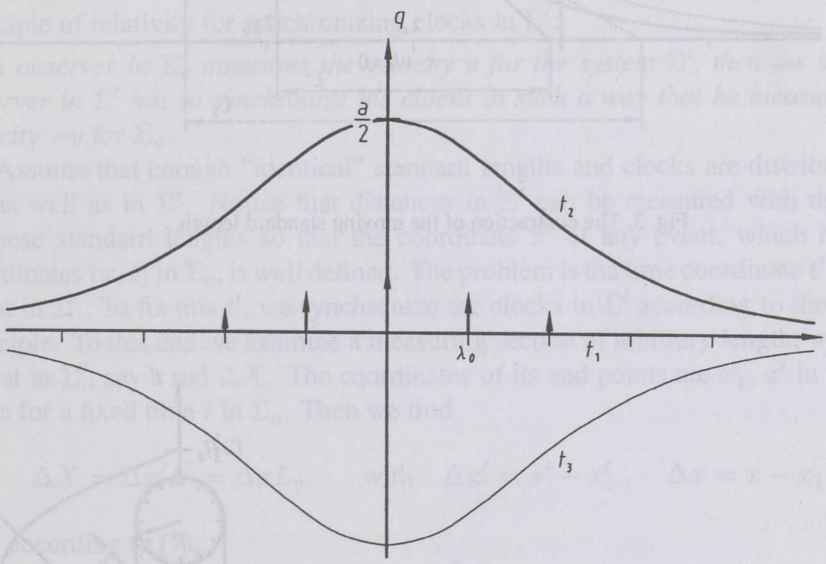

Fig. 2. Periodical oscillation of the breather solution (5) for some t values.

On the other hand, according to (6) the oscillation period T' of the moving clock, if it moves with a constant velocity v, is enlarged by the Lorentz factor (4). The hand of the clock counts the oscillations. Hence the moving clock goes behind according to the equation (see Fig. 4)

$$t' = t\sqrt{1 - v^2/c_o^2}\,. \tag{8}$$

By Eqs. (7) and (8) we have found for our crystal both fundamental kinematic effects of special relativity, Lorentz contraction, as well as time dilation. Nevertheless, this is not yet the special theory of relativity! First of all we distinguished the particular inertial system Σ_o, the system in which our crystal rests. We described the sine-Gordon equation in this reference system and derived the rest from this equation. Thus we have only shown that when standard lengths and clocks move against the inertial system Σ_o, then these standard lengths are shortened and these clocks go behind.

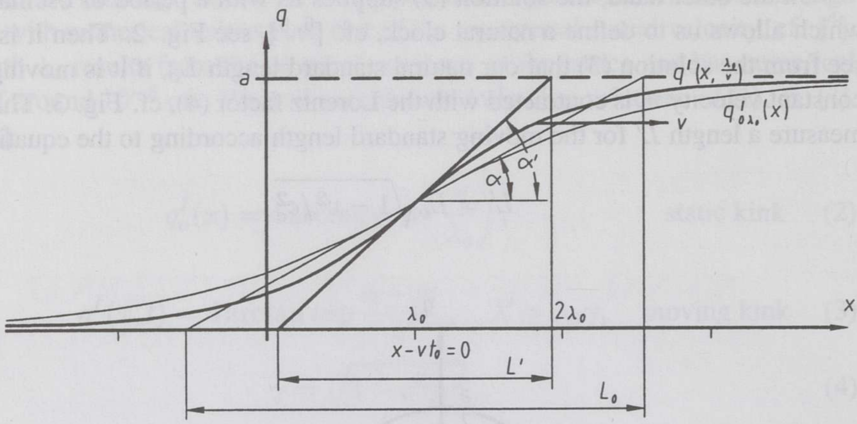

Fig. 3. The contraction of the moving standard length.

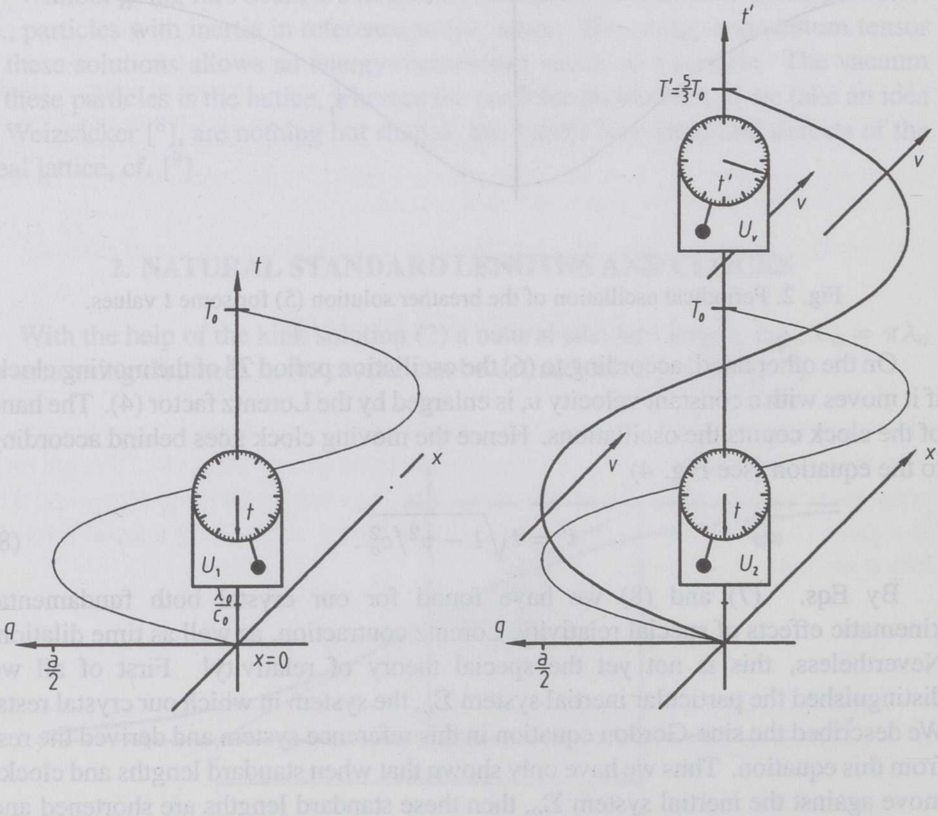

Fig. 4. The moving clock goes behind.

3. SYNCHRONIZATION

The crystalline solid defines our special reference system Σ_o with well-defined synchronized clocks (e.g., on the basis of the isotropic velocity c_o of the sine-Gordon equation). Let us consider the moving standard lengths L' (Fig. 3) and the moving clocks U_v with the oscillation period T' (Fig. 4) as the moving reference system Σ'. Notice, however, that we do not have a well-known velocity c_o' for synchronizing clocks in Σ'. Hence, we are not at all able to describe physical laws with respect to Σ'. To overcome this deficit, let us define the following elementary principle of relativity for synchronizing clocks in Σ':

If an observer in Σ_o measures the velocity v for the system Σ', then the moving observer in Σ' has to synchronize his clocks in such a way that he measures the velocity $-v$ for Σ_o.

Assume that enough "identical" standard lengths and clocks are distributed in Σ_o as well as in Σ'. Notice that distances in Σ' can be measured with the help of these standard lengths so that the coordinate x' of any event, which has the coordinates (x, t) in Σ_o, is well defined. The problem is the time coordinate t' of that event in Σ'. To fix this t', we synchronize the clocks in Σ' according to the above principle. To this end we examine a measuring section of arbitrary length, which is at rest in Σ', say a rod ΔX. The coordinates of its end points are x_1', x' in Σ' and x_1, x for a fixed time t in Σ_o. Then we find

$$\Delta X = \Delta x' L' = \Delta x L_o \quad \text{with} \quad \Delta x' = x' - x_1', \quad \Delta x = x - x_1 \tag{9}$$

and, according to (7),

$$\Delta x' = \frac{\Delta x}{\sqrt{1 - v^2/c_o^2}} . \tag{10}$$

Consider the usual initial condition for the coordinate systems of Σ_o and Σ', i.e., there is the event O with

$$O: \quad (x, t) = (x', t') = (0, 0) . \tag{11}$$

Assume that the left-hand end point of the rod is positioned at the coordinate origin of Σ' (Fig. 5), which moves with a velocity v in Σ_o, so that for an arbitrary time t in Σ_o

$$x_1 = vt , \quad x_1' = 0 . \tag{12}$$

For Eqs. (9)–(12) we can then write

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c_o^2}} . \tag{13}$$

There are moving clocks U_v^o and U_v^* at the left, resp. right-hand, end points of the rod. Consider the event A, where the right-hand end point of the rod has the

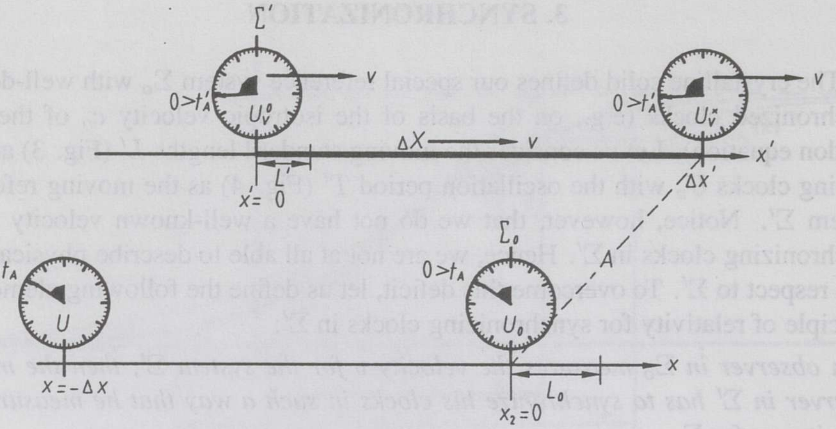

Fig. 5. The synchronization of the clocks U_v^o and U_v^* at the event A according to the elementary principle of relativity.

coordinate $x = 0$ in Σ_o. Then the left-hand end point has the coordinate $x_1 = -\Delta x$, see Fig. 5. We suppose the velocity $v = \Delta x/(-t_A)$ for the rod and the clocks U_v^o and U_v^*. Hence, the hand of the clock U_o at the coordinate origin of Σ_o for the event A points at $t_A = -\Delta x/v$,

$$\Sigma_o : A\left(x = 0, \quad t_A = \frac{-\Delta x}{v}\right). \tag{14}$$

On the other hand, at the event A the observer in Σ' measures the position $\Delta x'$ for the clock U_o in Σ_o. This clock arrives at $x' = 0$ for $T' = 0$. He measures the velocity $v' = (-\Delta x)/(-t'_A)$ for the clock U_o. According to our elementary principle, we demand $v' = -v$. Hence, the observer in Σ' measures the velocity $-v$ for the clock U_o in Σ_o if the hand of the clock U_v^* for the event A points at $t'_A = -\Delta x'/v$, see Fig. 5. Hence, using (10), we get

$$\Sigma' : A\left(x' = \Delta x' = \frac{\Delta x}{\sqrt{1 - v^2/c_o^2}}, \quad t'_A = \frac{-\Delta x}{v\sqrt{1 - v^2/c_o^2}}\right). \tag{15}$$

After the time lapse Δt_A in Σ_o with $\Delta t_A = \Delta x/v$ all clocks in Σ_o point at the time $t = 0$. The moving clocks in Σ' go behind by the Lorentz factor γ. Hence, during the time lapse Δt_A in Σ_o the hand of the clock U_v^* moves on by the amount $\Delta t'_A = \gamma \Delta t_A$, i.e., $\Delta t'_A = (\Delta x\sqrt{1 - v^2/c_o^2})/v$, and therefore it points at a time $t'_B := t'_A + \Delta t'_A$,

$$t'_B = \frac{-\Delta x}{v\sqrt{1 - v^2/c_o^2}} + \Delta x \frac{\sqrt{1 - v^2/c_o^2}}{v} = \frac{\Delta x}{v\sqrt{1 - v^2/c_o^2}}(-1 + 1 - v^2/c_o^2).$$

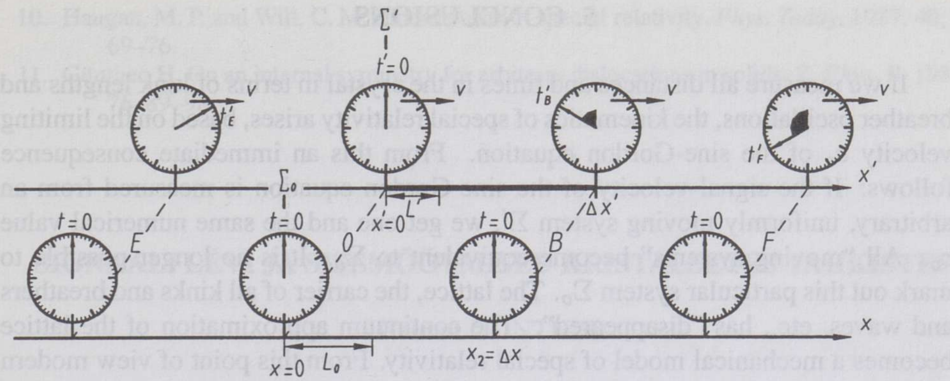

Fig. 6. The synchronization of the moving clocks for the time $t = 0$ in Σ_o.

According to (9) and (12) we have for $t = 0$ simply $\Delta x = x$. Hence, from our synchronization procedure we find: If a static clock at the position x in Σ_o indicates $t = 0$ (say the event B), the hand of the corresponding moving clock points to $t'_B = t'(x, 0)$ with (see Fig. 6)

$$t'(x,0) = \frac{-xv/c_o^2}{\sqrt{1-v^2/c_o^2}}\,. \tag{16}$$

4. LORENTZ TRANSFORMATION

Equation (16) describes the hand settings $t'(x, 0)$ of the moving clocks in Σ', which are observed in Σ_o at the time $t = 0$ and the position x. We know how these clocks run. They go behind by the Lorentz factor (4). Therefore, if they arrive at the position $x + vt$ after the time t, their hands have moved on by the amount $\gamma \cdot t$, i.e.,

$$t'(x+vt,t) = \frac{-xv/c_o^2}{\sqrt{1-v^2/c_o^2}} + t\sqrt{1-v^2/c_o^2} = \frac{t-(x+vt)v/c_o^2}{\sqrt{1-v^2/c_o^2}}\,.$$

Here x and t are arbitrary numbers. Hence, if we rewrite Eq. (13), we get as a whole

$$\left.\begin{aligned} x'(x,t) &= \frac{x-vt}{\sqrt{1-v^2/c_o^2}} \\ t'(x,t) &= \frac{t-vx/c_o^2}{\sqrt{1-v^2/c_o^2}} \end{aligned}\right\} \quad \text{Lorentz transformation} \tag{17}$$

5. CONCLUSIONS

If we measure all distances and times in the crystal in terms of kink lengths and breather oscillations, the kinematics of special relativity arises, based on the limiting velocity c_o of the sine-Gordon equation. From this an immediate consequence follows: If the signal velocity of the sine-Gordon equation is measured from an arbitrary, uniformly moving system Σ', we get one and the same numerical value c_o. All "moving systems" become equivalent to Σ_o. It is no longer possible to mark out this particular system Σ_o. The lattice, the carrier of all kinks and breathers and waves, etc., has "disappeared". The continuum approximation of the lattice becomes a mechanical model of special relativity. From this point of view modern tests of special relativity would no longer look for a supposedly ether drift (e.g., see [10]) but for a minimal length.

Let us additionally note that it is possible to formulate the equations for incompatible elastic strain created by arbitrary dislocations in a crystal as exact relativistic field equations. This incompatible strain of dislocations becomes a space-time Lorentz tensor. In particular, moving with a constant velocity through the crystal, the incompatible strain field of a dislocation can be calculated with the help of a Lorentz transformation from the static deformation field, cf. Günther [6, 11]. This case is highly analogous to the electromagnetic one.

Going back to H. Poincaré's belief: According to our approach it is really possible to start with the contraction hypothesis with respect to a particular reference frame Σ_o as an independent axiom in order to derive special relativity. To this end we have additionally assumed time dilation with respect to Σ_o, as well as our elementary principle of relativity. A. Einstein's universal constancy of the signal velocity then comes out as a result.

REFERENCES

1. Seeger, A. *Zur Dynamik von Versetzungen in Gitterreihen mit verschiedenen Gitterkonstanten*. Diplomarbeit T. H. Stuttgart, 1948/49.
2. Seeger, A. and Schiller, P. Kinks in dislocation lines and their effects on the internal friction in crystals. In *Physical Acoustics* (Mason, W. P., ed.). Academic Press, New York, 1966, **IIIA**, 361–495.
3. Eilenberger, E. *Solitons. Mathematical Methods for Physicists.* Springer, Berlin, 1981.
4. Eshelby, J. D. The interaction of kinks and elastic waves. *Proc. Roy. Soc. A*, 1962, **266**, 222–246.
5. Seeger, A. Solitons in crystals. In *Continuum Models of Discrete Systems* (Kröner, E. and Anthony, K.-H., eds.). University of Waterloo Press, Waterloo, 1980, 253–327.
6. Günther, H. On Lorentz symmetries in solids. *phys. stat. sol. (b)*, 1988, **149**, 101–109.
7. Günther, H. The crystalline structure as a basis for a reversed access to the special theory of relativity. *phys. stat. sol. (b)*, 1994, **185**, 335–340.
8. von Weizsäcker, C. F. Materie, Energie, Information. In *Die Einheit der Natur.* Hanser, München, 1982, 342–366.
9. Günther, H. *Grenzgeschwindigkeiten und ihre Paradoxa. Gitter · Äther · Relativität.* Teubner, Stuttgart, 1996.

10. Haugan, M. P. and Will, C. M. Modern tests of special relativity. *Phys. Today*, 1987, **40**, 5, 69–76.
11. Günther, H. On an internal symmetry for arbitrary dislocations in solids. *Z. Phys. B*, 1989, **76**, 89–98.

SIGNAALI LEVI JA SISEMÕÕTMISED KRISTALSETES TAHKISTES

Helmut GÜNTHER

Kristalses tahkises leviva signaali kirjeldamiseks on kasutatud siinus-Gordoni võrrandi solitonlahendeid, mille abil on defineeritud pikkuse ja aja skaala. Liikuvate koordinaatsüsteemide abil on selgitatud Lorentzi sümmeetriaid tahkistes ja esitatud teoreetilised kaalutlused erirelatiivsusteooria kontseptsioonide selgitamiseks mehaanikas.