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THEORETICAL AND PRACTICAL PROBLEMS OF HYGROTHERMALLY LOADED FIBRE REINFORCED COMPOSITES

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Abstract. Because composites are inhomogeneous and anisotropic, both of these properties can be used for tailoring their thermohygroelastic (THE) behaviour. The fundamental equations of THE fields are examined to gain insight into the potentials of tailoring. These fields can be shown to be coupled in dynamic cases. The construction of the target function for the tailoring is discussed and explicit forms for it are given in some simplified cases. Explicit results for balanced and symmetric laminates, consisting of commercially available plies, are also shown.

Key words: composites, tailoring, thermohygroelasticity.

1. INTRODUCTION

The examination of general coupled fields in composites gives hints of tailoring. Figure 1 shows a set of fields in continua and their possible combinations that are well known as coupled fields in physics and engineering. The overwhelming majority of scientific literature on composites mentions the heat and moisture effects as environmental ones (e.g., [1]), which is necessarily not valid. Generally, in the case of composites the elastic, thermal, and moisture concentration fields have the basic role in all the mechanical problems and are usually coupled. In dynamic cases the coupling between displacement and thermal fields is never negligible (see the Gough–Joule effect, or the elastothermodynamic damping [2]). In the following we always suppose this case and consider the problem as time dependent. The most often appearing couplings are the thermoelastic, hygroelastic, and thermohygroelastic (THE) ones. In special composites, e.g., in actuators, other couplings (piezoelectric, magnetostrictive) are frequent as well. The cross-coupling between thermal and moisture fields (Soret and Dufour effects) is also well known (e.g., [3]). When dealing with the tailoring of composites, their heterogeneous and microstructured nature must be taken into account. If the load and/or the variation of environmental influence is dynamic, then the analysis needs to be completed by the viewpoint of nonlinear dynamics. According to the above, theoretical and practical problems are discussed.

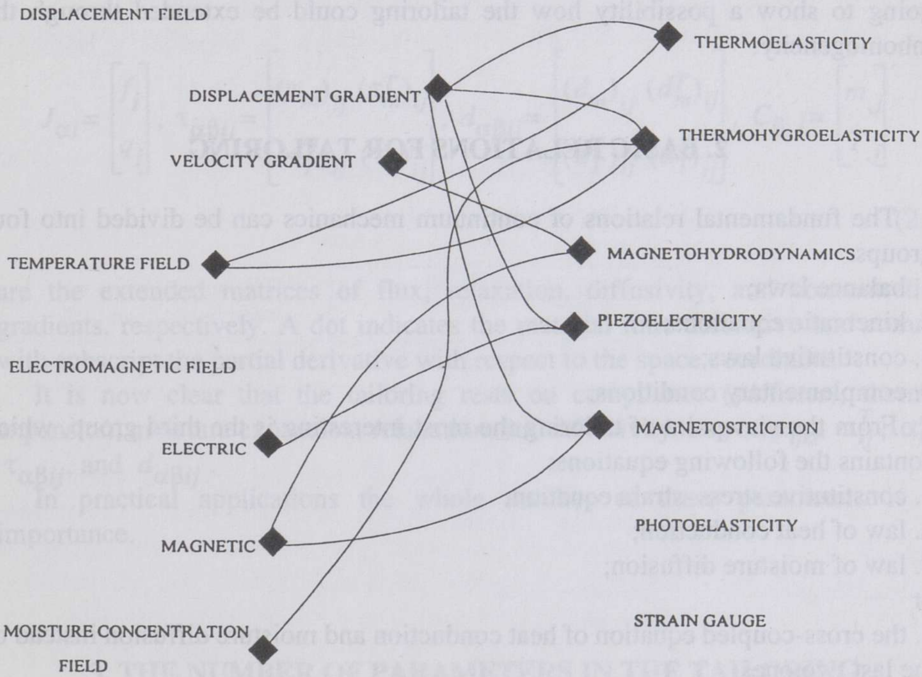

Fig. 1. Most frequent coupled fields of mechanics.

The heat conduction and moisture diffusion phenomena have a triple connection in case of composites:
1. total analogy between the Fourier and Fick laws;
2. cross-coupling described by the Dufour and Soret effects;
3. both can cause degradation and may lead to a failure.

On the other hand, the coupled handling of THE fields gives an extra opportunity for the tailoring. The purpose and possibility of tailoring are rather well known. The main target of tailoring is to avoid additional deformations or to cause just specified deformation due to given mechanical and environmental loads occurring in a specified way at a specified load ("fuse-effect"). These targets require optimally engineered expansion (thermal and moisture), stress distribution, temperature distribution, and moisture distribution, which in their turn require feasible or optimal directions and distributions of various thermomechanical properties.

Composite materials present a unique possibility to the designer because the applications of these material systems can be tailored or engineered to easily meet the particular performance requirements. This is possible due to the directional nature of the filamentary materials used as a reinforcement, as well as due to the wide variety of available fibre and matrix materials.

The conventional approach of tailoring is the use of the anisotropy. We are going to show a possibility how the tailoring could be extended through the inhomogeneity.

2. BASIC RELATIONS FOR TAILORING

The fundamental relations of continuum mechanics can be divided into four groups:
1. balance laws;
2. kinematic equations;
3. constitutive laws;
4. complementary conditions.

From the viewpoint of tailoring the most interesting is the third group, which contains the following equations:
1. constitutive stress–strain equation;
2. law of heat conduction;
3. law of moisture diffusion;
or
4. the cross-coupled equation of heat conduction and moisture diffusion instead of the last two ones.

We have to mention also the equation of state, but in our case it has a special role and we return to this question later.

Let us consider all of these equations in the most general form [4] but ignoring the temperature dependence of the material properties. Taking the constitutive equations of the anisotropic THE solids into account, we recall the general constitutive equation together with Fourier's heat conduction law and Fick's moisture transport relation

$$\varepsilon_{ij} = a_{ijkl}\sigma_{kl} + \alpha_{ij}^{T}\Delta T + \alpha_{ij}^{m}\Delta m, \tag{2.1}$$

$$q_i = -k_{ij}T_{,j}, \tag{2.2}$$

$$f_i = -(D_m)_{ij}m_{,j}. \tag{2.3}$$

Here $\varepsilon_{ij}, \sigma_{kl}, \Delta T, \Delta m, q_i, f_i$ denote the strain, stress, temperature, and moisture difference, heat and moisture flux, respectively, and a_{ijkl}, α_{ij}^T , α_{ij}^m , k_{ij}, and $(D_m)_{ij}$ are the space dependent material properties: compliance, coefficients of thermal and moisture expansion, thermal conductivity, and moisture diffusivity.

Instead of the two last equations, the coupled and modified relations based on the "second sound" phenomenon will be applied. For anisotropic THE materials

$$J_{\alpha i} + \tau_{\alpha\beta ij} \dot{J}_{\beta j} = -d_{\alpha\beta ij} C_{\beta, j} \tag{2.4}$$

with $\alpha, \beta = 1, 2$ and $i, j, k, l = 1, 2, 3$. Here

$$J_{\alpha i}= \begin{bmatrix} f_i \\ q_i \end{bmatrix}, \ \tau_{\alpha\beta ij}= \begin{bmatrix} (\tau_m)_{ij} & (\tau_m^T)_{ij} \\ (\tau_T^m)_{ij} & (\tau_T)_{ij} \end{bmatrix}, \ d_{\alpha\beta ij}= \begin{bmatrix} (d_m)_{ij} & (d_m^T)_{ij} \\ (d_T^m)_{ij} & (d_T)_{ij} \end{bmatrix}, \ C_{\beta, j}= \begin{bmatrix} m_{,j} \\ T_{,j} \end{bmatrix} \tag{2.5}$$

are the extended matrices of flux, relaxation, diffusivity, and concentration gradients, respectively. A dot indicates the material time derivative and comma with subscript the partial derivative with respect to the space coordinate.

It is now clear that the tailoring rests on compliance (stiffness), thermal expansion, moisture expansion, relaxation, and diffusivity, i.e., on a_{ijkl}, α_{ij}^T , α_{ij}^m , $\tau_{\alpha\beta ij}$, and $d_{\alpha\beta ij}$.

In practical applications the whole number of these parameters is of importance.

3. THE NUMBER OF PARAMETERS IN THE TAILORING

The compliance (stiffness) tensor has $3^4 = 81$ elements, but they are not independent of each other. On the basis of (2.1) one can reach the well-known lower number of stiffness parameters [5] that is briefly shown below.

The symmetry of the stress and strain tensors results in the symmetry in the indices $i \leftrightarrow j$ and $k \leftrightarrow l$, i.e.,

$$a_{ijkl} = a_{ijlk}, \ a_{ijkl} = a_{jikl}, \tag{3.1}$$

which reduces the number of parameters from 81 to 54 and then to 36.

Energy considerations require additional symmetries. From the definition of elastic potentials [5] it follows that

$$a_{ijkl} = a_{klij} \, . \tag{3.2}$$

This means that finally, in the case of a generally anisotropic material, the number of independent parameters is reduced from 36 to 21.

Generally the thermal and moisture expansion tensors have $3^2 = 9$ components. Due to the symmetry of the strain tensor, they are also symmetrical, i.e.,

$$\alpha_{ij}^{T} = \alpha_{ji}^{T} \text{ and } \alpha_{ij}^{m} = \alpha_{ji}^{m} . \tag{3.3}$$

This reduces the number of independent coefficients to only 6 in both tensors.

In special cases, e.g. general orthotropy, special orthotropy, and isotropy, further reductions are possible. These cases are shown in the table (see also [5]). The features of relaxation and diffusivity are similar to the above expansion tensors.

Number of independent coefficients

Material characteristic	Tensor: Compliance (stiffness) a_{ijkl} — Speciality	No.	Tensor: Thermal and moisture expansions α_{ij}^T and α_{ij}^m — Speciality	No.
General anisotropy		81		9
Anisotropy,	Stress tensor $\sigma_{ij} = \sigma_{ji}$	54		
considering symmetries	Strain tensor $\varepsilon_{ij} = \varepsilon_{ji}$	36	Strain tensor	6
	Elastic energy properties	21		
General orthotropy		9		3
Special orthotropy	Transverse isotropy	5		
	Plane stress	4		
Isotropy		2		1

A composite material can be defined as a combination of two or more materials (reinforcing elements, fillers, and composite matrix binder), differing in form or composition on a macroscale. The constituents retain their identities, i.e., they do not dissolve or merge completely into one another although they act together. Normally, the components can be physically identified and exhibit an interface between one another (e.g., glass and plastic in a glass fibre reinforced plastic). By varying the relative amounts of fibre and matrix or by varying the fibre direction one can try to match the coefficients to specified values in different parts of the composite material, i.e., tailor the material.

4. ANISOTROPY AND INHOMOGENEITY AS A POSSIBILITY FOR THE TAILORING

In the tailoring it is customary to use the anisotropy rather than the inhomogeneity. It is quite logical to ask what is the reason for such an approach in spite of the fact that in the case of composites it is possible to create not only anisotropic but also inhomogeneous properties. In addition, the difference between the anisotropy and inhomogeneity should be clearly stated. Briefly, both of them are space co-ordinate dependent properties, but the inhomogeneity is location dependent while the anisotropy is direction dependent. It must be clear how to express the difference between these two properties. In the case of heat conduction, for example, we have

$$q_i(\mathbf{r}, t) = -k_{ij}(\mathbf{r})T_{,j}(\mathbf{r}, t). \tag{4.1}$$

If

$k_{ij}(\mathbf{r}) = k_{ij}(\mathbf{r}_0) = k_{ij}^{\text{const}}$, then the material is homogeneous and anisotropic;

$k(\mathbf{r})\delta_{ij}$, then the material is inhomogeneous and isotropic;

$k_{ij}(\mathbf{r}_0)\delta_{ij} = \delta_{ij}k^{\text{const}}$, then the material is homogeneous and isotropic;

$k_{ij}(\mathbf{r})$, then the material is inhomogeneous and anisotropic.

Several definite material properties can be only inhomogeneous, never anisotropic. These properties include density, specific heat capacity, etc. Other properties can be both inhomogeneous and anisotropic, as for example stiffness, conductivity, etc. This difference depends on the character of the property. There are scalar properties, vector (first-order tensor) and tensor (second- and higher-order) ones. Scalar properties (zeroth-order tensors) couple scalar quantities and can not have direction (in the sense a vector has). Hence, the anisotropy is not possible. Properties which are second-order tensors (e.g., k_{ij}) couple vector quantities, i.e., first-order tensors (q_i and $T_{,j}$) to each other and can thus be different in different directions. Properties which are fourth-order tensors (e.g., a_{ijkl}) couple second-order tensors (strain and stress). Thus, also here the anisotropy is possible.

Let us approach the problem from the manufacturing point of view. The most important question is: Are we able to fabricate inhomogeneous materials or only anisotropic ones? Before answering we may try to reverse the question: Are we able to fabricate homogeneous material? The answer to this question is definitely "no". The only question is whether we are able to regulate or control the inhomogeneity. If the answer is "yes", then we have a possibility of tailoring on the basis of inhomogeneity too, not only on the basis of anisotropy.

The importance of the problem is obvious. If we can apply all the material parameters and their dependence both on direction and location to the tailoring, the opportunities are much wider.

No matter what is the specific manufacturing method, composite laminates consist in principle of unidirectional plies (layers) bonded together with pressure and heat. The plies used can be considered as generally orthotropic or transversely isotropic. They are also macroscopically homogeneous when unidirectional fibres are used as a reinforcement. All of the manufacturing processes allow us to vary the fibre direction of each ply. Hence, we can use also the ply angles as parameters in the tailoring. Moreover, by varying the fibre volume fraction from ply to ply we can easily create inhomogeneity in the thickness direction. Also, different fibre–matrix combinations can be used in different plies for this purpose.

In filament winding, for example, the fibre volume fraction and fibre direction can be varied continuously along the axial co-ordinate of a pipe or a cylinder.

5. THE TARGET FUNCTION OF TAILORING: CONSTRUCTION AND SOLUTION

Let us assume that prescription is given for displacement, strain, stress, temperature, and moisture distribution (or for some of them). That means,

$$\begin{aligned} \mathbf{u} &= \bar{\mathbf{u}}(\mathbf{r}), \\ \varepsilon &= \bar{\varepsilon}(\mathbf{r}), \\ \sigma &= \bar{\sigma}(\mathbf{r}), \\ T &= \bar{T}(\mathbf{r}), \\ m &= \bar{m}(\mathbf{r}) \end{aligned} \tag{5.1}$$

are given everywhere in the domain or, in addition to the boundary conditions, their values are specified somehow at certain points inside the domain. The specification may be that they obtain a prescribed value or a minimum/maximum value. The overbar emphasizes that the right-hand sides are just given functions of space co-ordinates.

The target function in its most general form is a function

$$F = F(\|\mathbf{u} - \bar{\mathbf{u}}\|, \|\varepsilon - \bar{\varepsilon}\|, \|\sigma - \bar{\sigma}\|, \|T - \bar{T}\|, \|m - \bar{m}\|), \tag{5.2}$$

the minimization of which guarantees that all the arguments are at their minimums with a certain distribution of the material properties

$$c_{ijkl}, \alpha_{ij}^{T}, \alpha_{ij}^{m}, \tau_{\alpha\beta ij}, d_{\alpha\beta ij} \tag{5.3}$$

and with a certain combination of ply angles. It should be noted, however, that the requirements to minimize all the arguments at the same time can be contradictory. In a pure thermal conduction problem we can, for example, simply choose

$$F = \|T - \bar{T}\| \tag{5.4}$$

and the design variable can be the distribution of the tensor k_{ij}, see [6]. Moreover, if only a property at a point is considered, we can use the absolute value instead of a norm in Eq. (5.4). In some cases, e.g., considering the coefficient of thermal expansion (CTE) in a balanced symmetric $[\theta/-\theta]_{ns}$ laminate (see [7]), an explicit expression can be found for the property in question in terms of the ply properties and the ply angles. Thus, the range in which the tailoring is possible can be studied easily.

For more complicated problems, constrained minimization methods must be used as described in [8–10]. The choice of the solution method depends not only on the type and content of the target function, but also on the computational resources available.

A typical feature, from the mathematical point of view, of the tailoring problem is that there can be either none, only one or several solutions as will be explained by the examples of the next section.

6. APPLICATIONS

Let us consider first the CTE of a balanced symmetric $[\theta/-\theta]_{ns}$ laminate manufactured from unidirectional specially orthotropic plies. Because the laminate is thin, we can assume that it is in a state of plane stress as are all of its plies. Hence, according to the table, seven parameters can be used for tailoring: the elastic constants of the ply (E_{11}, E_{22}, G_{12}, ν_{12}) and the CTEs of the ply (α_{11} and α_{22}), which can be varied by varying the fibre volume fraction, plus θ. The CTEs of the laminate in x- and y-directions depend on all these parameters. Of course, when θ equals zero and 90°, the CTEs in x-direction equal α_{11} and α_{22}, respectively. Some results calculated by using the expressions developed in [7] are shown in a dimensionless form in Fig. 2. As we can see from the graphs, we can tailor the CTE to a value smaller than the smallest of the individual ply and also to a value larger than the largest of the individual ply used.

Figures 3, 4, and 5 illustrate the dependence of the CTE, coefficient of moisture expansion (CME), and coefficient of thermal/moisture expansion (CT/ME) on the ply angle when some commercially available plies are used, i.e., ply properties are fixed and the only tailoring parameter is the ply angle. However, we see that the range of possibilities for tailoring is much broader than the engineering intuition would suggest, e.g., laminates having negative values of the CME can be created. In this kind of simplified tailoring one just chooses the ply angle corresponding to the specified value of the coefficient.

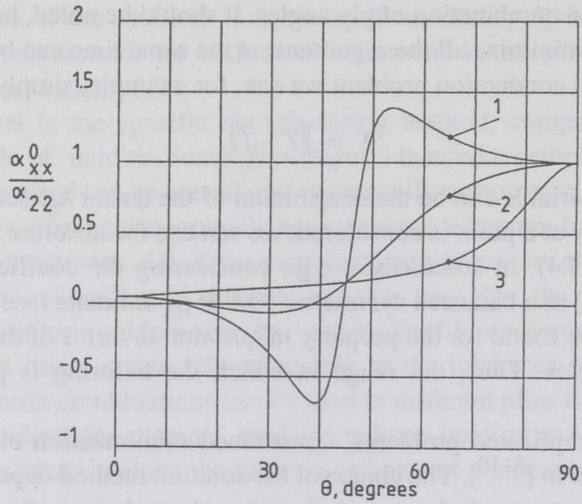

Fig. 2. Dimensionless longitudinal coefficient of thermal expansion of a $[\theta/-\theta]_{ns}$ laminate as a function of ply angle. Ply properties: $E_{11}/E_{22} = 10$, $\nu_{12} = 0.25$, $\alpha_{11}/\alpha_{22} = 0.1$; $G_{12}/E_{22} = 0.05, 0.5$, and 4 for curves 1, 2, and 3, respectively.

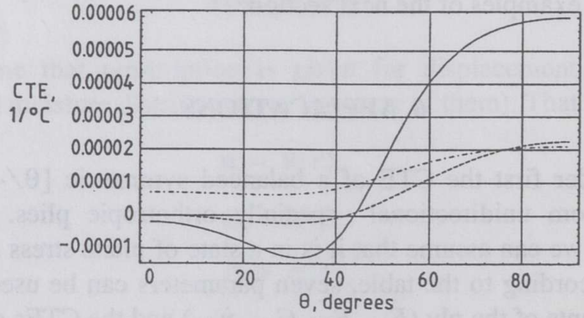

Fig. 3. Longitudinal coefficient of thermal expansion (CTE) as a function of ply angle for three $[\theta/-\theta]_{ns}$ laminates consisting of different commercial ply materials: Kevlar-epoxy (solid line), graphite-epoxy (dashed line), and E-glass-epoxy (dash-dot line).

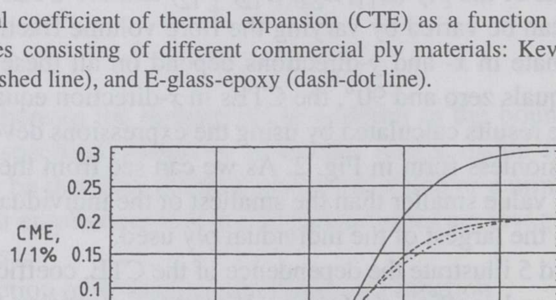

Fig. 4. Longitudinal coefficient of moisture expansion (CME) as a function of ply angle for three $[\theta/-\theta]_{ns}$ laminates consisting of different ply materials: Kevlar-epoxy (solid line), graphite-epoxy (dashed line), and E-glass-epoxy (dash-dot line).

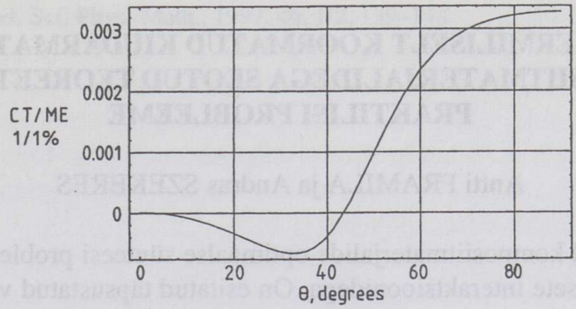

Fig. 5. Longitudinal coefficient of thermal/moisture expansion (CT/ME) as a function of ply angle for a $[\theta/-\theta]_{ns}$ graphite-epoxy laminate.

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REFERENCES

1. Tsai, S. W. *Composites Design*. Think Composites. Dayton, USA, 1987.
2. Milligan, K. B. and Kinra, V. K. Elastothermodynamic damping of fiber-reinforced metal-matrix composites. *J. Appl. Mech.*, 1995, **62**, 441–449.
3. Szekeres, A. and Engelbrecht, J. *Coupled Thermal and Moisture Fields with Application to Composites. Research Report Mech. 108/94*. Inst. of Cybernetics, Estonian Acad. Sci., Tallinn, 1994.
4. Szekeres, A. and Heller, R. Basic equations of hygro-thermo-mechanical materials and applications to composites. In *Pres. held at 2nd Thermal Structures Conference, October 18–20, 1994*. University of Virginia, Charlottesville, Virginia, 1994.
5. Daniel, I. M. and Ishai, O. *Engineering Mechanics of Composite Materials*. Oxford University Press, New York, 1994.
6. Autio, M. and Pramila, A. Inverse problems in thermoelasticity of composites. In *Proc. of 15th Canadian Congress of Applied Mechanics, May 28–June 2, 1995* (Tabarrok, B. and Dost, S., eds.). Victoria, B.C., 1995, 282–283.
7. Pramila, A. Extrema and zeros of coefficients of thermal expansion of a balanced symmetric laminate. *J. Composite Materials*, 1990, **24**, 786–794.
8. Pramila, A. Extrema of some thermoelastic properties of symmetric laminates. *Period. Polytech. Mech. Engng.*, 1992, **36**, 125–134.
9. Autio, M., Laitinen, M., and Pramila, A. Systematic creation of composite structures with prescribed thermomechanical properties. *Composites Engineering*, 1993, **3**, 249–259.
10. Laitinen, M., Pramila, A., and Szekeres, A. Extrema of coefficients of thermal/moisture expansion for thermo-hygro-elastic composites. In *32nd Annual Technical Meeting of the Society of Engineering Science, 29 Oct.–1 Nov., 1995* (Hui, D. and Michaelidis, S., eds.). New Orleans, 1995, 299–300.

HÜGROTERMILISELT KOORMATUD KIUDARMATUURIGA KOMPOSIITMATERJALIDEGA SEOTUD TEOREETILISI JA PRAKTILISI PROBLEEME

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On käsitletud komposiitmaterjalide optimaalse sünteesi probleeme, arvestades termohügroelastsete interaktsioonidega. On esitatud täpsustatud võrrandid termohügroefekti arvutamiseks ja näidatud, kuidas kiudarmatuuri suuna muutmisega saab sünteesida optimaalseid materjale.