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DETERMINATION OF PARAMETERS OF ELASTIC LAYER BY MEASURED DISPERSION CURVES OF ZERO-ORDER LAMB-TYPE WAVES

(*Presented by J. Engelbrecht*)

1. Introduction

An experimental method of determining parameters of thin elastic layer (the Poisson's ratio ν, the velocity of transverse wave c_t and the thickness $2d$) is proposed in papers [1, 2] based on the detection of Lamb-type waves. In these papers the dispersion curves of zero-order antisymmetric A_0 and symmetric S_0 Lamb-type waves are described by the following approximate formulas

$$\omega = T_1 k^2 + T_2 k^4 \quad \text{(for the } A_0 \text{ wave)},$$
$$\omega = T_3 k \quad \text{(for the } S_0 \text{ wave)}.$$

Here ω is the angular frequency, k is the wave number; T_1, T_2 and T_3 are coefficients.

The parameters of elastic layer ν, c_t and d from one side and the coefficients T_j $(j=1, 2, 3)$ from the other side can be simply presented one-to-one [2]. After the detection of the dispersion curves and using the signal processing techniques for determining the approximate T_j values, both the thickness $2d$ of the layer and its elastic constants ν and c_t can be estimated.

In this paper, more exact approximations are proposed for the dispersion curves of Lamb-type waves A_0 and S_0. This allows one to extend the range of frequency measurement (up to $k_t d \sim 1.5$) and to determine the parameters of the layer from the single Lamb-type wave, either A_0 or S_0. Two variants of formulas are given below (the first one is based on the A_0 wave and the second is based on the S_0 wave) for estimating ν, c_t and d, and the errors involved in such a procedure are being discussed.

2. Asymptotic formulas for describing the dispersion curves of the zero-order Lamb-type waves A_0 and S_0

As is well known, the dispersion equations for the antisymmetric A_l and symmetric S_l $(l=0, 1, 2, \ldots)$ Lamb-type waves propagating in a plane layer which is stress-free on both surfaces, have the form

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$$F=0, \quad E=0, \tag{1}$$

where

$$F=\varphi_1 \operatorname{th} p_l - \varphi_2 \operatorname{th} p_t, \quad E=\varphi_1 \operatorname{cth} p_l - \varphi_2 \operatorname{cth} p_t, \tag{2}$$

$$\varphi_1=(2-y^2)^2, \quad \varphi_2=4\sqrt{(1-y^2)(1-\gamma_0^2 y^2)},$$

$$p_l=\frac{z}{y}\sqrt{1-\gamma_0^2 y^2}, \quad p_t=\frac{z}{y}\sqrt{1-y^2},$$

$$\gamma_0=\frac{c_t}{c_l}\equiv\sqrt{\frac{1-2\nu}{2-2\nu}}, \quad y=\frac{c^{ph}}{c_t}, \quad z=k_t d,$$

$$\lambda=\frac{z}{y}\equiv kd, \quad k_t=\frac{\omega}{c_t}, \quad k=\frac{\omega}{c^{ph}}.$$

Here the following notations are used: c_l, c_t are the longitudinal and transverse velocities in the linear elasticity theory, ν is the Poisson's ratio, $2d$ is the thickness of the layer, k_t is the wave number of the transverse wave; c^{ph}, k and λ are the phase velocity, wave number and the wave halfthickness of a Lamb-type wave, respectively.

From Eqs. (1) the dependence $y(z)$ should be found.

Now we shall obtain the asymptotic description for the dispersion curves of the zero-order antisymmetric A_0 and symmetric S_0 Lamb-type waves at small z values ($z \ll 1$).

The A_0 wave. Using the notation $\lambda=z/y$, we shall search for the solution of the equation $F=0$ in the form

$$\lambda^4(z)=a_* z^2(1+a_1 z+a_2 z^2+a_3 z^3)+O(z^6). \tag{3}$$

By substituting (3) in the equation $F=0$, using the asymptotic expansions of $\operatorname{th} p_l$ and $\operatorname{th} p_t$ at small values of the arguments and by equating the coefficients at (identical) z powers, after simple but tedious calculations we obtain

$$a_*=\frac{3}{2}(1-\nu), \quad a_1=\sqrt{\frac{3}{2(1-\nu)}}\,\frac{17-7\nu}{15}, \tag{4}$$

$$a_2=\frac{1179-818\nu+409\nu^2}{2100(1-\nu)},$$

$$a_3=\sqrt{\frac{3}{2}(1-\nu)}\,\frac{5951-2603\nu+9953\nu^2-4901\nu^3}{126000(1-\nu)^2}.$$

We shall also present the approximate formula for the phase velocity. Substituting in (3) $y=z/\lambda$, and rejecting the O term, we shall write

$$y(z)=\frac{\sqrt{z}}{\sqrt[4]{a_*}\sqrt[4]{1+a_1 z+a_2 z+a_3 z^3}}. \tag{5}$$

The S_0 wave. Analogously, from the equation $E=0$ we obtain the asymptotic expansion for the wave halfthickness λ of the S_0 wave

$$\lambda^2(z)=s_* z^2(1+s_1 z^2+s_2 z^4+s_3 z^6)+O(z^{10}). \tag{6}$$

where

$$s_*=\frac{1-\nu}{2}, \quad s_1=\frac{\nu^2}{6(1-\nu)}, \quad s_2=\frac{\nu^2(6-10\nu+3\nu^2)}{180(1-\nu)^2}, \tag{7}$$

$$s_3=\frac{\nu^2(51-168\nu+183\nu^2-56\nu^3-8\nu^4)}{7560(1-\nu)^3}.$$

Corresponding to (5), the approximate formula of the phase velocity takes the form

$$y(z)=\frac{1}{\sqrt{s_*}\sqrt{1+s_1z^2+s_2z^4+s_3z^6}}. \tag{8}$$

Further, apart from the expansion (3) and (6), we shall use the formulas, in the right hand sides of which there are less terms than in (3) and (6). We shall name the n-th order ($n=0, 1, 2, 3$) approximations the following expansions for the A_0 and S_0 waves, respectively:

$$\lambda_n^4(z)=a_*z^2\sum_{j=0}^{n}a_jz^j \qquad (a_0=1), \tag{9}$$

$$\lambda_n^2(z)=s_*z^2\sum_{j=0}^{n}s_jz^{2j} \qquad (s_0=1), \tag{10}$$

where the coefficients a_*, s_*, a_j, s_j ($j=1, 2, 3$) are defined by the formulas (4) and (7).

3. Region of the Validity of the Formulas

The formulas (3) and (6) are obtained in the assumption $z\ll 1$. In fact, they can be used outside the domain restricted by this condition. The real limits of validity of these formulas, as well as of the aproximations of zero, first and second order, can be estimated by comparison of the values of λ obtained from the proposed formulas with the exact values, which are obtained from the equations $F=0$ and $E=0$.

In Fig. 1,2 and Table 1 the results of such a comparison are given. The computation is carried out in the case of the aluminium layer with parameters

$$c_l=6380 \text{ m/s}, \quad c_t=3100 \text{ m/s}, \quad \varrho_1=2.79\cdot 10^3 \text{ kg/m}^3, \quad \nu=0.3455. \tag{11}$$

As can be seen from the comparison of columns of Table 1, the third-order approximations ($n=3$) describe $\lambda(z)$ of the A_0 and S_0 waves very exactly up to $z\leqslant 1.5$. In this domain the error of the third-order approximations is less than 0.2%. The third-order approximation also gives a rather good description of $\lambda(z)$ of the A_0 waves for bigger z values. Because at $z=\pi/2$ the A_1 wave can be generated in the layer, we shall use, for determining the layer parameters, the domain $z\leqslant 1.5$ only.

In the region $z\leqslant 1.5$ the error of the second-order approximations is less than 2% for the A_0 wave and 0.6% for the S_0 wave; in the region $z\leqslant 1$ these errors are less than 0.9% and 0.04%, respectively.

As can be seen from Table 1, if, in calculating $\lambda(z)$, the 1% error can be accepted, then the region of validity of the approximation of the first order for the A_0 wave and of the zero-order of the S_0 wave is restricted by $z\leqslant 0.25$ and $z\leqslant 0.75$, respectively. In determining the parameters of the layer in papers [1, 2], approximations asymptotically equivalent to the ones described above, are used.

The computations of $\lambda(z)$ for different ν values [$\nu=0+0.1q$ ($q=0, 1, 2, 3, 4, 5$)] in the domain $0\leqslant z\leqslant 2$ have shown that up to $z\leqslant 1.5$ the approximations of the third order work so well that they can be considered practically exact. Therefore the approximations (3), (6) and (5), (8)

Table 1

The exact value of the wave halfthickness λ_* and its approximations λ_n of the A_0 and S_0 waves for an aluminius layer

z	A_0 wave					S_0 wave				
	λ_*	λ_3	λ_2	λ_1	λ_0	λ_*	λ_3	λ_2	λ_1	λ_0
0.1	0.326	0.326	0.326	0.326	0.315	0.057	0.057	0.057	0.057	0.057
0.2	0.477	0.477	0.477	0.475	0,445	0.115	0.115	0.115	0.115	0.114
0.3	0.604	0.604	0.604	0.597	0.545	0.172	0.172	0.172	0.172	0.172
0.4	0.719	0.719	0.719	0.707	0.630	0.229	0.229	0.229	0.229	0.229
0.5	0.829	0.829	0.827	0.808	0.704	0.287	0.287	0.287	0.287	0.286
0.6	0.934	0.934	0.932	0.903	0.771	0.345	0.345	0.345	0.345	0.343
0.7	1.037	1.037	1.033	0.994	0.833	0.404	0.404	0.404	0.403	0.400
0.8	1.139	1.139	1.132	1.082	0.890	0.463	0.463	0.463	0.462	0.458
0.9	1.239	1.239	1.230	1.166	0.944	0.522	0.522	0.522	0.521	0.515
1.0	1.338	1.339	1.327	1.248	0.995	0.582	0.582	0.582	0.581	0.572
1.1	1.437	1.438	1.423	1.328	1.044	0.643	0.643	0.643	0.641	0.629
1.2	1.536	1.537	1.518	1.406	1.090	0.705	0.705	0.705	0.701	0.687
1.3	1.636	1.636	1.613	1.483	1.135	0.769	0.768	0.767	0.764	0.744
1.4	1.734	1.735	1.707	1.558	1.178	0.834	0.833	0.821	0.824	0.801
1.5	1.833	1.835	1.800	1.632	1.219	0.901	0.900	0.896	0.887	0.858
1.6	1.931	1.934	1.894	1.704	1.259	0.971	0.968	0.963	0.950	0.915
1.7	2.031	2.034	1.987	1.775	1.298	1.045	1.039	1.032	1.014	0.973
1.8	2.130	2.134	2.079	1.846	1.336	1.124	1.114	1.102	1.079	1.030
1.9	2.229	2.235	2.172	1.915	1.372	1.210	1.191	1.175	1.145	1.087
2.0	2.329	2.336	2.264	1.984	1.408	1.305	1.273	1.250	1.212	1.144

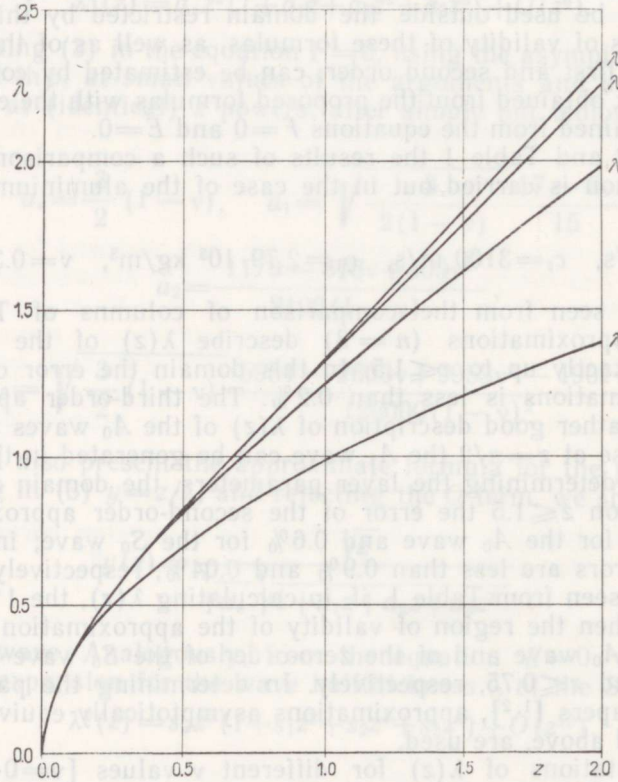

Fig. 1. The A_0 wave halfthickness approximations λ_n $(n=0, 1, 2, 3)$ (see formula (9)) for an aluminium layer.

Table 2

The relative phase velocity $y=c^{ph}/c_t$ of the A_0 and S_0 waves for different ν values computed according to asymptotic expansions (see formulas (5) and (8))

z	A_0 wave						S_0 wave					
	$\nu=0.0$	$\nu=0.1$	$\nu=0.2$	$\nu=0.3$	$\nu=0.4$	$\nu=0.5$	$\nu=0.0$	$\nu=0.1$	$\nu=0.2$	$\nu=0.3$	$\nu=0.4$	$\nu=0.5$
0.0	0.000	0.000	0.000	0.000	0.000	0.000	1.414	1.491	1.581	1.690	1.826	2.000
0.1	0.276	0.284	0.292	0.302	0.313	0.327	1.414	1.491	1.581	1.690	1.825	1.999
0.2	0.378	0.388	0.399	0.412	0.428	0.446	1.414	1.491	1.581	1.690	1.824	1.997
0.3	0.450	0.461	0.474	0.489	0.507	0.528	1.414	1.491	1.581	1.689	1.822	1.992
0.4	0.504	0.517	0.531	0.548	0.567	0.589	1.414	1.490	1.580	1.687	1.819	1.987
0.5	0.548	0.562	0.577	0.595	0.615	0.638	1.414	1.490	1.579	1.686	1.815	1.979
0.6	0.585	0.599	0.615	0.633	0.654	0.678	1.414	1.490	1.579	1.683	1.811	1.969
0.7	0.616	0.631	0.647	0.666	0.687	0.711	1.414	1.490	1.578	1.681	1.805	1.958
0.8	0.643	0.658	0.675	0.693	0.714	0.738	1.414	1.490	1.576	1.678	1.798	1.945
0.9	0.666	0.681	0.698	0.717	0.738	0.762	1.414	1.489	1.575	1.674	1.790	1.929
1,0	0.686	0.702	0.719	0.738	0.759	0.782	1.414	1.489	1.573	1.669	1.780	1.912
1.1	0.704	0.720	0.737	0.756	0.776	0.799	1.414	1.489	1.571	1.664	1.770	1.892
1.2	0.720	0.736	0.753	0.772	0.792	0.814	1.414	1.488	1.569	1.658	1.757	1.870
1.3	0.734	0.750	0.767	0.786	0.806	0.827	1.414	1.487	1.566	1.651	1.743	1.845
1.4	0.747	0.763	0.780	0.798	0.817	0.838	1.414	1.487	1.563	1.643	1.727	1.817
1.5	0.758	0.774	0.791	0.809	0.828	0.848	1.414	1.486	1.559	1.633	1.709	1.787
1.6	0.768	0.785	0.801	0.819	0.837	0.856	1.414	1.485	1.554	1.622	1.688	1.754
1.7	0.778	0.794	0.810	0.828	0.845	0.863	1.414	1.483	1.549	1.610	1.666	1.718
1.8	0.786	0.802	0.819	0.836	0.853	0.870	1.414	1.482	1.543	1.595	1.640	1.679
1.9	0.794	0.810	0.826	0.843	0.859	0.876	1.414	1.480	1.535	1.579	1.612	1.637
2.0	0.801	0.817	0.833	0.849	0.865	0.880	1.414	1.478	1.527	1.560	1.581	1.592

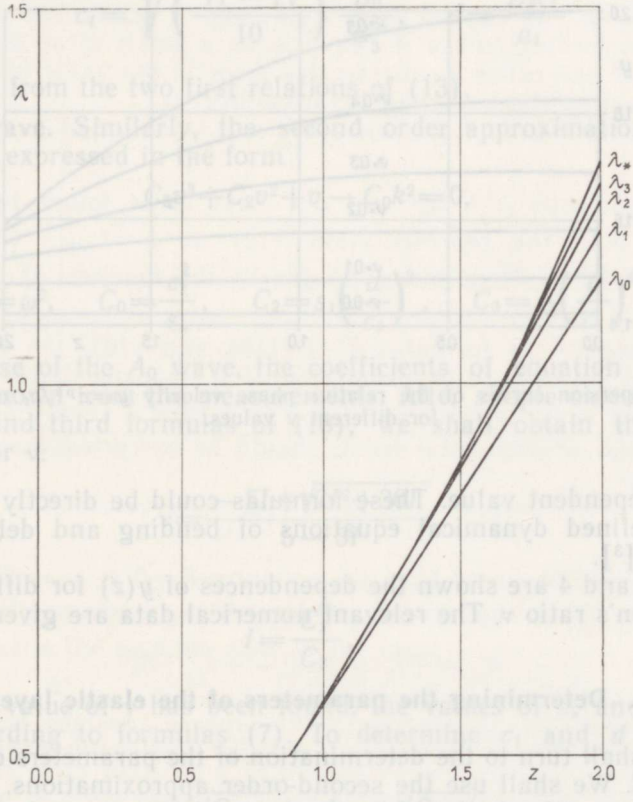

Fig. 2. The S_0 wave halfthickness approximations λ_n $(n=0, 1, 2, 3)$ (see formula (10)) and the exact value λ_* found from the equation $E=0$. At $z<0.85$ the approximation curves do not differ from the exact ones.

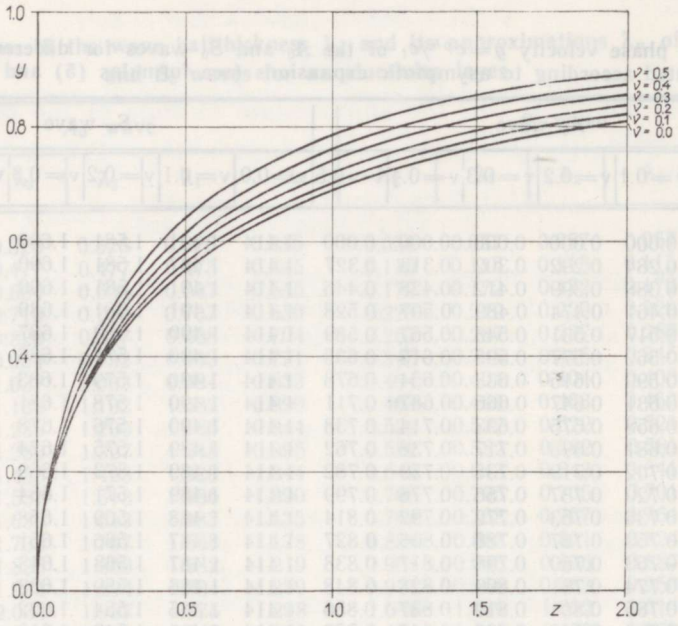

Fig. 3. The dispersion curves of the relative phase velocity $y=c^{ph}/c_t$ of the A_0 wave for different ν values.

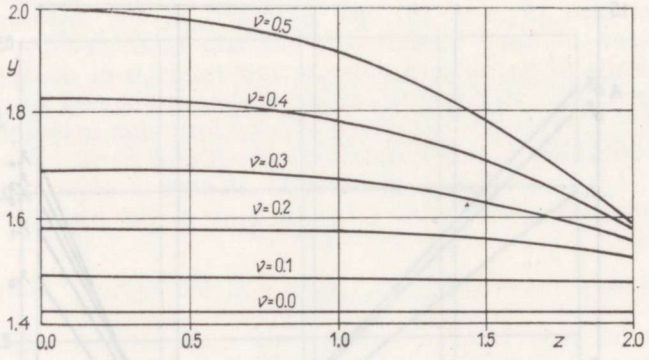

Fig. 4. The dispersion curves of the relative phase velocity $y=c^{ph}/c_t$ of the S_0 wave for different ν values.

have an independent value. These formulas could be directly obtained by using the refined dynamical equations of bending and delating plates proposed in [3].

In Fig. 3 and 4 are shown the dependences of $y(z)$ for different values of the Poisson's ratio ν. The relevant numerical data are given in Table 2.

4. Determining the parameters of the elastic layer

Now we shall turn to the determination of the parameters of the elastic layer ν, c_t, d. We shall use the second-order approximations.

The A_0 wave. We shall rewrite the formula (9) at $n=2$ in the dimension form

$$B_4\omega^4+B_3\omega^3+\omega^2-B_0k^4=0, \tag{12}$$

where

$$B_0=\frac{(c_t d)^2}{a_*}, \quad B_3=\frac{a_1 d}{c_t}, \quad B_4=\frac{a_2 d^2}{c_t^2}. \tag{13}$$

To determine the unknown parameters ν, c_t, d in (13), we shall present (with some discrepancy) the dependence of the wave number on frequency $k_{\exp}(\omega)$ which has been obtained from the experiment. Setting $k^4=k^4_{\exp}$, we shall find the values of coefficients B_0, B_3 and B_4 in the theoretical dependence (12). From the known values of B_0, B_3 and B_4 the parameters ν, c_t and d can easily be found. Thus, introducing the coefficient

$$D=\frac{B_3^2}{B_4} \tag{14}$$

from the second and third formulas of (13), taking into consideration (4), we shall obtain the square equation for determining ν. The solution of this equation is

$$\nu=\frac{409D-1666+\sqrt{1100820D-314930D^2}}{409D-686}. \tag{15}$$

When the Poisson's ratio ν is known, the transverse wave velocity c_t and the halfthickness of the layer d can be found according to the formulas

$$c_t=\sqrt[4]{\left(\frac{17-7\nu}{10}\right)^2\frac{B_0}{B_3^2}}, \quad d=\frac{B_3 c_t}{a_1} \tag{16}$$

which follow from the two first relations of (13).

The S_0 wave. Similarly, the second order approximation for the S_0 wave can be expressed in the form

$$C_3 v^3+C_2 v^2+v-C_0 k^2=0, \tag{17}$$

where

$$v=\omega^2, \quad C_0=\frac{c_t^2}{s_*}, \quad C_2=s_1\left(\frac{d}{c_t}\right)^2, \quad C_3=s_2\left(\frac{d}{c_t}\right)^4. \tag{18}$$

As in the case of the A_0 wave, the coefficients of equation (17) can be considered known from the measurements. After simple calculations from the second and third formulas of (18), we shall obtain the following expression for ν:

$$\nu=\frac{-5I+\sqrt{7I^2+30I}}{5-3I}, \tag{19}$$

where

$$I=\frac{C_2^2}{C_3}. \tag{20}$$

After the value of ν has been found, the values of s_* and s_1 are calculated according to formulas (7). To determine c_t and d we use the formulas

$$c_t=\sqrt{C_0 s_*}, \quad d=c_t\sqrt{C_2 s_1}. \tag{21}$$

Thus, contrary to the procedure outlined in papers [1, 2], here the parameters ν, c_t and d could be determined either from the A_0 or S_0 wave.

5. Computational errors and iterative process

We shall now consider the computational errors which are introduced with the proposed procedure. We shall assume that the dispersion curve can be obtained as exact as desired from the measured data, i.e. that for the A_0 wave $k^4_{\exp}(\omega)=k^4_*(\omega)$, and for the S_0 wave $k^2_{\exp}(\omega)=k^2_*(\omega)$ (here k_* is the exact value of the wave number obtained either from the equation $F=0$ or from the equation $E=0$, respectively).

The error of the Poisson's ratio ν, which is defined either from equation (15) (according to the A_0 wave) or from equation (19) (according to the S_0 wave), can be connected only with the error in determining the coefficients D or I (see equations (14) and (20), respectively). In its turn, the error of D and I is caused by the error of the solution obtained on the second order approximations, i.e., by the difference between $k(\omega)$ calculated from (9), (10) at $n=2$, and $k_*(\omega)$, which causes a notorious error in determining of either the values B_0, B_3 and B_1 or C_0, C_2 and C_3 by approximating the experimental curve by the curve given by either expression (12) or (17), respectively. In Fig. 5, 6 the dependences $\ln|\delta_A|$ and δ_S on ν are shown. The quantities δ_A and δ_S are defined by

$$\delta_A=\frac{\Delta\nu/\nu}{\Delta D/D}, \qquad \delta_S=\frac{\Delta\nu/\nu}{\Delta I/I}. \tag{22}$$

Here $\Delta\nu/\nu$ is the relative error of the evaluation of the Poisson's ratio ν; $\Delta D/D$ and $\Delta I/I$ is the relative error in evaluating coefficients D and I, respectively.

The A_0 wave. As can be seen from Fig. 5, when the value of ν is determined according to the A_0 wave, even a small error in determining the value of D can cause a considerable error in the value of ν. This error can be reduced by the following iterative process.

Let us assume that dependence $k^4_{\exp}(\omega)$ is obtained from the experiment. Then

(i) the coefficients $B_0^{(i)}$, $B_3^{(i)}$ and $B_4^{(i)}$ have to be found by comparing $k^4_{\exp(i)}$ with the representation (12) at $k=k_{\exp(i)}$ (if $i=0$, then $k^4_{\exp(0)}=k^4_{\exp}$). Here index (i) marks the number of the iteration $(i=0, 1, 2, \ldots)$;

(ii) according to the formulas (14), (15) and (16), the values of $\nu^{(i)}$, $c_t^{(i)}$ and $d^{(i)}$ have to be calculated. If it happens that $\nu^{(i)}>0.5$, then it should be put $\nu^{(i)}=0.5$ and if $\nu^{(0)}<0$ then it should be put $\nu^{(i)}=0$;

(iii) the initial experimental curve should be corrected according to the formula

$$k^4_{\exp(i+1)}=k^4_{\exp}-(k^4_{*(i)}-k^4_{(i)}). \tag{23}$$

Here $k_{*(i)}$ should be found from the equation $F=0$ at $\nu=\nu^{(i)}$, $c_t=c_t^{(i)}$ and $d=d^{(i)}$, while $k^4_{(i)}$ is the second order approximation

$$k^4_{(i)}=(B_4^{(i)}\omega^4+B_3^{(i)}\omega^3+\omega^2)/B_0^{(i)}. \tag{24}$$

The steps should be repeated until the process becomes established. By using the curve $k^4_{\exp(i)}(\omega)$ instead of the experimental curve $k^4_{\exp}(\omega)$ according to formula (23), we reduce the computational errors which are connected with the replacement of the $k^4_*(\omega)$ curve by its second-order approximation. It should be noted that in the considered frequency range the third-order approximation has sufficiently high precision. Here, we

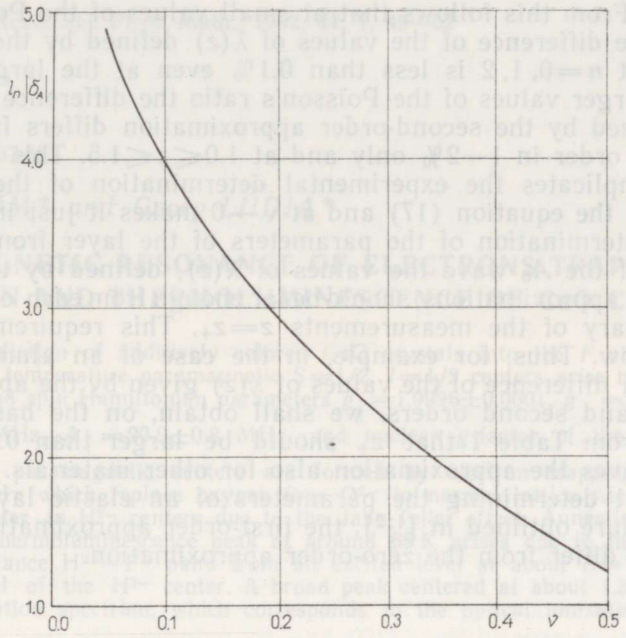

Fig. 5. The dependence $\ln|\delta_A|$ on ν for the A_0 wave (see formula (22)).

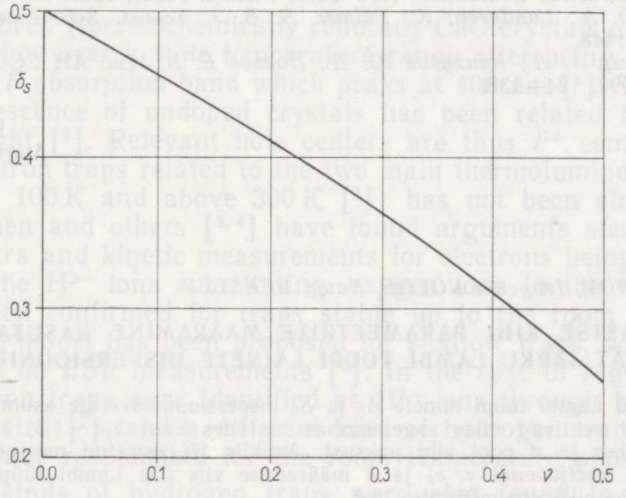

Fig. 6. The dependence δ_S on ν for the S_0 wave (see formula (22)).

are dealing not with approximation of $k(\omega)$, but of $k^4(\omega)$. Identifying $k^4_*(\omega)$ with the analogous curve defined by the third-order approximation, we shall rewrite the formula (24) in the form

$$k^4_{\exp(i+1)} = k^4_{\exp} - (B^{(i)}_3 B^{(i)}_4 a^{(i)}_3 / B^{(i)}_0 a^{(i)}_1 a^{(i)}_2)\,\omega^5. \qquad (25)$$

The S_0 wave. We shall turn now to the case of determining the parameters of the layer according to the dispersion curve of the S_0 wave. In this case the relative error in determining coefficient I on several percents is not frightful (see Fig. 6). Here, however, an obstacle of another kind arises. It is connected with coefficients s_1 and s_2. As can be seen from formulas (7) $s_2 \ll s_1 \ll 1$, and the coefficients s_1 and s_2 are propor-

tional to ν^2. From this follows that at small values of the Poisson's ratio $\nu(\nu \leqslant 0.1)$ the difference of the values of $\lambda(z)$ defined by the approximations (10) at $n=0, 1, 2$ is less than 0.1% even at the largest value of $z=1.5$. At larger values of the Poisson's ratio the difference of the value of $\lambda(z)$ defined by the second-order approximation differs from the that of the third order in 1—2% only and at $1.0 \leqslant z \leqslant 1.5$. This circumstance strongly complicates the experimental determination of the coefficients C_2 and C_3 in the equation (17) and at $\nu \to 0$ makes it just impossible.

At the determination of the parameters of the layer from the dispersion curve of the A_0 wave the values of $\lambda(z)$, defined by the first- and second-order approximations, should be far enough from each other near the upper boundary of the measurements $z=z_+$. This requirement restricts z_+ from below. Thus, for example, in the case of an aluminium layer, using the 2% difference of the values of $\lambda(z)$ given by the approximations of the first and second orders, we shall obtain, on the basis of (9) at $n=1, 2$ or from Table 1 that z_+ should be larger than 0.4. In such a manner behaves the approximation also for other materials. It should be noted that at determining the parameters of an elastic layer according to the procedure outlined in [1, 2], the first-order approximation must also significantly differ from the zero-order approximation.

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ELASTSE KIHI PARAMEETRITE MÄÄRAMINE KASUTADES NULLINDAT JÄRKU LAMBI TÜÜPI LAINETE DISPERSIOONIKÕVERAID

On esitatud Lambi tüüpi lainete A_0 ja S_0 dispersioonikõverate asümptootilised arendused. Kõverad kehtivad laias sagedusribas ulatudes kuni $k_t d \sim 1{,}5$, kus k_t on põikilaine lainenumber ja d pool kihi paksust. Artiklis [2] esitatud meetodi arenduses on antud Poissoni koefitsiendi ν, c_t ja d määramise viis ühe Lambi tüüpi laine (kas A_0 või S_0) dispersioonikõvera mõõtmise teel.

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ОТЫСКАНИЕ ПАРАМЕТРОВ УПРУГОГО СЛОЯ ПО ИЗМЕРЕННЫМ ДИСПЕРСИОННЫМ КРИВЫМ ВОЛН ЛЕМБА НУЛЕВОГО ПОРЯДКА

Выведены асимптотические разложения дисперсионных кривых волн Лемба A_0 и S_0, работающие в широком диапазоне частот вплоть до $k_t d \sim 1{,}5$ (здесь k_t волновое число поперечной волны, d — полутолщина слоя). Представлен способ отыскания коэффициента Пуассона ν, c_t и d по измеренной дисперсионной кривой волны A_0 или S_0.