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DYNAMIC DISTURBANCE DECOUPLING FOR DISCRETE TIME NON-LINEAR SYSTEMS: THE NON-SQUARE AND NON-INVERTIBLE CASE

(Presented by Ü. Jaaksoo)

1. Introduction

Consider a discrete time non-linear plant P described by equations of the form

$$x(t+1)=f(x(t), u(t), w(t)), \quad x(0)=x_0, \quad y(t)=h(x(t)), \tag{1}$$

where the states $x(\cdot)$ belong to an open part X of R^n, the controls $u(\cdot)$ belong to an open part U of R^m, the unmeasurable disturbances $w(\cdot)$ belong to an open part W of R^r, and the outputs $y(\cdot)$ belong to an open part Y of R^p. The mappings f and h are supposed to be real analytic.

In the Dynamic Disturbance Decoupling Problem (DDDP) one searches for a regular dynamic state feedback

$$z(t+1)=\psi(z(t), x(t), v(t)), \quad z(0)=z_0, \quad u(t)=\varphi(z(t), x(t), v(t)) \tag{2}$$

with the μ-dimensional compensator state $z(\cdot)\in Z\subset R^\mu$, with a new m-dimensional control $v(t)$, so that in the feedback modified dynamics

$$\begin{aligned} x(t+1)&=f(x(t), \varphi(z(t), x(t), v(t)), w(t)),\\ z(t+1)&=\psi(z(t), x(t), v(t)) \end{aligned} \tag{3}$$

the disturbances $w(t)$ do not influence the outputs $y(t)$. Here the regularity of (2) means that the dynamical system (2) with inputs $v(t)$ and outputs $u(t)$ is invertible, or, equivalently, that it defines a one-to-one (x, z) — dependent correspondence between the input variable v and the output variable u.

Throughout the paper we shall adopt a local vievpoint. However, contrary to the continuous time case, in the discrete time case the local study is impossible around an arbitrary initial state since even in one step the state evolution can move far from the initial point. For this reason we shall consider the DDDP locally around the equilibrium point of the system. Such an approach was also used by Nijmeijer [1] in studing the input-output decoupling problem. So, we are assumed to work in a neighbourhood of an equilibrium point of the system (1), that is around $(x^0, u^0, w^0)\in X\times U\times W$ such that $f(x^0, u^0, w^0)=x^0$. For the initial state $x(0)=x^0$, the constant input sequence $u(t)=u^0$, $t\geqslant 0$ and the constant disturbance sequence $w(t)=w^0$, $t\geqslant 0$, there exists the constant output sequence $y(t)=y^0=h(x^0)$, $t\geqslant 0$.

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In [2] the DDDP has been solved locally around the equilibrium point of the system (1) under two additional assumptions. The first assumption was that the original system (1) is square, i.e. the number of inputs m equals the number of outputs p. The second assumption was that the system (1) with $w(t)\equiv 0$ is invertible. The purpose of this paper is to give a complete regular local solution of the DDDP without any further assumptions on the discrete time non-linear system (1). So we will not assume here that $m=p$ nor the invertibility of (1). The solution of the DDDP we present here is an extension of the results earlier obtained by Kotta and Nijmeijer [2], which roughly indicate that the problem is locally solvable if and only if it is solvable by means of a certain compensator got from the inversion algorithm. Similar results for continuous time non-linear systems, linear in control, have been obtained by Huiberts, Nijmeijer and Van der Wegen [1].

2. Preliminary results

In this section we give preliminary results. The main tool in the solution of the DDDP is the so-called inversion (structure) algorithm for discrete time nonlinear systems without disturbances. This algorithm has been introduced by Kotta [3] for construction of a right inverse of a discrete time nonlinear system, and given in a more general and simple form by Kotta and Nijmeijer [2]. Both algorithms can be considered as generalizations of the algorithm obtained by Lee and Marcus [4] that was only applicable under some restrictive assumptions. These assumptions have been shown to be a necessary and sufficient condition for local static-state feedback input-output linearizability.

In the sequel we will present the inversion algorithm obtained by Kotta and Nijmeijer [2] for the system (1) with $w(t)=w^0$. Let us denote $f(x(t), u(t), w^0)$ by $F(x(t), u(t))$.

Step 1. Calculate

$$y(t+1)=h(F(x(t), u(t)),$$

and define

$$\varrho_1=\operatorname{rank}\frac{\partial}{\partial u}h(F(x,u))\Big|_{x=x^0,\ u=u^0}.$$

Let us assume that $\varrho_1=\text{const}$ is some neighbourhood O_1 of (x^0, u^0). Permute, if necessary, the components of the output so that the first ϱ_1 rows of the matrix $\frac{\partial}{\partial u}h(F(x,u))$ are linearly independent. Decompose $y(t+1)$ and $h(F(x,u))$ according to

$$y(t+1)=\begin{bmatrix}\tilde{y}_1(t+1)\\ \hat{y}_1(t+1)\end{bmatrix},\quad h(F(x,u))=\begin{bmatrix}\tilde{a}_1(x,u)\\ \hat{a}_1(x,u)\end{bmatrix},$$

where $\tilde{y}_1(t+1)$ and $\tilde{a}_1(x,u)$ consist of the first ϱ_1 components of $y(t+1)$ and $h(F(x,u)$, respectively. Since the last $p-\varrho_1$ rows of the matrix $\frac{\partial}{\partial u}h(F(x,u))$ are linearly dependent on the first ϱ_1 rows, we can write

$$\tilde{y}_1(t+1)=\tilde{a}_1(x(t), u(t)),$$

$$\hat{y}_1(t+1)=\hat{a}_1(x(t), u(t))=\hat{y}_1(x(t), \tilde{y}_1(t+1)).$$

Denote $\tilde{a}_1(x,u)$ by $A_1(x,u)$. Denote $\hat{y}_0(t)=h(x(t))$ and $\varrho_0=0$.

Step $k+1$ $(k \geqslant 1)$. Suppose that in Steps 1 through k, $\tilde{y}_1(t+1)$, $\tilde{y}_2(t+2), \ldots, \tilde{y}_k(t+k)$, $\hat{y}_k(t+k)$ have been defined so that

$$\tilde{y}_1(t+1)=\tilde{a}_1(x(t), u(t)),$$
$$\tilde{y}_2(t+2)=\tilde{a}_2(x(t), u(t), \tilde{y}_1(t+2)),$$
$$\cdots\cdots$$
$$\tilde{y}_k(t+k)=\tilde{a}_k(x(t), u(t), \{\tilde{y}_i(t+j), 1 \leqslant i \leqslant k-1, i+1 \leqslant j \leqslant k\}),$$
$$\hat{y}_k(t+k)=\hat{y}_k(x(t), \{\tilde{y}_i(t+j), 1 \leqslant i \leqslant k, i \leqslant j \leqslant k\}).$$

Suppose also that the matrix $A_k=[\tilde{a}_1^T \ldots \tilde{a}_k^T]$ has full rank equal to ϱ_k in some neighbourhood O_k of (x^0, u^0).

Compute

$$\hat{y}_k(t+k+1)=\hat{y}_k(F(x(t), u(t)), \{\tilde{y}_i(t+j+1), 1 \leqslant i \leqslant k, i \leqslant j \leqslant k\}) \stackrel{\triangle}{=}$$
$$\stackrel{\triangle}{=} a_{k+1}(x(t), u(t), \{\tilde{y}_i(t+j), 1 \leqslant i \leqslant k, i+1 \leqslant j \leqslant k+1\})$$

and define

$$\varrho_{k+1}=\operatorname{rank} \frac{\partial}{\partial u}\left[\begin{array}{c} A_k(\cdot) \\ a_{k+1}(\cdot) \end{array}\right]_{x=x^0, u=u^0, \tilde{y}^0=h(x^0)}.$$

Let us assume that $\varrho_{k+1}=$const is some neighbourhood O_{k+1} of (x^0, u^0). Permute, if necessary, the components of $\hat{y}_k(t+k+1)$ so that the first ϱ_{k+1} rows of the matrix

$$\frac{\partial}{\partial u}[A_k^T, a_{k+1}^T]^T$$

are linearly independent. Decompose $\hat{y}_k(t+k+1)$ and a_{k+1} according to

$$\hat{y}_k(t+k+1)=\left[\begin{array}{c} \tilde{y}_{k+1}(t+k+1) \\ \hat{y}_{k+1}(t+k+1) \end{array}\right], \quad a_{k+1}=\left[\begin{array}{c} \tilde{a}_{k+1} \\ \hat{a}_{k+1} \end{array}\right],$$

where $\tilde{y}_{k+1}(t+k+1)$ and $\tilde{a}_{k+1}$ consist of the first $\varrho_{k+1}-\varrho_k$ components of $\hat{y}_k(t+k+1)$ and a_{k+1}, respectfully. Since the last $p-\varrho_{k+1}$ rows of the matrix $\frac{\partial}{\partial u}[A_k^T, a_{k+1}^T]^T$ are linearly dependent on the first ϱ_{k+1} rows, we can write

$$\tilde{y}_1(t+1)=\tilde{a}_1(x(t), u(t)),$$
$$\cdots\cdots$$
$$\tilde{y}_{k+1}(t+k+1)=\tilde{a}_{k+1}(x(t), u(t), \{\tilde{y}_i(t+j), 1 \leqslant i \leqslant k, i+1 \leqslant j \leqslant k+1\}),$$
$$\hat{y}_{k+1}(t+k+1)=\hat{y}_{k+1}(x(t), \{\tilde{y}_i(t+j), 1 \leqslant i \leqslant k+1, i \leqslant j \leqslant k+1\}).$$

Denote

$$A_{k+1}=[A_k^T, \tilde{a}_{k+1}^T]^T.$$

End of the step $k+1$.

In the sequel we need a notion of regularity associated with the inversion algorithm which will be defined below.

Definition. *We call the equilibrium point* (x^0, u^0) *of the system* (1) *with* $w(t)=w^0$ *regular if, for an appropriate application of the inversion algorithm,*

$$\operatorname{rank}\frac{\partial}{\partial u}A_k(\cdot)=\varrho_k, \quad 1\leqslant k \tag{4}$$

in some neighbourhood of (x^0, u^0). *We call* (x^0, u^0) *strongly regular if* (4) *holds for each application of the algorithm.*

It has been proved by Kotta and Nijmeijer [2] that the integers $\varrho_1, \ldots, \varrho_k$ do not depend on the particular permutation of the components of $\hat{y}_k(t+k+1)$. Thus, using this algorithm around a strongly regular equilibrium point, we obtain a uniquely defined sequence of integers $0\leqslant\varrho_1\leqslant\ldots\leqslant\varrho_k\leqslant\ldots\leqslant\min(p, m)$. Let $\varrho^*=\max\{\varrho_k, k\geqslant 1\}$ and let α be defined as the smallest $k\in N$ such that $\varrho_k=\varrho^*$. In analogy with Moog [5], we call the ϱ_k's the invertibility indices of the system (1) with $w(t)=w^0$.

Next we shall show that around a regular equilibrium point the structure algorithm terminates in at most n steps. To prove it, we need the following Lemma.

Lemma 1. *If the rank of the matrix* $\partial A_\alpha(\cdot)/\partial u$ *is equal to* p (i. e. $\varrho_\alpha=p$) *in some neighbourhood* $\tilde{0}$ *of* $(x^0, u^0, y^0, \ldots, y^0)\in X\times U\times Y^{\alpha-1}$ *then for* $k=0, 1, \ldots, \alpha-1$ *on* $\pi_k(\tilde{0})$ (*where* $\pi_k : X\times U\times Y^{\alpha-1}\to X\times Y^k$ *is the projection along* $U\times Y^{\alpha-1-k}$ *on* $X\times Y^k$) *the following equalities hold*:

$$\operatorname{rank}\frac{\partial}{\partial x}\begin{bmatrix}\hat{y}_0(x)\\ \hat{y}_1(x, \tilde{y}_1(t+1))\\ \cdot\\ \cdot\\ \hat{y}_k(x, \{y_i(t+j), 1\leqslant i\leqslant k, i\leqslant j\leqslant k\})\end{bmatrix}=\sum_{i=0}^{k}(p-\varrho_i). \tag{5}$$

Proof. We shall prove the lemma only for the case $k=\alpha-1$; the proof for the other cases is analogous. Denote by $\hat{y}_{ik}(\cdot)$ the k th component of $\hat{y}_i(\cdot)$. Assume that

$$\operatorname{rank}\frac{\partial}{\partial x}\begin{bmatrix}\hat{y}_0(\cdot)\\ \ldots\ldots\\ \hat{y}_{\alpha-1}(\cdot)\end{bmatrix}<\sum_{i=0}^{\alpha-1}(p-\varrho_i),$$

and let, for example, (without loss of generality) the last row of the matrix $\partial(\hat{y}^T_0, \ldots, \hat{y}^T_{\alpha-1})^T/\partial x$ be linearly dependent on the other rows. Therefore, on the neighbourhood of $(x^*, y^*, \ldots, y^*)\in\pi_{\alpha-1}(\tilde{0})$ there exist functions $\gamma^*_{ik}(x, \{\tilde{y}_i(t+j), 1\leqslant i\leqslant\alpha-1, i\leqslant j\leqslant\alpha-1\})$ such that

$$\frac{\partial}{\partial x}\hat{y}_{\alpha-1,\, p-\varrho_\alpha-1}(\cdot)=$$

$$=\sum_{i=0}^{\alpha-2}\sum_{k=1}^{p-\varrho_i}\gamma^*_{ik}(\cdot)\frac{\partial}{\partial x}\hat{y}_{ik}(\cdot)+\sum_{k=1}^{p-\varrho_{\alpha-1}-1}\gamma^*_{\alpha-1,k}(\cdot)\frac{\partial}{\partial x}\hat{y}_{\alpha-1,k}(\cdot). \tag{6}$$

Because of the analyticity of the functions $\hat{y}_{ik}(\cdot)$ the equality (6) holds on some suitable neighbourhood of every point from the set $\pi_{\alpha-1}(\tilde{0})$, perhaps for the other functions $\gamma_{ik}(\cdot)$. Therefore, the equality (6) holds also around the point $(F(x, u), y^*, \ldots, y^*)$. Multiplying both sides of (6) to $\partial F(x, u)/\partial u$, and taking into account that

$$\frac{\partial}{\partial u}\hat{y}_{ij}(F(x,u),\cdot)=\frac{\partial}{\partial x}\hat{y}_{ij}(x,\cdot)\bigg|_{x=F(x,u)}\cdot\frac{\partial F}{\partial u}\bigg|_{(x,u)}$$

we obtain

$$\frac{\partial}{\partial u}\hat{y}_{\alpha-1,\,p-\rho_{\alpha-1}}(F(x,u),\cdot)=$$

$$=\sum_{i=0}^{\alpha-2}\sum_{k=1}^{p-\rho_i}\gamma_{ik}(\cdot)\frac{\partial}{\partial u}\hat{y}_{ik}(F(x,u),\cdot)+$$

$$+\sum_{k=1}^{p-\rho_{\alpha-1}-1}\gamma_{\alpha-1,k}(\cdot)\frac{\partial}{\partial u}\hat{y}_{\alpha-1,k}(F(x,u),\cdot).$$

Using the inversion algorithm the last equality results in

$$\frac{\partial}{\partial u}y_{\alpha,\,p-\rho_{\alpha-1}}(x,\{\tilde{y}_i(t+j),\ 1\leqslant i\leqslant\alpha-1,\ i\leqslant j\leqslant\alpha-1\})=$$

$$=\sum_{i=0}^{\alpha-2}\left(\sum_{k=1}^{\rho_{i+1}-\rho_i}\gamma_{ik}(\cdot)\frac{\partial}{\partial u}\tilde{y}_{i+1,k}(\cdot)+\sum_{k=\rho_{i+1}-\rho_i+1}^{p-\rho_i}\gamma_{ik}(\cdot)\frac{\partial}{\partial u}\hat{y}_{i+1,k}(\cdot)\right)+$$

$$+\sum_{k=1}^{p-\rho_{\alpha-1}-1}\gamma_{\alpha-1,k}(\cdot)\frac{\partial}{\partial u}\tilde{y}_{\alpha,k}(\cdot).\tag{7}$$

The left hand side of (7) is actually the last row of the matrix $\partial A_\alpha(\cdot)/\partial u$. The only difference is that $\tilde{y}_i(t+j)$ in this matrix are equal to $\tilde{y}_i(t+j-1)$ for $1\leqslant i\leqslant\alpha$, $i+1\leqslant j\leqslant\alpha$. This fact, of course, does not restrict the generality, because $\tilde{y}_i(t+j)$ can take arbitrary values around y_0. Recall that by the inversion algorithm $\partial\hat{y}_{ik}(\cdot)/\partial u=0$. Thus, the right hand side of (7) is the linear combination the $p-1$ first rows of the matrix $\partial A_\alpha(\cdot)/\partial u$ (where $\tilde{y}_i(t+j)=\tilde{y}_i(t+j-1)$). This contradiction proves the Lemma.

L e m m a 2. *Around the regular equilibrium point the structure algorithm terminates in at most n steps, i. e.*

$$\varrho^*=\varrho_n.$$

P r o o f. Assume at first that $\varrho^*=p$. Then by Lemma 1 we obtain

$$\operatorname{rank}\frac{\partial}{\partial x}\begin{bmatrix}\hat{y}_0(\cdot)\\ \cdots\\ \hat{y}_{\alpha-1}(\cdot)\end{bmatrix}=\sum_{i=0}^{\alpha-1}(p-\varrho_i)>\sum_{i=0}^{\alpha-1}1=\alpha.$$

From the other side, as $x\in X\subset R^n$, for every x we have

$$\operatorname{rank}\frac{\partial}{\partial x}\begin{bmatrix}\hat{y}_0(\cdot)\\ \cdots\\ \hat{y}_{\alpha-1}(\cdot)\end{bmatrix}\leqslant n.$$

In the case $\alpha>n$ this will give us a contradiction. Therefore, if $\varrho^*=p$, then $\alpha\leqslant n$.

In the general case ($\varrho^*<p$) we can extract a subsystem from the system (1) with $w=w^0$ that has ϱ^* outputs, and the result still holds.

3. Main result

In this section we give our main result. We define a compensator for a system (1) as follows. Let (x^0, u^0, w^0) be a strongly regular equilibrium point for (1) and let us apply the inversion algorithm to (1) with $w=w^0$. This yields at the nth step:

$$\tilde{Y}_n = A_n(x(t), u(t), \{\tilde{y}_i(t+j), 1\leq i\leq n-1, i+1\leq j\leq n\}), \tag{8}$$

$$\hat{y}_n(t+n) = \hat{y}_n(x(t), \{\tilde{y}_i(t+j), 1\leq i\leq n, i\leq j\leq n\}),$$

where $\tilde{Y}_n = [\tilde{y}_1^T(t+1), \tilde{y}_2^T(t+2), \ldots, \tilde{y}_n^T(t+n)]^T$ and the matrix $\partial A_n(\cdot)/\partial u$ has full row rank ϱ_n on a neighbourhood 0_n of (x^0, u^0). For $i=1, 2, \ldots, \varrho_n$, let $t+\gamma_i$ be the lowest time instant and $t+\varepsilon_i$ be the highest time instant in which y_i appears in (8). Then we can rewrite (8) as

$$[y_{\varrho_{k-1}+1}(t+k), \ldots, y_{\varrho_k}(t+k)]^T = \tag{9}$$
$$= \tilde{a}_k(x(t), u(t), \{y_i(t+j), 1\leq i\leq \varrho_{k-1}, \gamma_i+1\leq j\leq \min(k, \varepsilon_i)\}),$$
$$k=1, 2, \ldots, n.$$

After a possible permutation of inputs we may assume that the Jacobian matrix of the right hand side of (9) with respect to $u^1=(u_1, \ldots, u_{\varrho_n})^T$ around the point $(x^0, u^0, y^0, \ldots, y^0)$ has full row rank ϱ_n. Therefore, the equation (9) can be solved for $u^1(t)$ uniquely around the point $(x^0, u^0, y^0, \ldots, y^0)$ by applying the Implicit Function Theorem. Define $u^2=(u_{\varrho_n+1}, \ldots, u_m)^T$. Then, from (9), we obtain

$$u^1(t) = \varphi(x(t), \{y_i(t+j), 1\leq i\leq \varrho_n, \gamma_i\leq j\leq \varepsilon_i\}, u^2(t)) \tag{10}$$

which is such that for $k=1, 2, \ldots, n$

$$[y_{\varrho_{k-1}+1}(t+k), \ldots, y_{\varrho_k}(t+k)]^T \equiv \tag{11}$$
$$\equiv \tilde{a}_k(x(t), \varphi(\cdot), \{y_i(t+j), 1\leq i\leq \varrho_{k-1}, \gamma_i+1\leq j\leq \min(k, \varepsilon_i)\}).$$

Notice that $\varphi: V_1 \to V_2$ is analytic for some (possible small) neighbourhoods V_1 and V_2 of $(x^0, y^0, \ldots, y^0, u^{20})$ in $X^0\times(Y^0)^n\times U^{20}$ and u^{10} in U^{10}. This implies that (11) will hold as long as $(x(t), \{y_i(t+j), 1\leq i\leq \varrho_n, \gamma_i\leq j\leq \varepsilon_i\}, u^2(t)) \in V_1$ and defined by (10) $u^1(t)\in V_2$. Of course, the identity (11) is lost if we leave the neighbourhoods V_1 *resp.* V_2, which may happen for some t_F.

Now, construct the compensator for (1) in the following way. Let $z_i = (z_{i1}, \ldots, z_{i,\varepsilon_i-\gamma_i})^T$, $i=1, \ldots \varrho_n$ be a vector of dimension $\varepsilon_i-\gamma_i$, v^2 a vector of dimension $m-\varrho_n$, and consider the system

$$\begin{gathered} z_i(t+1) = A_i z_i(t) + B_i v_i(t), \\ i=1, \ldots, \varrho_n, \\ u^1(t) = \varphi(x(t), \{z_{ij}(t), 1\leq j\leq \varepsilon_i-\gamma_i, v_i(t), 1\leq i\leq \varrho_n\}, v^2(t)), \\ u^2(t) = v^2(t) \end{gathered} \tag{12}$$

with inputs $v^1=(v_1, \ldots, v_{\varrho_n})^T$ and v^2, outputs $u=(u^{1,T}, u^{2,T})^T$ and (A_i, B_i) in Brunovsky canonical form

$$A_i = \begin{bmatrix} 0 & 1_{\varepsilon_i-\gamma_i-1} \\ 0 \ldots & 0 \end{bmatrix}, \quad B_i = (0, \ldots 0 \; 1)^T.$$

Now we shall show that the compensator (12) is regular.

L e m m a 3. *The compensator* (12) *is invertible on a neighbourhood of* (x^0, u^0, y^0).

P r o o f. Obviously, (12) is invertible if and only if the system

$$z_i(t+1)=A_i z_i(t)+B_i v_i(t), \qquad i=1, \ldots, \varrho_n, \tag{13a}$$

$$u^1(t)=\varphi(x(t), z_1(t), \ldots, z_{\varrho_n}(t), v^1(t), v^{20}) \tag{13b}$$

is invertible. The system is invertible if it is both right and left invertible. Right invertibility of (13) means that given arbitrary u^{1*}, we can construct v^{1*} that yields this desired u^{1*}. From (13a) we obtain

$$v_i(t)=z_{i,\varepsilon_i-\gamma_i}(t+1)=\ldots=z_{i1}(t+\varepsilon_i-\gamma_i). \tag{14}$$

Denote the ith row of $\tilde{a}_k(\cdot)$ in the structure algorithm by $a_{ki}(\cdot)$. Taking into account that (13b) has been obtained around (x^0, u^0, y^0) as a solution of (9) with $u^2=u^{20}$, $y_i(t+\gamma_i+j-1)=z_{ij}(t)$, $j=1, \ldots, \varepsilon_i-\gamma_i$ and $y_i(t+\varepsilon_i)=v_i(t)$, we obtain

$$z_{i1}(t)=a_{1i}(x(t), u^1(t), u^{20}), \quad 1\leqslant i\leqslant \varrho_1,$$

and taking into account (14), we can construct for $i=1, \ldots, \varrho_1$ the new control $v_i^*(t)$ that yields u^{1*}:

$$v_i^*(t)=a_{1i}(x(t+\varepsilon_i-\gamma_i), u^{1*}(t+\varepsilon_i-\gamma_i), u^{20}), \qquad i=1, \ldots, \varrho_1. \tag{15}$$

Let us denote

$$\hat{z}_1(t)=\{z_{i2}(t): \varepsilon_i>2, \ i=1, \ldots, \varrho_1\},$$

$$\hat{v}_1(t)=\{v_i(t): \varepsilon_i=2, \ i=1, \ldots, \varrho_1\}.$$

Then we obtain

$$z_{i1}(t)=a_{2i}(x(t), u^1(t), u^{20}, \hat{z}_1(t), \hat{v}_1(t)), \quad \varrho_1+1\leqslant i\leqslant \varrho_2,$$

and taking into account (14), (15), we can construct for $i=\varrho_1+1, \ldots, \varrho_2$ the new control $v_i^*(t)$ that yields u^{1*}:

$$v_i^*(t)=a_{2i}(x(t+\varepsilon_i-\gamma_i), u^{1*}(t+\varepsilon_i-\gamma_i), u^{20}, \hat{z}_1(t+\varepsilon_i-\gamma_i),$$

$$\hat{v}_1^*(t+\varepsilon_i-\gamma_i)), \ \varrho_1+1\leqslant i\leqslant \varrho_2.$$

Applying the above arguments repeatedly, we prove that given arbitrary u^{1*}, we can construct v^{1*} that yields this desired u^{1*}. Hence the compensator (12) is right invertible on a neighbourhood of (x^0, u^0, y^0).

The left invertibility of (12) on a neighbourhood of (x^0, u^0, y^0) means that if whenever $v(0)\in Y^0$ and $v^*(0)\in Y^0$ are distinct controls, the corresponding output sequences differ, and it follows from the fact that the equation (9) can be solved uniquely for $u^1(t)$ around the point (x^0, u^0, y^0) by applying the Implicit Function Theorem.

Our main result can now be stated as follows.

T h e o r e m. *Consider the system* (1) *around a strongly regular equilibrium point* (x^0, u^0, w^0). *Apply the inversion algorithm to* (1) *with* $w=w^0$. *Then the DDDP for system* (1) *is locally finite time solvable around* (x^0, u^0, w^0) *if and only if for* $0\leqslant k\leqslant n-1$

$$\frac{\partial \hat{y}_k(f(x, u, w), \ \{\tilde{y}_i(t+j+1), \ 1\leqslant i\leqslant k, \ i\leqslant j\leqslant k\})}{\partial w}=0. \tag{16}$$

Moreover, if (16) holds, the DDDP can be solved around (x^0, u^0, w^0) by means of the compensator (12) with arbitrary initial state.

P r o o f. Sufficiency. If (16) holds, then applying the inversion algorithm to (1) with $w=w^0$ gives the same result as applying it to (1) where we consider w as a parameter. It is now easy to see that the compensator (12) applied to (1) locally around (x^0, u^0, w^0) yields

$$y_i(\gamma_i+j-1)=z_{ij}(0), \qquad j=1, \ldots, \varepsilon_i-\gamma_i$$

$$y_i(t+\varepsilon_i)=v_i(t), \qquad 0\leqslant t\leqslant t_F, \qquad i=1, \ldots, \varrho_n,$$

which is independent from $w(t)$. Furthermore, if (16) holds for $0\leqslant k\leqslant n-1$, then it holds for every $k\geqslant 0$. Therefore,

$$y_i(j) \quad \text{for} \quad (1\leqslant i\leqslant \varrho_n,\ 0\leqslant j\leqslant \gamma_i-1) \quad \text{and} \quad (\varrho_n+1\leqslant i\leqslant p,\ j\geqslant 0)$$

being the components of $\hat{y}_k(t+k)$, $k\geqslant 0$ are independent from $w(t)$ by assumption. Therefore, (12) solves the DDDP.

N e c e s s i t y. Let us assume that there exists a regular dynamic feedback control defined by (2) for (1) that locally finite time solves the DDDP. Furthermore, assume that (16) does not hold for $k=0$, that is

$$\frac{\partial h(f(x, u, w))}{\partial w} \neq 0.$$

Then at the first step of the inversion algorithm we have that $y(t+1)$ explicitly depends on w:

$$y(t+1)=h(f(x(t), u(t), w(t))) \tag{17}$$

Since (2) solves the DDDP for (1), this w-dependence should disappear if we plug (2) in (17). However, this is not possible, since (2) does not depend on w. Thus (2) must be such that it imposes the constraint

$$\frac{\partial h(f(x, u, w))}{\partial w} \stackrel{\triangle}{=} \zeta_1(x, u, w)=0.$$

But this would imply the nonregularity of (2). So we necessarily have that (16) holds for $k=0$. Next, assume that (16) does not hold for $k=1$. Then we obtain at the second step of the inversion algorithm applied to (1) (where we consider w as a parameter):

$$\frac{\partial \hat{y}_1(f(x, u, w),\ \tilde{y}_1(t+2))}{\partial w} \neq 0.$$

Using the same argument as above, we can see that this w-dependence will not disappear, unless (2) is constructed in such a way that the constraint

$$\frac{\partial \hat{y}_1(f(x, u, w),\ \tilde{y}_1(t+2))}{\partial w} = \zeta_2(x, u, w, \tilde{y}_1(t+2))=0$$

is imposed on the system. That will imply the existence of $k\in\{1, \ldots, \varrho_n\}$ such that v_k cannot be chosen arbitrarily and would contradict the regularity of (2). Therefore, (11) has to hold for $k=1$. Applying this algorithm repeatedly, we can show that (11) holds for $k=0, 1, \ldots, n-1$.

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Ülle KOTTA

HÄIRINGUTE DÜNAAMILINE KOMPENSEERIMINE DISKREETSETES MITTELINEAARSETES SÜSTEEMIDES: ERINEVA SISENDITE JA VÄLJUNDITE ARVUGA MITTEPÖÖRATAVATE SÜSTEEMIDE JUHT

On vaadeldud mitme sisendi ja mitme väljundiga diskreetse mittelineaarse süsteemi tarbeks sellise dünaamilise tagasiside kujul esitatud kompensaatori konstrueerimise ülesannet, mis tagaks suletud süsteemi väljundite invariantsuse mittemõõdetavate häiringute suhtes. Varasemad tulemused on üldistatud juhule, kus juhtimisobjekti sisendite ja väljundite arv võib olla erinev ning ta ei pruugi olla pööratav. Ülesande lahendus põhineb pööramisalgoritmil: tema terminites on esitatud ülesande lokaalse lahendatavuse tarvilikud ja piisavad tingimused regulaarse (pööratava) kompensaatori abil juhtimisobjekti tasakaalupunkti ümbruses. Nende tingimuste täidetuse korral on tuletatud kompensaatori võrrandid.

Юлле КОТТА

ДИНАМИЧЕСКАЯ КОМПЕНСАЦИЯ ВОЗМУЩЕНИЙ В ДИСКРЕТНЫХ НЕЛИНЕЙНЫХ СИСТЕМАХ: СЛУЧАЙ НЕКВАДРАТНЫХ И НЕОБРАТИМЫХ СИСТЕМ

В статье для дискретных нелинейных систем со многими входами и выходами рассматривается задача построения компенсатора в виде динамической обратной связи по состоянию, обеспечивающего инвариантность выхода замкнутой системы по отношению к неизмеряемым возмущениям. Ранние результаты обобщаются для случая необратимых систем с несовпадающим числом входов и выходов. Решение задачи основывается на алгоритме обращения системы: в его терминах представлены необходимые и достаточные условия разрешимости задачи с помощью регулярного (обратимого) компенсатора локально в окрестности точки равновесия системы. При выполнении условий разрешимости задачи найдены уравнения компенсатора.