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SOLITARY WAVES AND INTEGRABILITY IN THE RESONANT SYSTEM OF LONG AND SHORT WAVES

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Abstract. In dispersive media, the interaction between long and short waves can be strong when the resonance condition is satisfied between the group velocity of the short wave and the phase velocity of the long wave. In this article, we deal with such a resonant system, which is described by the coupled system of the Korteweg–de Vries equation and nonlinear Schrödinger equation. One-soliton solutions are found systematically by means of the modified Hirota method. It is shown that there exist a variety of numerical solutions including the oscillatory solitary waves. In connection with these solutions, the integrability of the equation in the sense of the Lyapunov exponent and the Painlevé property is discussed.

Key words: wave interaction, soliton, oscillatory solitary wave, integrability, Lyapunov exponent, Painlevé property.

1. INTRODUCTION

At two or more different wave modes coexisting in dispersive media, nonlinear wave interactions are known to play an important role in energy exchange between (or among) these wave modes. In this article, we deal with the interaction of the long and short waves that can be strong if the resonance condition is satisfied with respect to the wave velocities.

This interaction is treated as a special case of the three-wave interaction, when we consider three waves: a single long wave $(\Delta\mathbf{k},\ \omega(\Delta k))$ and two short waves, $(\mathbf{k}+\Delta\mathbf{k}/2,\ \omega(k+\Delta k/2))$ and $(\mathbf{k}-\Delta\mathbf{k}/2,\ \omega(k-\Delta k/2))$,

where the wave frequency ω is assumed to be a function of the wave number $k = |\mathbf{k}|$ and $\Delta k \ll k$ [1]. Since the resonance condition for the above three waves is given as

$$\omega(\Delta k) = \omega\left(k + \frac{\Delta k}{2}\right) - \omega\left(k - \frac{\Delta k}{2}\right), \tag{1}$$

the following resonance condition is obtained for the long and short waves:

$$\Delta\mathbf{k} \cdot \left.\frac{\partial\omega}{\partial\mathbf{k}}\right|_{\mathbf{k}} \simeq \omega(\Delta k), \tag{2}$$

where ω is assumed to be small for Δk in the dispersion relation $\omega = \omega(k)$. Since the phase velocity of the long wave is given by $\mathbf{v}_p = \omega(\Delta k)\Delta\mathbf{k}/\Delta k^2$ and the group velocity of the short wave $\mathbf{v}_g = (\partial\omega/\partial\mathbf{k})|_{\mathbf{k}}$, the resonance condition (2) is rewritten as

$$\mathbf{v}_p \cdot \mathbf{v}_g \simeq v_p^2. \tag{3}$$

Therefore, as is seen from the equivalent representation $v_g \cos\psi \simeq v_p$ to (3), the interaction is possible between the long and short waves propagating in different directions under an angle ψ. In particular, this condition is simplified to the one that the group velocity of the short wave is nearly equal to the phase velocity of the long wave ($v_g \sim v_p$) when both waves propagate in the same direction.

This resonance condition can be satisfied in water waves, plasma waves, and others in dispersive media, and several nonlinear interaction equations which describe the wave behaviour in a long time run have been proposed [1−6]. In the following, we will focus on the interaction equation that is given by the coupled Korteweg–de Vries and nonlinear Schrödinger equations, when both long and short waves propagate in the same direction.

2. INTERACTION EQUATION

Using the singular perturbation method under the assumption of weak nonlinearity, the following interaction equation can be obtained in the normalized form with two control parameters α and β[7]:

$$\mathrm{i}\frac{\partial S}{\partial t} \pm \frac{\partial^2 S}{\partial x^2} = SL, \quad \frac{\partial L}{\partial t} + \alpha\frac{\partial L}{\partial x} + \beta\frac{\partial^3 L}{\partial x^3} = \frac{\partial |S|^2}{\partial x}, \tag{4}$$

where S and L denote, respectively, the complex amplitude of the envelope of the short wave and the real long wave, while x and t are spatial and temporal coordinates in a frame of reference, moving with the phase velocity of

the long wave or with the group velocity of the short wave. The parameters α and β and the alternative of the $\pm$ signs in front of $\partial^2 S/\partial x^2$ depend upon the individual properties of the waves and the media concerned: the gravity and capillary waves in a single-layer fluid ($\alpha, \beta \leq 0$ and $+$ sign) [8,9], the gravity waves in a two-layer fluid ($\beta \leq 0$ and $-$ sign) [10−12], the ion acoustic and electron plasma waves ($\alpha \geq 0$, $\beta \leq 0$ and $+$ sign) [4,13], and so on [14−16]. However, since the case of $-$ sign can be obtained if t, L and β in Eq. (4) are replaced by $-t$, $-L$ and $-\beta$, in the following we will consider only the case of $+$ sign.

When both parameters α and β are equal to zero, Eq. (4) represents the case when the magnitude of the long wave is much less than that of the short wave ($|L| \ll |S|$) [9,16]. For this case, the equation is proved to be integrable or to have the n-soliton solution by means of the inverse scattering transform (IST) method [17,18]. On the other hand, when both α and β have finite values, Eq. (4) represents the case for which the magnitudes of the long and short waves are of the same order ($|L| \sim |S|$) [8,16]. It is expected for this case that in the long time run the asymptotic behaviour of waves may become chaotic since Eq. (4) for $\beta = 1$ is shown to be non-integrable by means of the IST [19], though a few solitary wave solutions have been analytically obtained [8,11,13,20]. However, the chaotic wave behaviour is not observed for the nearly integrable case, that is, in the vicinity of $\alpha = \beta = 0$, as far as continuous wave trains are considered [7].

In this article, we deal mainly with the solitary waves and their connections with the integrability of Eq. (4) in the (α, β) parameter space. Therefore, the soliton solutions are first examined and compared with the numerical solutions. Then they are discussed from the viewpoint of the integrability in the sense of the Lyapunov exponent and the Painlevé property.

3. HIROTA METHOD

3.1. Hirota binary form

It is convenient to use the modified Hirota method [21] to obtain systematically the soliton solutions of Eq. (4). For this end, S and L are assumed in the following forms in terms of F, G and Q:

$$S = \frac{G}{F^a}, \qquad L = \frac{Q}{F}, \tag{5}$$

$G(x,t)$ is assumed to be complex, while $F(x,t)$ and $Q(x,t)$ are real. Substituting the expressions (5) into the first equation in Eq. (4), we obtain

$$\frac{1}{F^{a+1}}[(\mathrm{i}\,\mathrm{D}_{\mathrm{a,t}} + \mathrm{D}^2_{\mathrm{a,x}})G \cdot F] = \frac{G}{F^a}\left[a(a+1)\frac{\mathrm{D}^2_{\mathrm{1,x}} F \cdot F}{2F^2} + \frac{Q}{F}\right].$$

On the other hand, the second equation in Eq. (4) is rewritten as

$$\frac{\partial W}{\partial t}+\frac{\alpha}{2}\Big(\frac{\partial W}{\partial x}\Big)^2+\beta\frac{\partial^3 W}{\partial x^3}=|S|^2-C^2,$$

where $L=\partial W/\partial x$ and C is an integral constant which satisfies the boundary condition $L\to 0$ and $|S|\to C$ as $|x|\to\infty$. Substituting (5) into the above equation, gives

$$\frac{-a(a+1)}{2F^2}[((1+\nu)\mathrm{D}_{1,\mathrm{x}}\mathrm{D}_{1,\mathrm{t}}+\beta\mathrm{D}^4_{1,\mathrm{x}})F\cdot F]+$$

$$+\frac{a(a+1)}{2F^4}\Big[\Big(\frac{a(a+1)}{4}\alpha+3\beta\Big)(\mathrm{D}^2_{1,\mathrm{x}}F\cdot F)^2+\nu F^2\mathrm{D}_{1,\mathrm{x}}\mathrm{D}_{1,\mathrm{t}}F\cdot F\Big]=$$

$$=\frac{GG^*}{F^{2a}}-C^2,$$

where the parameter ν is introduced to be determined later. Decoupling these resulting equations for F and G, the following two significant cases of the Hirota binary equations are found to be possible for $a=1$ and $a=2$:

(i) $a=1$: $(\alpha+6\beta=0)$

$$(\mathrm{i}\,\mathrm{D}_{\mathrm{a,t}}+\mathrm{D}^2_{\mathrm{a,x}})G\cdot F=0,$$

$$(\mathrm{D}_{1,\mathrm{x}}\mathrm{D}_{1,\mathrm{t}}+\beta\mathrm{D}^4_{1,\mathrm{x}})F\cdot F=\frac{2}{a(a+1)}(C^2F^2-F^{2-2a}GG^*),\qquad(6)$$

(ii) $a=2$: $(\alpha+2\beta\neq 0)$

$$(\mathrm{i}\,\mathrm{D}_{\mathrm{a,t}}+\mathrm{D}^2_{\mathrm{a,x}})G\cdot F=0,$$

$$[(1+\nu)\mathrm{D}_{1,\mathrm{x}}\mathrm{D}_{1,\mathrm{t}}+\beta\mathrm{D}^4_{1,\mathrm{x}}]F\cdot F=0,$$

$$\Big[\frac{a(a+1)}{4}\alpha+3\beta\Big](\mathrm{D}_{1,\mathrm{x}}{}^2F\cdot F)^2+\qquad(7)$$

$$+\nu F^2\mathrm{D}_{1,\mathrm{x}}\mathrm{D}_{1,\mathrm{t}}F\cdot F=\frac{2}{a(a+1)}(C^2F^4-F^{4-2a}GG^*).$$

In the above expressions, the binary operator D is defined as

$$\mathrm{D}^{\mathrm{n}}_{\mathrm{a,x}}\mathrm{D}^{\mathrm{m}}_{\mathrm{b,t}}F\cdot G\equiv\Big(\frac{\partial}{\partial x}-a\frac{\partial}{\partial x'}\Big)^n\Big(\frac{\partial}{\partial t}-b\frac{\partial}{\partial t'}\Big)^m F(x,t)G(x',t')\Big|_{x=x',t=t'}.$$

Once (6) or (7) is solved for F and G, then L is given by $L = Q/F = -a(a+1)\partial^2 \log F/\partial x^2$ in terms of F, since $\mathrm{D}^2_{1,x} F \cdot F = 2F^2 \partial^2 \log F/\partial x^2$ and Q is given as $-a(a+1)\mathrm{D}^2_{1,x} F \cdot F/(2F)$.

3.2. Soliton solutions

When we choose F and G as exponential functions of x and t, depending upon (I)$C = 0$ or (II)$C \neq 0$, the following one-soliton solutions are found in both cases (i) and (ii):

(I-A) $C = 0$ and $a = 1$: $(\alpha + 6\beta = 0)$

$$S = p\sqrt{2(\lambda - 4\beta p^2)}\,\mathrm{sech}(p(x - \lambda t))\exp\Big(\mathrm{i}\,\frac{\lambda}{2}(x - Vt)\Big),$$

$$L = -2p^2\,\mathrm{sech}^2(p(x - \lambda t)), \tag{8}$$

where $F = 1 + \exp[2p(x - \lambda t)]$, $G = p\sqrt{2(\lambda - 4\beta p^2)}\exp(p(x - \lambda t))\exp(\mathrm{i}(\lambda/2)(x - Vt))$ in (6), and $V = \lambda/2 - 2p^2/\lambda$ (p, λ and β: arbitrary).

(I-B) $C = 0$, $a = 2$ and $\nu = -3p^2(\alpha + 2\beta)$: $(\alpha + 2\beta \neq 0)$

$$S = 3p^2\sqrt{-2(\alpha + 2\beta)}\,\mathrm{sech}(p(x - \lambda t))\tanh(p(x - \lambda t))$$

$$\exp\Big(\mathrm{i}\,\frac{\lambda}{2}(x - Vt)\Big), \tag{9}$$

$$L = -6p^2\,\mathrm{sech}^2(p(x - \lambda t)),$$

where $F = 1 + \exp(2p(x - \lambda t))$, $G = 6p^2\sqrt{-2(\alpha + 2\beta)}\exp(p(x - \lambda t))[1 - \exp(p(x - \lambda t))]\exp(\mathrm{i}(\lambda/2)(x - Vt))$ in (7), and $\lambda = (3\alpha + 2\beta)p^2$ and $V = \lambda/2 + 2/(3\alpha + 2\beta)$ (α, β and p: arbitrary).

(I-C) $C = 0$, $a = 2$ and $\nu = 0$: $(\alpha + 2\beta \neq 0)$

$$S = 3p^2\sqrt{2(\alpha + 2\beta)}\,\mathrm{sech}^2(p(x - \lambda t))\exp\Big(\mathrm{i}\,\frac{\lambda}{2}(x - Vt)\Big),$$

$$L = -6p^2\,\mathrm{sech}^2(p(x - \lambda t)), \tag{10}$$

where $F = 1 + \exp(2p(x - \lambda t))$, $G = 12p^2\sqrt{2(\alpha + 2\beta)}\exp(2p(x - \lambda t))\exp(\mathrm{i}(\lambda/2)(x - Vt))$ in (7) and $\lambda = 4\beta p^2$ and $V = \lambda/2 - 2/\beta$ (α, β and p: arbitrary).

(II-A) $C \neq 0$ and $a = 1$: $(\alpha + 6\beta = 0)$

$$S = C\tanh(p(x - \lambda t))\exp\Big(\mathrm{i}\,\frac{\lambda}{2}(x - \frac{\lambda}{2}t)\Big),$$

$$L = -2p^2 \operatorname{sech}^2(p(x - \lambda t)), \tag{11}$$

where $F = 1 + \exp(2p(x - \lambda t))$, $G = C[1 - \exp(2p(x - \lambda t))] \exp(\mathrm{i}(\lambda/2)(x - \lambda t/2))$ in (6), and $C^2 = 2p^2(4\beta p^2 - \lambda)$ (β, λ and p: arbitrary).

(II-B) $C \neq 0$, $a = 2$ and $\nu = -4p^2(\alpha + 2\beta)$: $(\alpha + 2\beta \neq 0)$

$$S = C\left[1 - \frac{3}{2}\operatorname{sech}^2(p(x - \lambda t))\right] \exp\left(\mathrm{i}\,\frac{\lambda}{2}\left(x - \frac{\lambda}{2}t\right)\right),$$

$$L = -6p^2 \operatorname{sech}^2(p(x - \lambda t)), \tag{12}$$

where $F = 1 + \exp(2p(x - \lambda t))$, $G = C[1 - 4\exp(2p(x - \lambda t)) + \exp(4p(x - \lambda t))] \exp(\mathrm{i}(\lambda/2)(x - \lambda t/2))$ in (7), and $C^2 = 8p^4(\alpha + 2\beta)$ and $\lambda = -4p^2(\alpha + \beta)$ (α, β and p: arbitrary).

The above one-soliton solutions are equivalent to those obtained in the reduced ordinary differential equation (ODE) from Eq. (4) through the following travelling-wave transformation:

$$S = f(\zeta) \exp\left(\mathrm{i}\,\frac{\lambda}{2}(x - Vt)\right), \quad L = g(\zeta), \tag{13}$$

where $\zeta = x - \lambda t$ [22]. Then, Fig. 1 shows the parameter regions of α and β where the above solutions (8)–(12) can exist. For the case (I)$C = 0$, Fig. 1a shows that (8), (9) and (10) can exist in the region (I-A), (I-B) and (I-C), respectively. On the other hand, for the case (II)$C \neq 0$, Fig. 1b shows that (11), (12) can exist in the region (II-A) and (II-B), respectively.

It should be noted that the solutions (9), (10) and (12) become meaningless when α and β tend to zero, since the short wave S vanishes in this limit and does not reduce to the solutions (8) or (11) for $\alpha = \beta = 0$. Therefore, only the solutions (8) and (11) for $\alpha + 6\beta = 0$ are uniformly valid in the parameter space (α, β). However, multi-soliton solutions or the n-soliton solution cannot be found in the binary form (6) except for $\alpha = \beta = 0$.

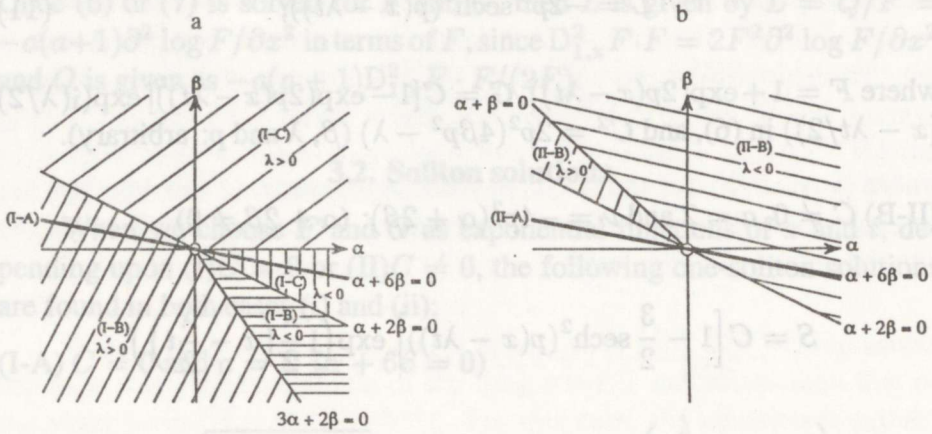

Fig. 1. Parameter regions of α and β in which exact solutions can exist: a – (I-A), (I-B) and (I-C) are the regions where the solutions (8), (9) and (10) are, respectively, possible for $C = 0$; b – (II-A) and (II-B) are the regions where the solutions (11) and (12) are, respectively, possible for $C \neq 0$ [22].

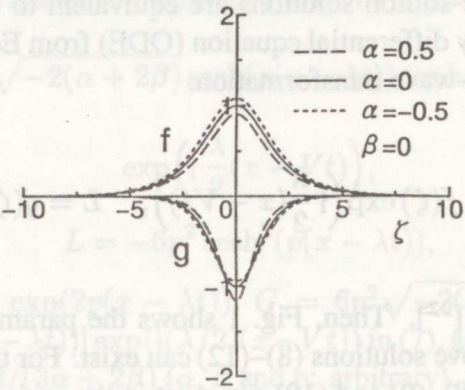

Fig. 2. Profiles of the solitary waves for $\alpha = 0.5, 0$ and -0.5 when $C = 0, \beta = 0$, $\lambda = 1$ and $V = -0.5$, where the exact solutions (8) are shown for $\alpha = 0$ [22].

4. DISCUSSION

Although the exact solutions are shown in the preceding section by means of the modified Hirota method, it is found that there exist a variety of numerical solitary wave solutions [22]. These numerical solitary waves are obtained in the reduced ODE through (13) by means of the shooting method [23]. Typical examples of these solutions are shown in the following figures. In Fig. 2 for $\alpha = -0.5, 0$ and 0.5, when $\beta = 0$, $\lambda = 1$ and $V = -0.5$, the solitary waves with similar profiles are shown in the short wave envelope f

and the long wave g, where the solutions for $\alpha \neq 0$ are found numerically to be reduced to the one for $\alpha = 0$ in the limit of $\alpha \to \pm 0$. On the other hand, in Fig. 3 for $\alpha = 0$, $\beta = -1$ and $\lambda = 1$, different types of wave profiles in f are successively shown depending upon the values of V, while the wave profiles similar to each other are observed in g, where the exact solution (9) is given for $V = -0.5$.

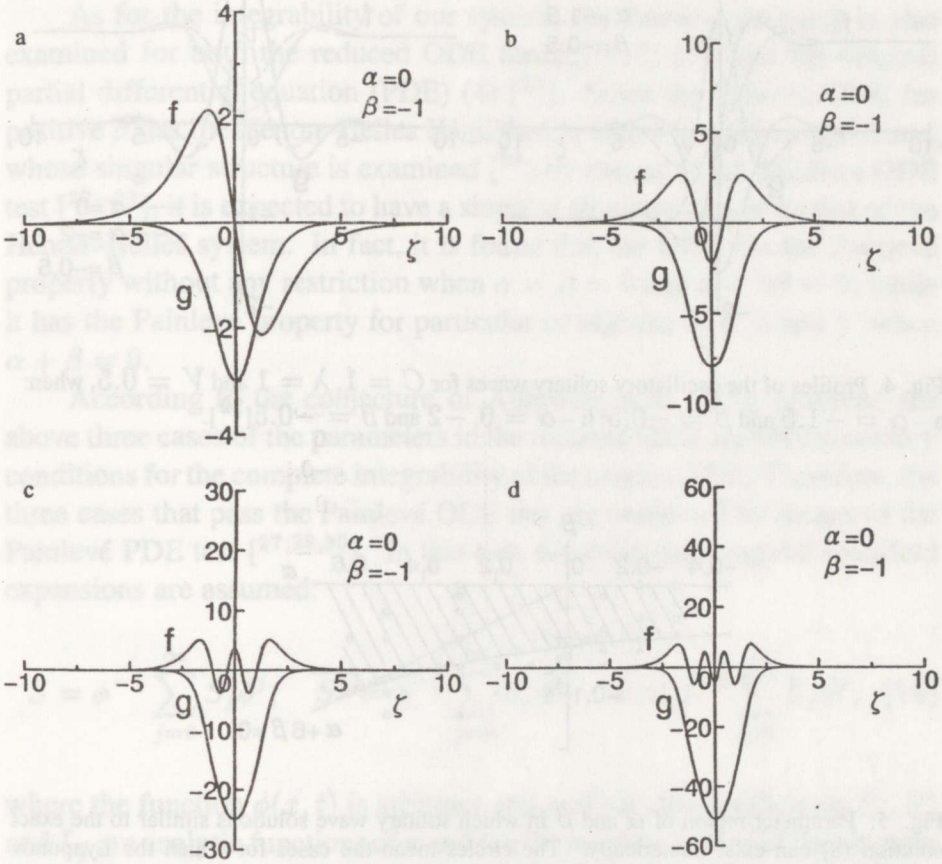

Fig. 3. Profiles of the solitary waves for different values of V when $\alpha = 0, \beta = -1$ and $\lambda = 1$: a – exact solution (9) for $V = -0.5$; b – numerical solutions for $V = -1.4420974\cdots$; c – $V = -3.486315\cdots$; d – $V = -5.693620\cdots$ [22].

We can obtain the oscillatory solitary wave solutions for some region of the parameter α when $\beta < -\lambda^2/(8C^2)$ and $\lambda > 0$. Typical examples are shown in Fig. 4a for $\alpha = -1.5, \beta = -0.5, \lambda = 1, V = 0.5$ and $C = 1$, where the oscillatory phase jump profiles for f are found, while for g both concave and convex oscillatory profiles are observed, depending upon the

boundary conditions for infinitely large $|\zeta|$. Another type of oscillatory solitary wave solutions is shown in Fig. 4b for $\alpha = 0, -2$, $\beta = -0.5$, $\lambda = 1$, $V = 0.5$ and $C = 1$, where oscillatory dark solitary wave profiles are found for f, while only oscillatory convex solitary waves are observed for g.

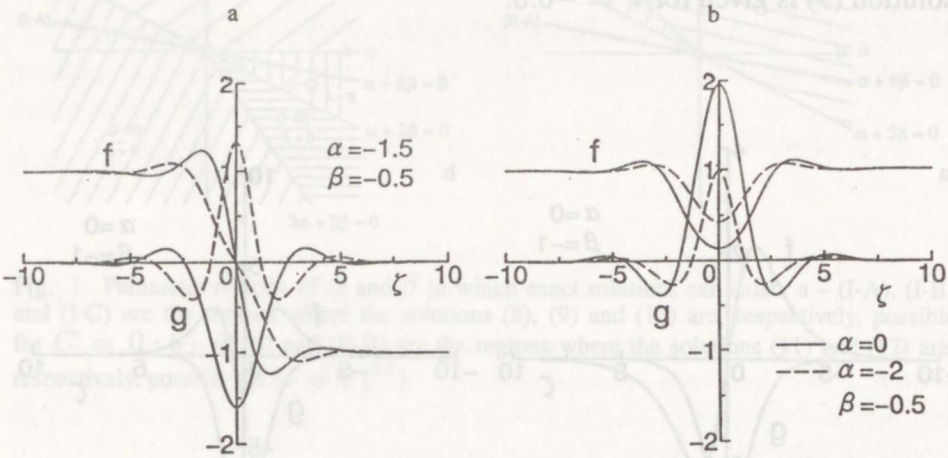

Fig. 4. Profiles of the oscillatory solitary waves for $C = 1, \lambda = 1$ and $V = 0.5$, when: a – $\alpha = -1.5$ and $\beta = -0.5$; b – $\alpha = 0, -2$ and $\beta = -0.5$[22].

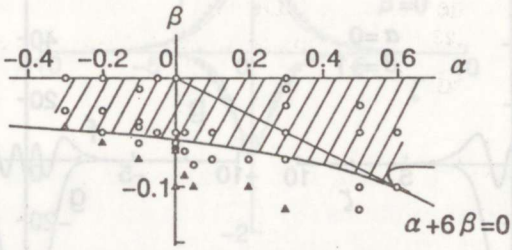

Fig. 5. Parameter region of α and β in which solitary wave solutions similar to the exact solution (8) can exist numerically. The circles mean the cases for which the Lyapunov exponents converge to zero after sufficiently long time, while the triangles mean the cases for which the Lyapunov exponents remain finite positive values [22].

In these exact and numerical solutions, except for $\alpha = \beta = 0$, it is interesting to know whether these solitary waves are stable or not to the interactions. Figure 5 might help to understand this. In this figure, numerical solitary waves with a wave profile closely similar to that of the exact solution (8) are found in the hatched region including $\alpha = 0$ or $\alpha + 6\beta = 0$ ($\beta \leq 0$). In addition to this, the circles mean that the numerically calculated Lyapunov exponents [22,24] converge to zero after a "sufficiently" long time (at most $t = 120$), while triangles denote the

case when they remain finite positive values. Resulting from this, though the calculations have been made for a limited number of initial conditions, our system (4) is expected to be integrable in the sense of the Lyapunov exponent in the region marked by the circles, which is closely related to the hatched region, while it is non-integrable in the region marked by the triangles or for negative values of β with larger magnitude. Therefore, the solitary waves with a wave profile similar to (8) may be stable to the interaction among them in the region with small magnitude of α and β, that is, in the near-integrable region.

As for the integrability of our system, the Painlevé property is also examined for both the reduced ODE through (13) [22] and the original partial differential equation (PDE) (4) [25]. Since the reduced ODE for positive β has the Hénon–Heiles Hamiltonian with adjustable coefficients, whose singular structure is examined [26] by means of the Painlevé ODE test [27–29], it is expected to have a singular structure similar to that of the Hénon–Heiles system. In fact, it is found that the ODE has the Painlevé property without any restriction when $\alpha = \beta = 0$ and $\alpha + 6\beta = 0$, while it has the Painlevé property for particular conditions of β, λ and V when $\alpha + \beta = 0$.

According to the conjecture of Ablowitz et al. [29], however, the above three cases of the parameters in the reduced ODE are only necessary conditions for the complete integrability of the original PDE. Therefore, the three cases that pass the Painlevé ODE test are examined by means of the Painlevé PDE test [27,28,30]. In this test, the following singular manifold expansions are assumed:

$$S = \phi^{-\gamma} \sum_{j=0}^{\infty} S_j \phi^j, \quad S^* = \phi^{-\gamma} \sum_{j=0}^{\infty} S_j^* \phi^j, \quad L = \phi^{-2} \sum_{j=0}^{\infty} L_j \phi^j, \quad (14)$$

where the function $\phi(x,t)$ is arbitrary and analytic, the coefficients S_j, S_j^* and L_j are analytic functions of x and t in the neighbourhood of the singular manifold $\phi(x,t) = 0$, and * denotes the complex conjugate. Substituting the above expressions into Eq. (4) and equating like power of ϕ, we obtain the equations with respect to the coefficients S_j, S_j^* and L_j. According to the test, we claim that S, S^* and L are single-valued about the (movable) singular manifold $\phi(x,t) = 0$ and all the compatibility conditions with respect to S_j, S_j^* and L_j are satisfied. As a result of this, significantly for $\gamma = 1$ and 2, it is found that Eq. (4) passes the test for the case $\alpha = \beta = 0$ without any restrictions, whereas it does not pass the test for the other two cases $\alpha + 6\beta = 0$ and $\alpha + \beta = 0$ without the imposition of the restrictions with respect to ϕ. Thus, only the case for $\alpha = \beta = 0$ is completely integrable, which is consistent with the result by the IST.

It is noted that the test is not successful in the near-integrable region, since the singular manifold expansions of (14) generally become non-uniformly valid when β tends to zero. This is due to the existence of a small parameter β in the highest-order derivative term in Eq. (4). On the other hand, we should remark for $\alpha + 6\beta = 0$ ($\beta \neq 0$), that the crucial condition which permits the Painlevé property is considerably relaxed for the finite time. That is, since the condition is shown to be $\theta_t - \theta\theta_x = 0$ for $\theta = \phi_t/\phi_x$, an arbitrary analytic solution of θ can exist for the finite time. This means the possibility of the finite time "integrability" of the evolution equation. Thus, at least, for $\alpha + 6\beta = 0$, the soliton interaction is expected to be elastic for the finite time, while only the one-soliton state can survive for the infinitely long time. This is also seen from the fact that the multi-soliton solution cannot be found in the Hirota binary form (6). Therefore, the Lyapunov exponent might not converge numerically to zero for $\alpha + 6\beta = 0$ in the near-integrable region after the infinitely long time (much longer than the "sufficiently" long time $t = 120$ in the numerical calculation).

In addition to this, since the numerical calculations of the Lyapunov exponents are not made for all initial conditions, stochastic regions which may be thin in the phase space might exist for the parameters of α and β in the nearly integrable region (Fig. 5).

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ÜKSIKLAINED NING PIKKADE JA LÜHIKESTE LAINETE RESONANTSSÜSTEEMI INTEGREERITAVUS

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Dispergeeruvas keskkonnas võivad pikad ja lühikesed lained olla tugevas vastastikmõjus, kui lühikeste lainete grupikiiruse ja pikkade lainete faasikiiruse resonantsitingimus on täidetud. Käesolevas artiklis on seesugust resonantssüsteemi kirjeldatud Kortewegi–de Vriesi ja mittelineaarse Schrödingeri võrrandiga. Ühesolitonised lahendid on leitud süstemaatiliselt, kasutades modifitseeritud Hirota meetodit. On tõestatud paljude numbriliste lahendite, sh. ka ostsilleeruvate solitonide olemasolu. Integreeritavust on analüüsitud Ljapunovi eksponendi mõttes, samuti on käsitletud Pailevé omadust.

УЕДИНЕННЫЕ ВОЛНЫ И ИНТЕГРИРУЕМОСТЬ ДЛИННЫХ И КОРОТКИХ ВОЛН В РЕЗОНАНСНОЙ СИСТЕМЕ

Такао ЙОШИНАГА

Взаимодействие диспергирующих длинных и коротких волн является сильным, если удовлетворено условие резонанса групповой скорости коротких и фазовой скорости длинных волн. В данной статье рассмотрена резонансная система из связанных уравнений Кортевега–де Вриза и нелинейного уравнения Шредингера. Односолитонное решение найдено модифицированным методом Хироты и доказано существование многих других решений, включая осциллирующие уединенные волны. В связи с этими решениями проанализирована интегрируемость в смысле экситона Ляпунова, а также свойство Пайнлевэ.