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ON MEAN-FIELD FREE ENERGY AND ORDER PARAMETERS OF AN INTERBAND PAIR-TRANSFER SUPERCONDUCTOR

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Abstract. It is shown that the phase transition in a two-band pair-transfer superconductor can be described by one order parameter appearing as a linear combination of coupled order parameters associated with interacting bands. The corresponding effective one-variable free energy is found.

Key words: interband model, superconducting order parameters, free energy, relaxation times.

A two-band model of superconductivity where the superconducting phase transition is caused or at least favoured by the interband pair-transfer interaction was introduced in [1–4]. After the discovery of high-T_c superconductivity, this trend in theory gained in importance (see for review [5]). In an interband pair-transfer model the superconducting phase is described by two coupled order parameters, the equilibrium values of which turn simultaneously to zero as temperature rises and passes through T_c. The mean-field free energy, which depends on these order parameters as on independent variables, was derived in [6]. In the present note it is shown that at least the equilibrium properties of the model under consideration can be described by an effective free energy depending on only one order parameter. Such approach means that we neglect the fast relaxing deviations from the equilibrium state of the system. The effective one-variable free energy is shown to be of canonical form of the Landau expansion for a second-order phase transition.

In the vicinity of the superconducting transition the expansion of the mean-field free energy is powers of homogeneous order parameters and $T - T_c$ for a two-band superconductor where the phase transition is caused by the interband pair-transfer interaction has the following form [6]:

$$F = F_0 + F_2 + F_4 \ , \tag{1}$$

$$F_2 = |W|\left\{\sum_\sigma (\eta_\sigma - A_\sigma\Theta)|\lambda_\sigma|^2 + \frac{w}{2}[4-(\eta_1 A_2 + \eta_2 A_1)\Theta](\lambda_1\lambda_2^* + \lambda_1^*\lambda_2)\right\} , \tag{2}$$

$$F_4 = -|W|\left\{\sum_\sigma \frac{3}{2}\nu_\sigma|\lambda_\sigma|^4 + \frac{w}{2}(\nu_1\eta_2|\lambda_1|^2 + \nu_2\eta_1|\lambda_2|^2)(\lambda_1\lambda_2^* + \lambda_1^*\lambda_2)\right\} \tag{3}$$

with

$$\eta_\sigma = 2|W|\sum_{\vec{k}} \tilde{\epsilon}_\sigma^{-1}(\vec{k})th\frac{\tilde{\epsilon}_\sigma(\vec{k})}{2k_BT_c} , \tag{4}$$

$$A_\sigma = \frac{|W|}{k_BT_c}\sum_{\vec{k}} ch^{-2}\frac{\tilde{\epsilon}_\sigma(\vec{k})}{2k_BT_c} , \tag{5}$$

$$\nu_\sigma = 4|W|^3\sum_{\vec{k}}\left[\frac{1}{\tilde{\epsilon}^3(\vec{k})}th\frac{\tilde{\epsilon}_\sigma(\vec{k})}{2k_BT_c} - \frac{1}{2k_BT_c\tilde{\epsilon}_\sigma^2(\vec{k})}ch^{-2}\frac{\tilde{\epsilon}_\sigma(\vec{k})}{2k_BT_c}\right] , \tag{6}$$

and $\Theta = (T-T_c)/T_c$, $w = W/|W|$, $\tilde{\epsilon}_\sigma(\vec{k}) = \epsilon_\sigma(\vec{k}) - \mu$. Here $\tilde{\epsilon}_\sigma(\vec{k})$ is the energy of an electron in the band numbered by σ ($\sigma = 1, 2$); μ is the chemical potential; W is the constant of interband pair-transfer coupling. The variables $\lambda_{1,2}$ are superconducting order parameters of the bands $\epsilon_{1,2}$; F_0 is independent of $\lambda_{1,2}$. In [6], the free energy (1)–(3) was derived as a function of variables $\delta_1 = -2W\lambda_1$ and $\delta_2 = 2W\lambda_2$. The equilibrium values of $\lambda_{1,2}$ minimizing the mean-field free energy are given self-consistently by the quantities $\sum_{\vec{k}} < a_{2,1-\vec{k}\downarrow}a_{2,1\vec{k}\uparrow} >$ [6], where $a_{\sigma\vec{k}s}$ is the annihilation operator of an electron. The temperature of superconducting phase transition T_c is determined in the present model by the equation

$$\eta_1\eta_2 = 4 \ . \tag{7}$$

It must be pointed out that the approximate free energy (1)–(3) reveals instability in case of too large values of $\lambda_{1,2}$ (except a special region of the space of order parameters). This formal problem arises in connection of the circumstance that the expansion of the exact mean-field free energy (the stability of which was proved in [6]) has been interrupted at the fourth-order terms.

The second-order phase transition into the superconducting state described by the free energy (1)–(3) takes place for the both signs of w. In case $w = +1$, the difference of the phases of nonzero equilibrium values of λ_1 and λ_2 determining the superconducting state equals to π, and in case $w = -1$ these phases coincide [4,7]. In the pseudospin formalism the superconducting state for $w = +1$ corresponds to an antiferromagnetic-type ordering and for $w = -1$, to a ferromagnetic-type ordering of pseudospins in the subspace of electron pairs [8].

The free energy (1)–(3) differs significantly from usual Landau-type expansions. First, the coefficients before $|\lambda_\sigma|^2$ do not change the sign if temperature passes T_c. In addition, the second-order contribution F_2 into the free energy includes the products ($\lambda_1\lambda_2^*$ and $\lambda_1^*\lambda_2$) of order parameters of different bands. Such products are included also in the fourth-order contribution F_4, however, just the interaction between λ_1 and λ_2 in F_2 leads to the phase transition in the present model. In some aspects the structure of the free energy (1)–(3) resembles the free energy in the Kittel model of antiferroelectricity [9].

One can diagonalize the quadratic form F_2 by means of an orthogonal transformation:

$$\begin{cases} \lambda_1 &= \lambda_r \cos\varphi + \lambda_s \sin\varphi \\ \lambda_2 &= -\lambda_r \sin\varphi + \lambda_s \cos\varphi \end{cases} \tag{8}$$

with

$$\tan\varphi = \frac{1}{2} w\eta_2 \left[-1 + (4(\eta_1+\eta_2))^{-1}(\eta_1^2 A_2 - \eta_2^2 A_1)\Theta\right], \tag{9}$$

where $\tan\varphi$ has already been expanded into the series in powers of Θ, taking only the lowest-order correction into account. The change of variables (8) replaces $\lambda_{1,2}$ with $\lambda_{s,r}$ and, as a result, one obtains F_2 in the following form (up to the terms proportional to Θ):

$$F_2 = |W|[\omega_s|\lambda_s|^2 + \omega_r|\lambda_r|^2], \tag{10}$$

$$\omega_s = (\eta_1+\eta_2)^{-1}(\eta_1 A_2 + \eta_2 A_1)\Theta, \tag{11}$$

$$\omega_r = \eta_1 + \eta_2 - (\eta_1+\eta_2)^{-1}[(2\eta_1+\eta_2)A_2 + (2\eta_2+\eta_1)A_1]\Theta, \tag{12}$$

where now the new variables λ_s and λ_r are separated. In Eq. (10), the coefficient ω_s decreases as T decreases and changes its sign if temperature passes T_c. The coefficient ω_r remains positive and its temperature dependence (ω_r increases as T decreases) is rather unessential.

The nonzero equilibrium values $\overline{\lambda_{1,2}}$ of preliminary order parameters corresponding to the minima of the free energy (1)–(3) if $T < T_c$ are given by the expressions [6]

$$|\overline{\lambda_{1,2}}|^2 = -\eta_{2,1}\Xi\Theta \ , \tag{13}$$

$$\Xi = \frac{\eta_1 A_2 + \eta_2 A_1}{\eta_1^2 \nu_2 + \eta_2^2 \nu_1} \ . \tag{14}$$

By applying the change of variables (8) to Eq. (13), one obtains the expected results for the equilibrium values of $\lambda_{s,r}$ if $T < T_c$:

$$|\overline{\lambda_s}|^2 = -(\eta_1 + \eta_2)\Xi\Theta = |\overline{\lambda_1}|^2 + |\overline{\lambda_2}|^2 \ , \tag{15}$$

$$|\overline{\lambda_r}|^2 = 0 \ . \tag{16}$$

For $T > T_c$, $\overline{\lambda_s} = \overline{\lambda_r} = 0$. Thus, in terms of new variables, only λ_s appears in the role of an order parameter for the phase transition under consideration.

The normal-phase relaxational dynamics of the system is determined by the following linear equations

$$\Gamma_s \frac{\partial \lambda_s}{\partial t} = -\frac{\partial F_2}{\partial \lambda_s^*} = -|W|\omega_s \lambda_s, \tag{17}$$

$$\Gamma_r \frac{\partial \lambda_r}{\partial t} = -\frac{\partial F_2}{\partial \lambda_r^*} = -|W|\omega_r \lambda_r \tag{18}$$

with the dissipation coefficients $\Gamma_s, \Gamma_r > 0$. By using $\lambda_{s,r}(t) = \lambda_{s,r}(0)\exp(-t/\tau_{s,r})$, we obtain from Eqs. (17) and (18) two relaxation times

$$\tau_s^{-1} = |W|\Gamma_s^{-1}\omega_s, \tag{19}$$

$$\tau_r^{-1} = |W|\Gamma_r^{-1}\omega_r, \tag{20}$$

one (τ_s) exhibiting a critical slow-down near T_c, the other (τ_r) remaining finite. In fact, the characteristic times $\tau_{s,r}$ correspond to different timescales as $T \to T_c$, i.e., $\tau_s >> \tau_r$.

Whereas the equilibrium properties of the model have been determined by the "soft" variable λ_s and the dependence of F on the "rigid" variable λ_r is obviously unimportant in this respect, it is quite natural to introduce an effective mean-field free energy depending on only one argument, λ_s. Also, in this way we exclude from consideration such deviations of order parameters $\lambda_{1,2}$ from the equilibrium position which are characterized by a small relaxation time. By using Eq. (8) together with the condition $\lambda_r = 0$, i.e., replacing λ_r by its equilibrium value, one finds on the basis of Eqs. (3) and (10) that the restricted free energy $\tilde{F}$ equals to

$$\tilde{F} = F_0 + a\Theta|\lambda_s|^2 + b|\lambda_s|^4 \tag{21}$$

with positive coefficients a and b:

$$a = |W|\frac{\eta_1 A_2 + \eta_2 A_1}{\eta_1 + \eta_2}, \tag{22}$$

$$b = |W|\frac{\eta_1^2 \nu_2 + \eta_2^2 \nu_1}{2(\eta_1 + \eta_2)^2}. \tag{23}$$

The expression (21) is a standard form of free energy expansion for a second-order phase transition. The minimization of the free energy (21) leads at $T < T_c$ to the expression (15) for $\overline{\lambda}_s$. The equilibrium thermodynamic quantities near T_c for the present model obtained in [10] on the basis of the free energy as a function of two order parameters follow naturally also from Eq. (21).

The effective free energy (21) is obtained in connection with the condition $\lambda_r = 0$. The latter together with the transformation (8) leads to the relations $\lambda_1 = \lambda_2 \tan\varphi$ and $|\lambda_s|^2 = |\lambda_1|^2 + |\lambda_2|^2$. By means of these relations one can express the effective free energy (21) also as a function of a variable λ_1 or λ_2.

Note that the formal problem in connection with too large values of $\lambda_{1,2}$ in Eqs. (1)–(3) mentioned above has been removed in Eq. (21), whereas the instability of the free energy (1)–(3) is connected just with nonzero values of λ_r omitted in Eq. (21) as unessential. In fact, the introduction of the effective free energy (21) means that we have restricted the space of order parameters $\lambda_{1,2}$ to the region where the free energy expansion (1)–(3) is stable for arbitrary values of λ_1 and λ_2. From Eq. (10) it is obvious that only the quadratic means of fluctuations of λ_s diverge near T_c. Consequently, the exclusion of λ_r from the free energy (21) is justified also in respect to critical thermodynamic fluctuations. The effects of fluctuations in the present model with the account of space-inhomogeneity of λ_s and λ_r will be considered elsewhere.

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TSOONIDEVAHELISE PAARIÜLEKANDEGA ÜLIJUHI VABAENERGIAST JA KORRASTUSPARAMEETRITEST

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On näidatud, et faasisiiret kahetsoonilises paariülekandega ülijuhis saab kirjeldada kasutades ühte korrastusparameetrit, mis kujutab endast tsoonidega seotud paardunud korrastusparameetrite lineaarkombinatsiooni. On leitud vastav ühe muutujaga efektiivne vabaenergia.