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GROWTH AND DECAY OF WAVES IN MICROSTRUCTURED SOLIDS

Paolo CERMELLI and Franco PASTRONE

Dipartimento di Matematica, Università di Torino, Via C. Alberto 10, 10123 Torino, Italy; e-mails: cermelli@dm.unito.it, pastrone@dm.unito.it

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Abstract. The behaviour of microstructured bodies is briefly discussed, with the final aim to investigate wave propagation and exhibit the influence of the microstructure on the wave characteristics. Two simple models (vector and scalar one-dimensional microstructures) are introduced and, with some additional constitutive assumptions which take account of dissipation, we prove the existence of a threshold effect. This effect is such that the microstructure can be dissipative if the disturbance is "small", but can amplify the amplitudes of incoming waves above the threshold, whose existence and value depend on the constitutive properties of the microstructure and the nonlinearity of the model.

Key words: microstructured solids, waves, amplitude evolution.

1. INTRODUCTION

Many composite or complex materials (for instance, liquid crystals, microfractured materials, polymers, and so on) may be described in terms of microstructures (see for instance Capriz [1], Gurtin and Podio-Guidugli [2], Kunin [3], and Mindlin [4]). The procedure applied here to construct the model is based on the use of the second law of thermodynamics to obtain constitutive restrictions (cf. Ericksen [5] and Leslie [6] for liquid crystals). In case of a vector microstructure, we have previously investigated a strongly nonlinear (smooth) viscous friction dissipation function (see Cermelli and Pastrone [7, 8]). A threshold effect for the propagation of disturbances may be described, which results mainly from the anisotropy caused by the orientation of the microstructure ahead of the wave front. In case of scalar microstructures, the simplest model is the one-dimensional body, together with suitable constitutive assumptions on the microstructure: for instance, if the microstructure is dissipative (in the sense of a dry friction model), and the body is elastic or viscoelastic, a strong absorbing effect on the propagation of disturbances is shown. The presence of a threshold for the amplitude of the incoming wave, in case of shocks, allows for undamped propagation of initially large disturbances, namely a wave with constant amplitude, while initially small amplitude shocks are dissipated, according to the classical rules of viscoelasticity. Since the critical value depends on the state of the deformation ahead of the wave, a self-adaptive behaviour arises. Our model could also be used to explain the long-range absence of decay observed in seismology (cf. the "dilaton" introduced by Engelbrecht [9, 10]. The basic ingredient of the model is a dry-friction type of dissipation in the microstructure. Natural generalizations of these models are microstructures of affine, Cosserat, pseudo-rigid type, etc. In any model, the interaction between the micro- and the macrostructure and the constitutive properties of the microstructure must be carefully investigated, since the possibility of phenomena like solitons or amplification of waves above a well determined threshold value for the initial amplitude depends strongly both on such properties and the nonlinearity of the model.

2. GENERAL FEATURES

A microstructured body is a model derived from classical continuum mechanics that is capable of describing the behaviour of many composite or complex materials (i.e., liquid crystals, microfractured materials, polymers), when phenomena occurring at different length scales must be taken into account. Among others, we refer to Capriz [1], Ericksen [5], and Gurtin and Podio-Guidugli [2] for the introduction of the basic ideas as well as the derivation of balance equations and constitutive relations.

In general, a body with a microstructure is a classical three-dimensional deformable body $\mathcal{B} \in \mathbb{R}^3$, with a field $\delta : \mathcal{B} \times \mathbb{R} \to Lin$ added, where Lin is some linear space; the field δ describes the mechanical characteristics of the microstructure, in the sense of Capriz [1]. In other words, it is a measure of the displacement of the fine structure of the body. The deformation of the body is determined when the triple $(\mathbf{F}, \delta, \mathbf{G})$ is known, where $\mathbf{F}$ is the usual deformation gradient, $\mathbf{G}$ the (microscopic) gradient field: $\mathbf{G} = \nabla\delta$; the gradients are evaluated with respect to some material coordinate $\mathbf{x}$.

The stress fields are introduced after the definition of the expended power on arbitrary processes and, according to Gurtin and Podio-Guidugli [2], we obtain a gross structure stress, namely the classical macroscopic stress, a fine structure stress, namely the microscopic stress, and an interaction force between the gross and fine structures.

The usual assumption that the expended power is invariant under translations and rotations leads to balance laws, such as the macroscopic and microscopic momentum balances and the macroscopic angular momentum balance. The requirement that the laws of thermodynamics must be satisfied allows us to obtain constitutive relations between the stress and strain fields.

We assume the existence of a total free energy, and force any constitutively determined field to depend on the list $(\mathbf{F}, \delta, \mathbf{G}, \dot{\mathbf{F}}, \dot{\delta}, \dot{\mathbf{G}})$, so that we deal with dissipative materials of the rate type.

Since we want to study wave propagation according to the theory of singular surfaces [11], we work in terms of jumps of the relevant fields at this surface: for a field $\Phi : \mathcal{B} \times \mathbb{R} \to Lin$ we define $[\![\Phi]\!] = \Phi^- - \Phi^+$, where Φ^- and Φ^+ are the values of the field at the two sides of the singular surface, separating the perturbed portion of the body from the unperturbed state. The problem is to determine the propagation condition, the velocity of the waves and the amplitude evolution, i.e., all the characteristics of the travelling waves and, more interestingly, the reciprocal influence of the gross and fine structures in relation to the decay and growth of the amplitudes.

If we denote a motion of $\mathcal{B}$ by $\mathbf{y} : \mathcal{B} \times \mathbb{R}^+ \to \mathbb{R}^3$, so that $\mathbf{F} = \nabla \mathbf{y}$ is the macroscopic deformation gradient and $\dot{\mathbf{y}}$ the macroscopic velocity, and denote by $\mathbf{T}$ the first Piola–Kirchhoff stress, $\mathbf{p} = \varrho \dot{\mathbf{y}}$ the linear momentum, ϱ the mass density, and let $\mathbf{S}$ be the microscopic stress, $\mathbf{Q}$ the microstructural momentum (we assume that $\mathbf{Q} = \mathbf{I}\dot{\delta}$, with $\mathbf{I}$ an "inertia tensor"), $\mathbf{k}$ the interaction (or internal) force, and finally neglect external forces, the equations of motion are given by the macroscopic momentum balance

$$\int_{\partial W} \mathbf{T}\mathbf{n}\, da = \frac{d}{dt} \int_W \mathbf{p}\, dv, \tag{1}$$

for any region $W \subset \mathcal{B}$, with $\mathbf{n}$ the outward unit normal to ∂W, the microscopic momentum balance

$$\int_{\partial W} \mathbf{S}\mathbf{n}\, da + \int_W \mathbf{k}\, dv = \frac{d}{dt} \int_W \mathbf{Q}\, dv, \tag{2}$$

and the macroscopic angular momentum balance, which depends on the model and the microstructure. The local forms of the balance laws (1) and (2) read

$$\operatorname{Div} \mathbf{T} = \dot{\mathbf{p}}, \qquad \operatorname{Div} \mathbf{S} + \mathbf{k} = \dot{\mathbf{Q}}, \tag{3}$$

while the angular momentum balance yields in general an algebraic relation.

We could also write down the constitutive relations, the dissipation inequality and the jump conditions (compatibility relations) in a local form. However, in this general framework the results would be formal and, at this point of our research, with poor physical meaning.

Some of the more interesting models of microstructured solids are: (i) crystalline microstructures: $\delta : \mathcal{B} \to GL(\mathbb{R}^3)$ is a field of lattice bases for a Bravais lattice; (ii) liquid crystals: $\delta : \mathcal{B} \to S^2$ is a field of orientations for the rod-like crystal molecules; (iii) Cosserat shells: $\delta : \Sigma \to \mathbb{R}^3$ is the "thickness" of the shell

Σ, a two-dimensional manifold; (iv) microfractured materials, where $\delta : \mathcal{B} \to \mathbb{R}^3$ is normal to the plane of the microscopic crack.

In order to obtain some preliminary and meaningful results, we must specify the microstructure, introducing vector and scalar models; moreover, we will make some further constitutive assumptions, to be able to model some real material: the applications we have in mind should be applicable to microfractured materials. Many results can be found in [7, 8].

We notice finally that it would be important to compare the results obtained here with those following from homogenization methods (cf. Maugin [12]).

3. VECTOR MICROSTRUCTURES

The microstructural field will now be denoted by $\mathbf{d} : \mathcal{B} \to \mathbb{R}^3$. As shown in Cermelli and Pastrone [7], the balance equations have the form

$$\begin{aligned} &\operatorname{Div} \mathbf{T} = \dot{\mathbf{p}}, \\ &\operatorname{Div} \mathbf{S} + \mathbf{k} = \mathbf{I}\ddot{\mathbf{d}}, \\ &\operatorname{Skw}(\mathbf{T}\mathbf{F}^\top + \mathbf{S}\mathbf{G}^\top + \mathbf{d} \otimes \mathbf{k}) = 0, \end{aligned} \tag{4}$$

where Skw is the skew-symmetric part of a tensor. Assuming that the total energy density φ, the stress fields $\mathbf{T}$ and $\mathbf{S}$, and the interactive force $\mathbf{k}$ depend on the list $(\mathbf{F}, \mathbf{G}, \mathbf{d}, \dot{\mathbf{G}}, \dot{\mathbf{d}})$, with the further assumption that the body is dissipative in its microscopic part, the dissipation inequality in its standard form forces these fields to satisfy the following constitutive restrictions:

$$\varphi = \varphi(\mathbf{F}, \mathbf{G}, \mathbf{d}), \quad \mathbf{T} = \frac{\partial \varphi}{\partial \mathbf{F}}, \quad \mathbf{S} = \frac{\partial \varphi}{\partial \mathbf{G}} + \hat{\mathbf{S}}, \quad \mathbf{k} = -\frac{\partial \varphi}{\partial \mathbf{d}} + \hat{\mathbf{k}}, \tag{5}$$

where $\hat{\mathbf{S}}$ and $\hat{\mathbf{k}}$ depend on the full list of variables $(\mathbf{F}, \mathbf{G}, \mathbf{d}, \dot{\mathbf{G}}, \dot{\mathbf{d}})$ and satisfy the restriction

$$-\hat{\mathbf{S}} \cdot \dot{\mathbf{G}} + \hat{\mathbf{k}} \cdot \dot{\mathbf{d}} \leq 0. \tag{6}$$

As shown in [7], it is sensible to consider quasi-linear constitutive relations for the dissipative components of the stress:

$$\hat{\mathbf{k}} = -\mathbf{L}_1[\dot{\mathbf{d}}] - \mathbf{L}_2[\dot{\mathbf{G}}], \qquad \hat{\mathbf{S}} = \mathbf{L}_3[\dot{\mathbf{d}}] + \mathbf{L}_4[\dot{\mathbf{G}}], \tag{7}$$

where $\mathbf{L}_i$ are linear in the arguments in brackets and depend on $(\mathbf{F}, \mathbf{G}, \mathbf{d})$.

Let a singular surface S (with normal velocity V and unit normal $\mathbf{n}$) move in the body across which the second derivatives of the kinematical fields suffer a discontinuity. According to standard notations, the compatibility conditions read

$$\begin{aligned} &[\![\ddot{\mathbf{x}}]\!] = \mathbf{a}, \quad [\![\dot{\mathbf{F}}]\!] = -\tfrac{1}{V}\mathbf{a} \otimes \mathbf{n}, \quad [\![\nabla \mathbf{F}]\!] = \tfrac{1}{V^2}\mathbf{a} \otimes \mathbf{n} \otimes \mathbf{n}, \\ &[\![\ddot{\mathbf{d}}]\!] = \mathbf{b}, \quad [\![\dot{\mathbf{G}}]\!] = -\tfrac{1}{V}\mathbf{b} \otimes \mathbf{n}, \quad [\![\nabla \mathbf{G}]\!] = \tfrac{1}{V^2}\mathbf{b} \otimes \mathbf{n} \otimes \mathbf{n}. \end{aligned} \tag{8}$$

Notice that by the micromomentum balance, the microtraction must be continuous across S, i.e., $[\![\mathbf{S}]\!]\mathbf{n} = 0$, and this yields the propagation condition

$$\mathbf{D}\mathbf{b} = 0, \tag{9}$$

where $\mathbf{D} = \mathbf{D}(\mathbf{F}, \mathbf{G}, \mathbf{d}, \mathbf{n})$ is the *dissipation tensor*, the positive semidefinite tensor given by

$$\mathbf{u} \cdot \mathbf{D}\mathbf{w} = (\mathbf{u} \otimes \mathbf{n}) \cdot \mathbf{L}_4[\mathbf{w} \otimes \mathbf{n}], \tag{10}$$

for any $\mathbf{u}, \mathbf{w}$ in $\mathbb{R}^3$.

Then the following theorem, whose proof is given in [7], holds.

Theorem. *Assume that*: (i) *the state ahead of the wave is uniform* (*i.e.*, $\mathbf{F} = 1$, $\mathbf{G} = 0$, *and* $\mathbf{d} =$ const.); (ii) *the dissipation coefficients are symmetric*; (iii) *the dissipation coefficients do not depend on the macroscopic deformation gradient* $\mathbf{F}$; (iv) *the generalized Legendre–Hadamard condition holds*:

$$(\mathbf{u} \otimes \mathbf{n}) \cdot \partial_{\mathbf{F}}\mathbf{T}[\mathbf{u} \otimes \mathbf{n}] + 2(\mathbf{w} \otimes \mathbf{n}) \cdot \partial_{\mathbf{G}}\mathbf{T}[\mathbf{u} \otimes \mathbf{n}] + (\mathbf{w} \otimes \mathbf{n}) \cdot \partial_{\mathbf{G}}\mathbf{S}_0[\mathbf{w} \otimes \mathbf{n}] > 0, \quad \forall\, \mathbf{n}, \mathbf{u}, \mathbf{w} \neq 0, \tag{11}$$

with $\mathbf{u}$, $\mathbf{w}$ *arbitrary vectors and* $\mathbf{S}_0 = \mathbf{S} - \hat{\mathbf{S}}$ *being the equilibrium microstress.*

Then, if $\text{rank}(\mathbf{D}) = N \leq 3$, *there exist* $(6 - N)$ *propagation speeds* V_K^2, *with* $K = 1, \ldots, 6 - N$, *not all necessarily distinct.*

If $\text{rank}(\mathbf{D}) = 3$, no acceleration waves may propagate in the microstructure: $\mathbf{b} = 0$. Acceleration waves may still propagate in the body, and in principle there exist three propagation velocities. The associated perturbation in the microstructure is of the third order, completely determined since $\mathbf{D}$ is invertible. The equation of evolution of the amplitude is derived in [7] and we obtain a Riccati-type equation

$$(\varrho J \sigma^2)' + (\mathcal{D} - \mathcal{A}) J \sigma^2 + \mathcal{B} J \sigma^3 = 0, \tag{12}$$

where σ is the amplitude of the acceleration waves, $\mathbf{a} = \sigma \mathbf{u}$, with $\mathbf{u}$ the unit eigenvector of the equation of propagation, J is the surface area element of the singular surface, and $\mathcal{A}$, $\mathcal{B}$, $\mathcal{D}$ are explicitly given in [7].

If $\text{rank}(\mathbf{D}) < 3$, we have again a Riccati-type equation for the evolution of the amplitude.

In both cases, $\mathcal{A}$ characterizes the influence of the ground state on the wave propagation: it is a high-order nonlinear interaction term between the micro- and macrostructures; $\mathcal{B}$ represents the nonlinearity of the elastic body and acts to favour shock-wave formation; $\mathcal{D}$ is a dissipative term, it is positive and induces exponential decay of small amplitude waves.

Notice the presence of competition between the coefficients $\mathcal{A}$ and $\mathcal{D}$ on the growth and decay of amplitudes.

4. ONE-DIMENSIONAL MODEL

A second example of growth–decay of the wave amplitude is given by scalar microstructures without inertia in one-dimensional bodies.

If x is the material coordinate in a reference configuration, we denote by $y = y(x,t)$ the deformation, so that $u(x,t) = y_x(x,t) - 1$ is the displacement gradient and $v = y_t$ the macrovelocity, where the subscript means derivation with respect to the corresponding variable.

The microstructure is here a scalar field denoted by $d = d(x,t)$ and the corresponding microvelocity is $w = d_t$. Moreover, we denote by T, S, and k here the unidimensional counterparts of **T**, **S**, and **k**.

In order to analyse microscopic dissipation, we assume that the microstress is negligible, $\mathbf{S} = 0$. Then, as shown in [8], if we take the dissipative internal force $\hat{k}$ be of bounded variation with respect to w and continuous away from $w = 0$, while the stress T is differentiable with respect to its arguments, by a standard theorem we can write

$$\hat{k} = E(w)w + \Phi(w), \tag{13}$$

with $E > 0$, and $\Phi(w)$ a set-valued function given by

$$\begin{aligned} \Phi &\in [-\Phi_c, \Phi_c] && \text{if } w = 0, \\ \Phi &= \Phi_c \operatorname{sgn}(w) && \text{if } w \neq 0, \end{aligned} \tag{14}$$

with Φ_c a constitutively determined threshold value for the stress. The first term in the relation for $\hat{k}$ represents viscous dissipation, while the second acts as a dry-friction term.

Let $\varphi = \varphi(u,d)$ be the free energy. In our applications, in order to obtain explicit results, we assume it to be quadratic in the displacement gradient

$$\varphi = \varphi(u,d) = \frac{1}{2}c^2(u - \lambda d)^2 + \phi(d), \tag{15}$$

with c, λ given constants ($\lambda > 0$) and ϕ a given function. For fixed $d = d_0$, this is a typical one-well convex energy function, with the equilibrium state $u_0 = \lambda d_0$. This form of the energy has been chosen because it is the simplest quadratic-type energy inducing a hysteretic behaviour in quasi-static loading cycles.

The field equations are, taking the mass density to be $\varrho = 1$,

$$\begin{aligned} u_t &= v_x, \\ v_t &= c^2 u_x - \lambda c^2 d_x + A v_{xx} + B w_x, \\ 0 &= -\lambda c^2 u + \lambda^2 c^2 d + \phi'(d) + C v_x + E w + \Phi(w). \end{aligned} \tag{16}$$

A strong singularity is a moving point $\xi = \xi(t)$ in $\mathcal{B}$, across which the fields u, v, and w are discontinuous, while y and d are continuous; a weak singularity occurs when u_x, u_t, and d_t are discontinuous across ξ.

If we denote by $V = \dot{\xi}$ the velocity of the singularity, then the jump equation $V[\![v]\!] + [\![T]\!] = 0$ holds: at a weak discontinuity, this implies $[\![T]\!] = 0$. In this case a weak singularity propagates according to the jump relations (with the further assumption that the state ahead of the singularity is at rest):

$$\begin{aligned} Av_x^- + Bw^- &= 0, \\ Cv_x^- + Ew^- &= -\partial_d\varphi(u,d) - \Phi_c \mathrm{sgn}(w^-). \end{aligned} \tag{17}$$

Conversely, a strong singularity, namely a shock wave, must satisfy the jump conditions

$$\begin{aligned} -A[\![u_t]\!] + BV[\![d_x]\!] &= (c^2 - V^2)[\![u]\!], \\ (-A[\![u_t]\!] + B[\![d_x]\!])[\![u]\!]V^2 &\leq 0, \\ -C[\![u_t]\!] + EV[\![d_x]\!] &= -\lambda c^2[\![u]\!] - \partial_d\varphi(u^+,d) - \Phi, \end{aligned} \tag{18}$$

where the second relation follows from the dissipation inequality: $-([\![\varphi]\!] - \langle\sigma\rangle[\![u]\!])V \leq 0$, with $\langle\sigma\rangle = \frac{1}{2}(\sigma^+ + \sigma^-)$ the average of σ across ξ.

The first equation, together with the second inequality, implies $(c^2 - V^2)[\![u]\!]^2 V \leq 0$, and assuming (without loss in generality) $V > 0$, this implies that either $[\![u]\!] = 0$, in which case no shock may propagate, or $V^2 > c^2$, so that the shock is supersonic, or finally

$$V^2 = c^2, \tag{19}$$

so that the complete list of jump conditions has the form

$$\begin{aligned} -AV^{-1}[\![u_t]\!] + B[\![d_x]\!] &= 0, \\ -CcV^{-1}[\![u_t]\!] + Ec[\![d_x]\!] &= -\lambda c^2[\![u]\!] - \partial_d\varphi(u^+,d) - \Phi. \end{aligned} \tag{20}$$

We assume that the state ahead of the wave is in equilibrium below the threshold value

$$-\Phi_c < \partial_d\varphi(u^+,d) < \Phi_c.$$

Let now

$$[\![u]\!] = \alpha, \quad [\![d_x]\!] = \gamma, \quad k_{\mathrm{eq}}(u,d) = -\partial_d\varphi(u,d),$$

and consider the special case of an energy function quadratic in strain: the jump conditions imply

$$Ec\gamma = -\lambda c^2\alpha + k_{\mathrm{eq}}(u^+,d) - \Phi.$$

We want to prove that the presence of a dry-friction term in the microstructure has a strong absorbing effect on the propagation of disturbances, in that the shock amplitude is necessarily bounded from above. On the other hand, the presence of the threshold-activation mechanism allows for undamped propagation of initially large disturbances, so that above-threshold shocks persist without decay (for an interesting analogy in the case of networks of beams see [13]).

In [8] we have examined the cases corresponding to the presence of viscous dissipation, namely when the coefficients A, B, C, and E are not all vanishing. For brevity sake, we sketch here only the case in which $A = B = C = E = 0$, so that the only source of dissipation in this model is the dry-friction term $\Phi = \Phi(w)$ depending on the microvelocity. Then, if $V = +c$, the propagation condition reduces to

$$\lambda c^2 \alpha = k_{\text{eq}}(u^+, d) - \Phi, \tag{21}$$

so that we have two possibilities:

(i) if $-\Phi_c < k_{\text{eq}}(u^-, d) = -\lambda c^2 \alpha + k_{\text{eq}}(u^+, d) < \Phi_c$,
then the microstructure is not activated: the internal force k_{eq} does not reach the threshold value, and the amplitude α of the shock is constant but otherwise arbitrary (determined by the initial conditions only);

(ii) if $k_{\text{eq}}(u^+, d) - \lambda c^2 \alpha \leq -\Phi_c$ or $\Phi_c \geq k_{\text{eq}}(u^+, d) - \lambda c^2 \alpha$,
then the microstructure is activated, but the shock amplitude is *completely determined* by the state ahead of the wave through the relation

$$\lambda c^2 \alpha = k_{\text{eq}}(u^+, d) - \Phi_c \text{sgn}(\gamma).$$

In this case the microstructure has a strong absorbing effect on the amplitude of a shock singularity propagating into a rest state, and this property might be used as a device to absorb vibrations. Notice that the threshold value itself for the shock α is determined by the state ahead of the wave, and may be modified by deforming the material or varying the microstructural density.

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LAINETE VÕIMENDUMINE JA SUMBUMINE MIKROSTRUKTUURIGA MATERJALIDES

Paolo CERMELLI ja Franco PASTRONE

On uuritud materjali mikrostruktuuri mõju deformatsioonilainete levile tahketes kehades ja esitatud kaks lihtsat mudelit, üks vektoriaalse ja teine skalaarse mikrostruktuuri kirjeldamiseks. Nende mudelite raames on näidatud nn. läveefekti olemasolu. Vähese intensiivsusega laine sumbub mikrostruktuuri mõjul, kuid lained, mille intensiivsus ületab läve, võivad võimenduda. Võimendusefekt sõltub mikrostruktuuri omadustest ja materjali mittelineaarsusest.