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INTERACTION OF MOVING SOLITONS IN A DISPERSIVE MEDIUM AND REGIMES OF THEIR RADIATIONLESS MOTION

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Abstract. Soliton interactions in a strongly dispersive one-dimensional medium are discussed. It is shown that the interactions between two moving solitons in the dispersive medium can change their character from repulsive to attractive depending on contraction properties of the solitons. As a result, the formation of the soliton complex which can move without radiation becomes possible. This universal phenomenon is demonstrated in the framework of dispersive variants of the sine-Gordon and double sine-Gordon models described by equations with additional fourth derivatives. The bound soliton states are exact travelling solutions of the equations. Conditions of the formation of the soliton complex are specified and its internal dynamics is studied in terms of collective variables.

Key words: soliton, dispersive medium, soliton interaction, radiationless motion.

1. INTRODUCTION

The first step in describing the microstructured solids consists usually in taking spacial dispersion of the medium into account. A natural cause of the dispersion is a discrete structure of real crystals. Nonlinear wave dynamics in discrete models of solids reveals many specific features known as the discreteness effects. Some of them are consequences of the translational invariance of the lattices; therefore they exist only in discrete systems. However, some of the effects, originating from dispersive properties of the medium, can hold in the long wave limit and hence these have a more universal character than the others.

Twelve years ago Peyrard and Kruskal observed by a numerical simulation almost radiationless motion of the 4π-soliton in the highly discrete sine-Gordon system [1]. It was a surprise since a stationary motion of a single 2π-soliton was impossible in this highly dispersive system because of a strong radiation emitted by the moving 2π-soliton [2]. The authors of [1] tried to explain the formation of the soliton complex of two identical 2π-solitons by exploiting the fact of the presence of the Peierls potential in the lattice under consideration. However, a similar effect of the bunching of solitons in the weak discrete sine-Gordon model was known much earlier, beginning with the work [3]. There were also attempts [4, 5] at an explanation of the phenomenon based on the use of the soliton perturbation theory applied to the continuous analogue of the system. The authors of [4] were the first to point out that one needs to add dispersive terms to the usual sine-Gordon model to obtain the steady multisoliton solutions. Recently Alfimov et al. [6] have shown numerically the existence of a freely-moving soliton complex in the continuous nonlocal sine-Gordon model. All these facts prompted that the formation of the bound soliton complex is a universal property of the dispersive media. Indeed, in a previous work [7] we found that the radiationless motion of the complex can be described explicitly in the framework of the dispersive sine-Gordon equation (dSGE) with a fourth spacial derivative.

The goal of the present work was to show that solitons in dispersive media possess an internal structure and their interaction strongly depends on intrinsic properties such as the flexibility. Due to this dependence two identical moving solitons begin to attract each other when their velocity exceeds the threshold value. We present exact analytical solutions corresponding to the soliton complex for two variants of the dispersive sine-Gordon equations and the dispersive double sine-Gordon equation. Together with a discussion of dynamic and intrinsic properties of the single 2π-soliton, the knowledge of exact solutions allows us to formulate the ansatz for the description of dynamics and interaction of two solitons in the dispersive medium. As a result, we find the condition for the formation of a soliton complex, calculate its dynamic characteristics, and study its stability with respect to decay into two solitons.

2. DISPERSIVE SINE-GORDON EQUATIONS

A large variety of physical processes in microstructured solids is described by the discrete sine-Gordon equation. We write it in the dimensionless form following the notations of [1]:

$$\frac{\partial^2 u_n}{\partial \tau^2} + 2u_n - u_{n-1} - u_{n+1} + \frac{1}{d^2}\sin(u_n) = 0, \tag{1}$$

where u_n is, e.g., the displacement of the atom and d is the discreteness parameter. The Hamiltonian of the system is written as

$$H = \sum_n \left\{ \frac{1}{2}u_\tau^2 + \frac{1}{2}(u_{n+1} - u_n)^2 + \frac{1}{d^2}(1 - \cos(u_n)) \right\}. \quad (2)$$

In studying the long wave processes, one can substitute xd for n, td for τ, and the second difference by the series:

$$u_{n-1} + u_{n+1} - 2u_n \approx u_{xx} + \beta u_{xxxx} + \frac{2}{5}\beta^2 u_{xxxxxx} + ..., \quad (3)$$

where the dispersive factor, $\beta \equiv 1/(12d^2)$, is supposed to be small. Neglecting the sixth derivative and higher-order ones, we obtain the dispersive sine-Gordon equation (1dSGE):

$$u_{tt} - u_{xx} - \beta u_{xxxx} + \sin(u) = 0. \quad (4)$$

With the same accuracy the Hamiltonian of the system is transformed into the following expression:

$$H = C \int_{-\infty}^{\infty} \frac{1}{2}\{u_t^2 + u_x^2 - \beta u_{xx}^2 + 2(1 - \cos(u))\}dx, \quad (5)$$

where $C = 16/d$. Note that the dispersive term in the Hamiltonian can affect a soliton interaction in the sine-Gordon system and change strongly the solitonic dynamic properties.

Equation (4) can be obtained from a Lagrangian of the following form:

$$L = C \int_{-\infty}^{\infty} \frac{1}{2}\{u_t^2 - u_x^2 + \beta u_{xx}^2 - 2(1 - \cos(u))\}dx. \quad (6)$$

Total field momentum of the system is as follows:

$$P = -C \int_{-\infty}^{\infty} u_t u_x dx. \quad (7)$$

Linear waves of Eq. (4) have the spectrum $\omega(k) = \sqrt{1 + k^2 - \beta k^4}$. Formally there exists a critical wave number k_0 at which $\omega(k_0) = 0$. This means that the equilibrium state $u = 0$ is unstable with respect to the short wavelength perturbations. Recalling the spectrum of linear excitations of the discrete system, one understands the artificial origin of this instability. Note that the same problem takes place in the Boussinesq equation [8] and there is another form of the Boussinesq equation especially proposed to avoid the instability. Following this idea, we can write another dispersive sine-Gordon equation (2dSGE)

$$u_{tt} - u_{xx} - \beta u_{ttxx} + \sin(u) = 0. \quad (8)$$

It has a dispersion relation of the form $\omega(k) = \sqrt{(1+k^2)/(1+\beta k^2)}$ and hence a stable state with $u = 0$.

Starting from the more complicated discrete model described by the double sine-Gordon equation,

$$\frac{\partial^2 u_n}{\partial t^2} + 2u_n - u_{n-1} - u_{n+1} + \frac{1}{d^2}(\sin(u_n) + h\sin(u_n/2)) = 0, \tag{9}$$

we can derive by the above procedure the dispersive analogue (dDSGE) of Eq. (9):

$$u_{tt} - u_{xx} - \beta u_{xxxx} + \sin(u)\cos(u) + h\sin(u) = 0. \tag{10}$$

This equation can be used in various applications. We mention here only its relevance in describing the nonlinear dynamics in low-dimensional magnets. In this context the parameter h denotes the amplitude of the magnetic field. Concluding this section we note that soliton dynamics in many physical systems such as plasmas [9], nonlinear transmission lines [10], anharmonic crystal lattices with next-neighbour interactions [11], and so on, obey the equations with the same strong dispersion in the wave motion.

3. REMARKS ON DYNAMICS OF 2π-SOLITON

Dynamics of a single 2π-soliton in the discrete sine-Gordon model and in the case of very weak dispersion for Eq. (4) has been studied to some extent [12, 13]. The results are summarized as: (i) because of a strong interaction with continuum waves the moving soliton quickly loses its velocity; (ii) its motion is accompanied by significant radiation mainly in backward direction; (iii) the dispersion influences the soliton shape and yields an internal soliton motion. The existence of the internal mode of a soliton oscillation in the discrete sine-Gordon model was shown numerically in [14]. It is difficult to prove this fact analytically for Eq. (4) because an exact solution for the 2π-soliton is absent. However, such a solution is easily found for Eq. (8): $u_s = 4\arctan\{\exp(x)\}$. The problem of the spectrum of small oscillations $v(x,t) = u(x,t) - u_s = f(x)\exp(i\omega t)$ is reduced to solving the eigenvalue problem:

$$\left[-(1-\beta\omega^2)\frac{d^2}{dx^2} + 1 - \frac{2}{\cosh^2(x)}\right] f = \omega^2 f. \tag{11}$$

Besides the translation mode with $\omega = 0$, Eq. (11) has the internal mode which corresponds to an oscillation of the soliton width. Frequency of the mode is a function of the parameter β:

$$\omega_1 = \frac{1}{\beta}\frac{\Delta^2 - 9}{\Delta^2 - 1}, \quad \Delta(\beta) = \frac{6\beta + \sqrt{17\beta^2 - 10\beta + 9}}{1+\beta}.$$

At small β the frequency behaves very similar to that of the discrete model and has to coincide with the corresponding dependence in Eq. (4). Thus, due to their internal structure, the 2π-solitons can interact nontrivially in the dispersive medium [14].

4. EXACT SOLITON SOLUTIONS OF DISPERSIVE EQUATIONS

Now we present exact solutions describing a radiationless motion of soliton complexes in the dispersive equations (4), (8), and (10), respectively:

$$u_{4\pi} = 8\arctan\left\{\exp\left(\sqrt{\frac{2}{3}}\frac{x - V_0 t}{\sqrt{1 - V_0^2}}\right)\right\}, \quad V_0(d) = \pm\sqrt{1 - \frac{1}{3d}};$$

$$u_{4\pi} = 8\arctan\left\{\exp\left(\sqrt{\frac{2}{3}}\frac{x - V_1 t}{\sqrt{1 - V_1^2}}\right)\right\}, \quad V_1(d) = \pm\left(\sqrt{1 + \frac{1}{36d^2}} - \frac{1}{6d}\right);$$

$$u_{2\pi} = 4\arctan\left\{\exp\left(\sqrt{\frac{2}{3} + h}\,\frac{x - V_* t}{\sqrt{1 - V_*^2}}\right)\right\}, \quad V_*(d) = \pm\sqrt{1 - \frac{1}{3d}\left(1 + \frac{3}{2}h\right)}.$$

We have written the soliton complex solutions in the Lorentz-invariant-like form. However, the velocity of the complex is not an arbitrary parameter of the solution but it depends strongly on the parameter β (or d). In the 1dSGE the velocity dependence on the discreteness parameter d differs less than 5% from the numerical one of the highly discrete sine-Gordon model (cf. with Fig. 12 in [1]). It should be noted that, despite the strong connection between the results, the continuous model does not admit the Peierls potential which was a principal ingredient in the phenomenological theory of Peyrard and Kruskal [1]. In the 2dSGE the velocity tends to zero when d vanishes. In the dDSGE the soliton complex consists of two π-domain walls. In this system the threshold value of the discrete parameter d at which the complex begins to move is a function of the magnetic field h. We interpret all these complexes as specific bound states of two identical solitons which can attract each other in the dispersive medium.

5. COLLECTIVE VARIABLE ANSATZ FOR DYNAMICS OF A SOLITON COMPLEX

Our analytical approach to the description of a soliton complex formation in a dispersive medium is based on the use of the collective variable ansatz. It is constructed with taking account of the translational and internal degrees of freedom of the soliton as well as interactions between solitons and solitons with radiation:

$$u(x,t) = u^{(s)}(x,t;q,X,R) + u^{(r)}(x,t), \tag{12}$$

where $u^{(r)}(x,t)$ is a part of the solution corresponding to radiation and $u^{(s)}$ is the solitonic part:

$$u^{(s)} = 4\arctan\{\exp(q(x-X-R))\} + 4\arctan\{\exp(q(x-X+R))\}. \tag{13}$$

Here $q = q(t)$, $X = X(t)$, and $R = R(t)$ are functions of time. In the adiabatic approximation functions $q(t)$ and $X(t)$ describe the internal and translational motions of solitons, respectively, and the function $R = R(t)$ corresponds to the changing distance between solitons. Note that stationary states are included in the ansatz. In particular, the exact solution of the 1dSGE corresponds to the expression (12) with $u^{(r)} = 0$ and $R = 0$, $X = V_0 t$, and

$$V_0 = \sqrt{1 - \sqrt{\frac{4\beta}{3}}}, \qquad q_0 = (3\beta)^{-1/4}. \tag{14}$$

In the present work we apply the ansatz only to the 1dSGE. The results concerning two other dispersive equations will be published elsewhere. After a substitution of the expressions (12), (13) in the Lagrangian (6) and performing the integration, we obtain the Lagrangian for three collective variables and function $u^{(r)}(x,t)$. This expression is very complicated for arbitrary large $R(t)$ so that we consider here only the case of small R. Let us neglect the radiation for the moment. Then the Lagrangian function with accuracy to order $O(R^2)$ can be written

$$L = L_0 + R^2[T_1 - U_1(q, X_t^2)], \tag{15}$$

where the zero-order Lagrangian is

$$L_0 = C\left\{\frac{\pi^2}{12}\frac{q_t^2}{q^3} + \frac{\beta}{3}q^3 - \frac{1}{3q} - q + qX_t^2\right\}, \tag{16}$$

T_1 contains all terms proportional to the time derivatives of the function q, and U_1 equals

$$U_1(q, X_t^2) = \frac{C}{3}\left\{q(X_t^2 - 1) + \frac{1}{5q} + \frac{7}{5}\beta q^3\right\}. \tag{17}$$

From Eq. (15) we see that one of the Lagrange equations takes the form $(\partial L/\partial R) = 0$. It is transformed into $RU_1(q, X_t^2) = 0$ for stationary states. In the next section we derive equations of motion for the soliton bound state parameters when taking the radiation into account.

6. EQUATIONS OF MOTION FOR COLLECTIVE COORDINATES

It is easy to find all the Lagrange equations by a variation of the Lagrangian (15). There is also another way to obtain the equations without neglecting the radiation terms. For this purpose one has to substitute the ansatz of general form (12) into the original 1dSGE

$$u_{tt}^{(s)} - u_{xx}^{(s)} - \beta u_{xxxx}^{(s)} + \sin(u^{(s)} + u^{(r)}) + u_{tt}^{(r)} - u_{xx}^{(r)} - \beta u_{xxxx}^{(r)} = 0 \quad (18)$$

and perform the integration of the equation with the solution derivatives with respect to parameters X and q, namely, $(\partial u^s/\partial X)$ and $(\partial u^s/\partial q)$. In the zero-order approximation with respect to small R the Lagrange equations are the following:

$$\frac{\pi^2}{6}\frac{q_{tt}}{q^3} - \frac{\pi^2}{4}\frac{q_t^2}{q^4} + 1 - X_t^2 - \beta q^2 - \frac{1}{3q^2} = -q^{-2}\int_{-\infty}^{\infty} \theta \cosh^{-1}(\theta) D u^{(r)} d\theta, \quad (19)$$

$$8\frac{d}{dt}(qX_t) = \int_{-\infty}^{\infty} \cosh^{-1}(\theta) D u^{(r)} d\theta, \quad (20)$$

where $\theta = q(x - X)$ and the operator D has the form

$$D = \frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} - \beta\frac{\partial^4}{\partial x^4} + \cos(u^{(s)}). \quad (21)$$

In general, the right-hand sides of Eqs. (19), (20) are not equal to zero. This occurs only when radiation is absent as in the case of the exact moving solutions. The role of radiation in nonstationary processes is very important and does not reduce only to dissipation of a soliton energy as in the case of the 2π-soliton. The total energy of the dispersive model is naturally conserved but a distribution of the energy between two subsystems, the soliton complex and the radiation, appears to be nontrivial. However, let us study Eqs. (19), (20), at first, with zero r.h.s.

7. ADIABATIC APPROXIMATION IN SOLITON COMPLEX DYNAMICS

Below we assume that $u(x,t) = 8\arctan\{\exp[q(t)(x - X(t))]\}$, where functions $q(t)$ and $X(t)$ obey the equations

$$\frac{\pi^2}{6}\frac{q_{tt}}{q^3} - \frac{\pi^2}{4}\frac{q_t^2}{q^4} + 1 - X_t^2 - \beta q^2 - \frac{1}{3q^2} = 0, \quad (22)$$

$$qX_t = p = \text{const}. \quad (23)$$

The second equation is a consequence of the fact that the coordinate X is cyclic as is seen from the Lagrangian L_0. These equations can be completely solved because their first integral is easily found as

$$\widetilde{E} = qX_t^2 + \frac{\pi^2}{12}\frac{q_t^2}{q^3} + q - \frac{\beta}{3}q^3 + \frac{1}{3q}. \tag{24}$$

It is interesting to note that Eq. (24) can be rewritten as a standard energy expression of a point particle placed in an effective field

$$\frac{\pi^2}{12}q_t^2 + q^2\left(\frac{1}{3} + p^2\right) - \widetilde{E}\,q^3 + q^4 - \frac{\beta}{3}q^6 = 0. \tag{25}$$

At first we seek stationary solutions of Eq. (22) and find that constant solutions q and $X_t = V$ are connected by the relation

$$V^2 = 1 - \beta q^2 - \frac{1}{3q^2} \tag{26}$$

or by the inverse two-valued dependence

$$\beta q_\pm^2 = \frac{1 - V^2}{2} \pm \sqrt{\frac{(1 - V^2)^2}{4} - \frac{\beta}{3}}. \tag{27}$$

Note that one of the parameters, q or V, can be considered as free yet. The function $V(q)$ has a maximum V_0 in the point $q = q_0$ which coincides with parameters (14) of the exact solution. Other values of parameters of Eq. (26) or Eq. (27) do not yield exact solutions of Eq. (4). However, they reflect the fact that the 1dSGE has two branches of stationary solutions $u^{(s)}(x - Vt)$ in the form different from the ansatz (13). Although the adiabatic approach gives obviously a qualitative description of the stationary solutions, its main conclusions are believed to be correct.

The total momentum calculated for the stationary solutions equals $P = 2Cq_\pm V$ and the total energy is as follows:

$$E = 2Cq_\pm\left(1 - \frac{2}{3}\beta q_\pm^2\right), \tag{28}$$

where $q_\pm(V)$ is given by Eq. (27). Taking the soliton-particle analogy into account [15–17], we can find a quasi-particle spectrum for the moving soliton. It is given by the two-valued function: $E = E_\pm(P)$. The upper branch of the dependence, corresponding to upper signs in Eq. (28), belongs to absolutely unstable states which are placed in a maximum of the effective potential in Eq. (25). Stationary states of the lower branch correspond to a minimum of the effective potential and hence are stable in the framework of Eqs. (22), (23). This conclusion is also proved by a direct analysis of stability [7]. Omitting details of calculations, we present the final expression for small oscillations near the stationary states: $y(t) = q(t) - q_\pm = y_0 \exp(i\Omega t)$, where the squared frequency $\Omega^2 = \frac{12}{\pi^2}q_\pm^2(1 - 2\beta q_\pm^2)$. In the point

$q_m = (2\beta)^{-1/2}$ both branches meet and reach the energy maximum. The energy appears to be also a two-valued function of the velocity $E = E^{\pm}(V)$. The lower branch $E_-(P)$ corresponds to a part of the dependence $E = E^+(V)$ after the point of the energy maximum and a whole branch $E = E^-(V)$. Both branches $E = E^{\pm}(V)$ coalesce in the point V_0 demonstrating a typical bifurcation behaviour.

The bifurcation nature of the exact solution becomes clear when we take account of the additional dependence of the solutions on the distance parameter R and consider the equation $(\partial L/\partial R) = 0$ together with the pair Eq. (22) and Eq. (23). We are easily convinced that the exact solution is uniquely defined as one of two equations, $U_1(q, V) = 0$ (see Eq. (17)) and Eq. (26). Discussing the stability of the soliton complex in the framework of three equations for collective variables, we note that the unstable mode includes now the dependence of a distance between solitons in the soliton complex increasing in time. This means the initial stage of the decay of the complex. It begins when the energy of interaction between solitons as a function of the mutual distance R changes its character from attractive to repulsive.

In reality the formation of the stationary solution proceeds with the participation of the radiation processes and the energy of two interacting solitons is changed until they reach the form of the exact radiationless soliton complex. If the initial velocity of solitons is lower than the critical value V_0, they repulse each other. In the opposite case, $V > V_0$, solitons attract each other by changing their widths and decreasing the velocity until it becomes equal to V_0 when they form the bound state.

In conclusion, we would like to emphasize that this phenomenon seems to be a universal effect in nonlinear strongly dispersive nonintegrable systems bearing soliton excitations. It can be observed not only in computer simulations but also in real experiments in low-dimensional microstructured solids.

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SOLITONIDE INTERAKTSIOON DISPERGEERUVAS KESKKONNAS JA NENDE RADIATSIOONIVABAD LIIKUMISREŽIIMID

Mihhail BOGDAN ja Arnold KOSSEVITŠ

On analüüsitud solitonide interaktsiooni tugevasti dispergeeruvas ühemõõtmelises keskkonnas ning näidatud, et interaktsioon võib olla kas repellori või atraktori tüüpi sõltuvalt solitonide struktuuri omadustest. Sellest tingituna on võimalik solitonide kompleks, mis saab levida ilma radiatsioonita. Näide käsitleb siinus-Gordoni ja kaksiksiinus-Gordoni võrrandi analüüsi, arvestades neljandat järku tuletiste abil modelleeritud lisadispersiooni. On leitud täpsed leviva laine tüüpi lahendid ja määratud solitonide kompleksi tekketingimused.