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ON *M*-IDEALS OF COMPACT OPERATORS AND LORENTZ SEQUENCE SPACES

(Presented by G. Vainikko)

1. The Lorentz sequence spaces $d(w,p)$ resemble the l_p-spaces in many respects (see e. g. [1]). Let $E\subset l_p(\Gamma)$ and $F\subset l_q(\Gamma')$ or $F\subset d(w,q)$ be infinite dimensional closed subspaces. Recently we have proved in [2] that the subspace $K(E,F)$ of compact operators is an uncomplemented M-ideal in the space $L(E,F)$ of the bounded linear operators, if E and F have the compact approximation property and $1<p\leqslant q<\infty$. In this note we are interested in the case $p>q$.

We shall show that in contrast to the case $p\leqslant q$, for $p>q$ the situation is different in the cases when $F\subset l_q(\Gamma')$ or $F\subset d(w,q)$. Namely, if $F\subset l_q(\Gamma')$, then $K(E,F)=L(E,F)$ (this fact can be proved similarly to the Pitt theorem in [1], p. 76, see also Corollary 1 below), i. e. $K(E,F)$ is a trivial M-ideal in $L(E,F)$. If $F\subset d(w,q)$, then two alternative cases are presented: (1) $K(E,F)$ is a trivial M-ideal in $L(E,F)$ for all E and F (Corollary 2), and (2) $K(l_p,d(w,q))$ is not an M-ideal in $L(l_p,d(w,q))$ (cf. Corollary 4). In the second case, $K(E,F)$ is an HB-subspace of $L(E,F)$ if one of the subspaces E or F has the compact approximation property [2], and it may happen that $K(E,F)=L(E,F)$, because $l_p(\Gamma)$ and $d(w,q)$ are «saturated» with the subspaces isomorphic to l_p and l_q, respectively.

We shall see also that the situation for $K(d(v,p)^*,d(w,q))$ is similar to the case $K(l_{p'},d(w,q))$, where p' is defined by $1/p+1/p'=1$.

2. In the sequel all spaces will be either real or complex; X and Y will denote Banach spaces, and we shall suppose $1<p,\ q<\infty$ unless stated otherwise.

Let $w(w_i)=(w_i)_{i\geqslant1}$ be a non-increasing sequence of positive numbers such that $w_1=1$, $\lim_{i\to\infty}w_i=0$ and $\sum_{i=1}^{\infty}w_i=\infty$. The Lorentz sequence space $d(w,q)$ is the Banach space of all sequences of scalars $x=(a_1,a_2,\ldots)$ for which the norm is

$$\|x\|=\sup_{\pi}\Big(\sum_{i=1}^{\infty}w_i|a_{\pi(i)}|^q\Big)^{1/q}<\infty,$$

where π ranges over all the permutations of the natural numbers **N**. Background material on Lorentz sequence spaces can be found e. g. in [1].

A subspace Y of X is called an M-ideal [3] if its annihilator $Y^{\perp}$ is complemented in X^* by a subspace G such that for each $f\in X^*$

$$\|f\|=\|g\|+\|h\|, \tag{1}$$

whenever $f=g+h$, $g\in G$, $h\in Y^{\perp}\setminus\{0\}$. When (1) is replaced by

$$\|f\|>\|g\|,\quad \|f\|\geqslant\|h\|,$$

then Y is called an HB-subspace [4].

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Let $B(a, r)$ denote the closed ball in X with center a and radius r. A subspace Y of X is called a semi M-ideal (cf. [5], Section 6) if for every two balls $B(a_1, r_1)$ and $B(a_2, r_2)$ in X with the properties $B(a_1, r_1) \cap B(a_2, r_2) \neq \emptyset$ and $B(a_i, r_i) \cap Y \neq \emptyset$ for $i=1, 2$, one has for all $\varepsilon > 0$

$$B(a_1, r_1+\varepsilon) \cap B(a_2, r_2+\varepsilon) \cap Y \neq \emptyset.$$

Every M-ideal is a semi M-ideal, but the converse is false in general [5].

A sequence $(x_n) \subset X$ is said to dominate a sequence $(y_n) \subset Y$ if there exists $T \in L([x_n], [y_n])$ (we denote by $[x_n]$ the closed linear subspace of X spanned by $\{x_1, x_2, \ldots\}$) such that $Tx_n = y_n$, $n \in \mathbf{N}$. In this case we shall write $(x_n) > (y_n)$ or $(y_n) < (x_n)$.

We have the following simple

P r o p o s i t i o n 1. *Let* (e_k) *and* (f_k) *be two sequences in Banach spaces. Suppose* X *is reflexive and assume that every sequence* $(x_n) \subset X$ *such that* $x_n \to 0$ *weakly and* $\|x_n\| \not\to 0$ *has a subsequence* $(x_{n_k}) < (e_k)$. *Suppose also that every sequence* $(y_n) \subset Y$ *such that* $y_n \to 0$ *weakly and* $\|y_n\| \not\to 0$ *has a subsequence* $(y_{n_k}) > (f_k)$. *If* (e_k) *dominates all its subsequences, but* (f_k) *is not dominated by* (e_k), *then* $K(X, Y) = L(X, Y)$.

P r o o f. Assume that there exists $T \in L(X, Y) \setminus K(X, Y)$. Then there are $(x_n) \subset X$ and $\alpha > 0$ such that $x_n \to 0$ weakly and $\|Tx_n\| \geqslant \alpha$, $n \in \mathbf{N}$. Then $Tx_n \to 0$ weakly and $\|x_n\| \geqslant \alpha / \|T\|$. By passing to subsequences of (x_n), and then of (Tx_n), we may assume that $(x_n) < (e_n)$ and $(Tx_n) > (f_n)$. This leads us to $(e_n) > (f_n)$, which is a contradiction.

Let (e_k) and (f_k) be the unit vector bases in l_p and l_q, respectively. Then $(e_k) > (f_k)$ if and only if $p \leqslant q$. It is well known also (see e. g. [1], pp. 7, 53) that every sequence $(x_n) \subset l_p$ such that $x_n \to 0$ weakly and $\|x_n\| \not\to 0$ has a subsequence which is equivalent to (e_k). Therefore, from Proposition 1 we get

C o r o l l a r y 1. *Let* E *and* F *be closed subspaces of* $l_p(\Gamma)$ *and* $l_q(\Gamma')$, *respectively. If* $p > q$, *then* $K(E, F) = L(E, F)$.

Let (e_k) and (f_k) be now the unit vector bases in l_p and $d(w, q)$, respectively, and $p > q$. If $w = (w_i) \in l_{(p/q')} = l_{(p/(p-q))}$, then for every choice of scalars $a_1, \ldots, a_n$, $n \in \mathbf{N}$, we have

$$\left(\sum_{i=1}^{n} w_i |a_i|^q\right)^{1/q} \leqslant \left(\sum_{i=1}^{n} w_i^{(p/q)'}\right)^{1/((p/q)'q)} \left(\sum_{i=1}^{n} |a_j|^p\right)^{1/p}.$$

Hence $(e_k) > (f_k)$. If $w = (w_i) \notin l_{(p/q)'}$, then there is $(b_i) \in l_{p/q}$, $b_1 \geqslant b_2 \geqslant \ldots > 0$, so that $\sum w_i b_i = \infty$. Consequently, there is no $C > 0$ such that

$$\left\| \sum_{k=1}^{n} (b_k)^{1/q} f_k \right\| \leqslant C \left\| \sum_{k=1}^{n} (b_k)^{1/q} e_k \right\|$$

for all $n \in \mathbf{N}$. Hence, (f_k) is not dominated by (e_k).

C o r o l l a r y 2. *Let* E *and* F *be closed subspaces of* $l_p(\Gamma)$ *and* $d(w, q)$ *respectively. If* $p > q$ *and* $w \notin l_{p/(p-q)}$, *then* $K(E, F) = L(E, F)$.

For the p r o o f it is sufficient to show that in Proposition 1 the space F may be taken instead of Y. Consider $(y_n) \subset F$ such that $y_n \to 0$ weakly and $\|y_n\| \not\to 0$. By passing to a subsequence we may assume (cf. [1], p. 7) that (y_n) is equivalent to a normalized block basis (u_n) of the unit vector basis (f_k) of $d(w, q)$. Let

$$u_n = \sum_{k=m_n+1}^{m_{n+1}} a_k f_k, \quad n \in \mathbf{N}. \tag{2}$$

If $\lim a_k=0$, then (u_n) contains a subsequence which is equivalent to the unit vector basis of l_q (see [1], p. 177). But this basis dominates (f_k). Hence, there exists $(y_{n_k})>(f_k)$.

Assume now that $a_k \not\to 0$. Then $|a_{j_k}|\geqslant\varepsilon$ for a sequence $j_1<j_2<\dots$ of integers and a positive number ε. By passing to the subsequence of those u_n whose expansion (2) contains a_{j_k} we may assume that

$$\forall n\in\mathbf{N} \quad \exists k(n)\in\{m_n+1,\dots,m_{n+1}\} \quad |a_{k(n)}|\geqslant\varepsilon.$$

Since (f_k) is an unconditional basis with the unconditional constant equal to 1, we have for every choice of scalars $b_1,\dots,b_N$, $N\in\mathbf{N}$,

$$\|\sum_{n=1}^{N} b_n u_n\|=\|\sum_{n=1}^{N}\sum_{k=m_n+1}^{m_{n+1}} a_k b_n f_k\|\geqslant\varepsilon\|\sum_{n=1}^{N} b_n f_{k(n)}\|/2=\varepsilon\|\sum_{n=1}^{N} b_n f_n\|/2.$$

Hence, (y_n) contains a subsequence $(y_{\tilde{n}_k})>(f_k)$. This completes the proof.

3. To prove that $K(l_p,d(w,q))$ is not an M-ideal for $p>q$ and $w\in l_{p/(p-q)}$, we need

Proposition 2. *Let X have a basis (e_k) and let Y have an unconditional basis (f_k) with the unconditional constant equal to* 1. *Suppose that there is $T\in L(X,Y)$ such that $Te_k=f_k$, $k\in\mathbf{N}$ (i. e. $(e_k)>(f_k)$). If there are $D\in K(X,Y)$ $\delta>0$ and $N\in\mathbf{N}$ such that $\|D\|\leqslant\|T\|<\delta$ and*

$$\|P^nT+D\|\geqslant\delta \quad \forall n\geqslant N,$$

where P^n is defined by $P^n y=\sum_{k=n+1}^{\infty} a_k f_k$ for $y=\sum_{k=1}^{\infty} a_k f_k\in Y$, then $K(X,Y)$ is not a semi M-ideal in $L(X,Y)$.

Proof. Let $P_n=I_Y-P^n$ be the natural projections. Since $\|P_nD-D\|\to0$, we may suppose that $D=P_mD$ for an integer $m\geqslant N$. Let $B_1=B(P^mT+D,\|T\|)$ and $B_2=B(P^mT-D,\|T\|)$. Then $B_1\cap B_2\ni P^mT$, $B_1\cap K(X,Y)\ni D$ and $B_2\cap K(X,Y)\ni(-D)$. Assume for contradiction that there is

$$A\in B(P^mT+D,\|T\|/2+\delta/2)\cap B(P^mT-D,\|T\|/2+\delta/2)\cap K(X,Y);$$

we may suppose that $A=P_nA$ for some $n\geqslant m$. Then

$$\begin{aligned}2\delta&\leqslant\|2(P^nT+D)\|\leqslant\\&\leqslant\|P^nT+D-P_mA\|+\|P^nT+D+P_mA\|=\\&=\|(P_m+P^n)(P^mT+D-A)\|+\|-P^nT+D+P_mA\|\leqslant\\&\leqslant\|T|/2+\delta/2+\|-P^mT+D+A\|\leqslant\\&\leqslant\|T\|+\delta<2\delta.\end{aligned}$$

This is a contradiction, so $K(X,Y)$ cannot be a semi M-ideal.

Proposition 2 improves Lemma 2.4 in [4], where $X=Y$, $(e_k)=(f_k)$ is unconditionally monotone, $T=I_X$ and D has a finite matrix.

Let us mention the following simple consequence of Proposition 2 which permits to have new examples of spaces $K(l_1,Y)$ that are not semi M-ideals in $L(l_1,Y)$.

Corollary 3. *Let Y have an unconditional basis (f_k) with the unconditional constant equal to* 1 *such that*

$$\sup_k\|f_k\|<\inf_k\|f_1+f_k\|.$$

Then $K(l_1,Y)$ is not a semi M-ideal in $L(l_1,Y)$.

For the proof one can take $D\in K(l_1,Y)$ defined by $D(a_1,a_2,\dots)=\sum_{k=1}^{\infty} a_k f_1$, $(a_1,a_2,\dots)\in l_1$, $\delta\in(\sup\|f_k\|,\inf\|f_1+f_k\|]$ and $N=1$.

The conditions of Corollary 3 are satisfied e. g. for $Y=l_p$ and $Y=d(w, p)$, $1\leqslant p<\infty$. Let us note that the example $K(l_1, l_p)$ is due to Saatkamp [6]. For the real case of the examples $K(l_1, Y)$ that are not semi M-ideals, cf. [7].

Corollary 4. *If $p>q$ and $w\in l_{p/(p-q)}$, then $K(l_p(\Gamma)\ d(w, q))$ is not a semi M-ideal in $L(l_p(\Gamma),\ d(w, q))$; $K(l_p(\Gamma),\ d(w, q))$ is an uncomplemented HB-subspace of $L(l_p(\Gamma), d(w, q))$.*

Proof. Let (e_k) and (f_k) be the unit vector bases in l_p and $d(w, q)$, respectively. Let T be the operator which sends e_k to f_k, $k\in\mathbf{N}$. Above we have seen that $T\in L(l_p, d(w, q))$. Since $e_k\rightarrow 0$ weakly and $\|f_k\|\not\rightarrow 0$, T is not compact. Since $K(l_p, d(w, q))\neq L(l_p, d(w, q))$, $d(w, q)$ has an unconditional basis and l_p is complemented in $l_p(\Gamma)$, we have according to [8] and [9] that $K(l_p(\Gamma), d(w, q))$ is not complemented in $L(l_p(\Gamma), d(w, q))$. It is an HB-subspace according to [10] or [2].

Since l_p is 1-complemented in $l_p(\Gamma)$, it follows from the definition of semi M-ideal that $K(l_p(\Gamma), d(w, q))$ is not a semi M-ideal, if $K(l_p, d(w, q))$ is not a semi M-ideal (in the corresponding spaces of the bounded linear operators). Let us prove that $K(l_p, d(w, q))$ is not a semi M-ideal.

We have $\|T\|=W^{(p-q)/(pq)}$, where $W=\sum_{i=1}^{\infty} w_i^{p/(p-q)}$. Define $D\in K(l_p, d(w, q))$ by $De_1=\|T\|f_1$, $De_2=De_3=\ldots=0$. Then $\|D\|=\|T\|$. Let

$$x_n=(\underbrace{0, \ldots, 0}_{n},\ w_2^{1/(p-q)},\ w_3^{1/(p-q)}, \ldots),\quad n\in\mathbf{N},$$

and let $a>W^{1/p}$. Then

$$\|ae_1+x_n\|=(a^p+W-1)^{1/p}=b$$

in l_p,

$$\|(P^nT+D)(ae_1+x_n)\|=(a^qW^{(p-q)/p}+W-1)^{1/q}=c$$

in $d(w, q)$, and for $\delta=c/b$, we have $\delta>\|T\|$ and $\|P^nT+D\|\geqslant\delta$, $n\in\mathbf{N}$. We conclude by applying Proposition 2.

4. Let $d(v, p)$ and $d(w, q)$ be two Lorentz sequence spaces. Note that the dual space $d(v, p)^*$ is canonically identified with a sequence space (see e. g. [1]) but we do not need the explicit representation of $d(v, p)^*$ in the sequel. If we suppose $p'\leqslant q$, i. e $p\leqslant(p-1)q$, then we have similarly to the case $K(l_{p'}, d(w, q))$ the following result.

Proposition 3. *Let $E\subset d(v, p)^*$ and $F\subset d(w, q)$ be infinite dimensional closed subspaces having the compact approximation property. If $p\leqslant(p-1)q$, then $K(E, F)$ is an uncomplemented M-ideal in $L(E, F)$.*

Proof. Since E and F have complemented subspaces isomorphic to $l_{p'}$ and l_q, respectively (cf. [11] and [1], p. 177), $K(E, F)$ is not complemented in $L(E, F)$ by [12]. To prove that $K(E, F)$ is an M-ideal we apply Theorem 3 in [2] (or Theorem 4 in [13]) to the sequences of natural projections (associated to the unit vector bases) and to the functions $N_1(a, b)=(a^{p'}+b^{p'})^{1/p'}$ and $N_2(a, b)=(a^q+b^q)^{1/q}$, $a, b\geqslant 0$, using also the following simple lemma in the case $N(a, b)=(a^p+b^p)^{1/p}$.

Lemma. *Let N be a function on $[0, \infty)\times[0, \infty)$ such that $N(a, b)\leqslant N(c, d)$ for $a\leqslant c$ and $b\leqslant d$. Let N^* be the function on $[0, \infty)\times[0, \infty)$ defined by $N^*(c, d)=\sup\{ac+bd: N(a, b)\leqslant 1\}$. Let $P\in L(X, X)$ be a projection such that $\|P\|=\|Q\|=1$, where $Q=I_X-P$. If*

$$\|Px+Qy\|\leqslant N(\|x\|, \|y\|),\quad x, y\in X,$$

then

$$N^*(\|P^*x^*\|, \|Q^*y^*\|)\leqslant\|P^*x^*+Q^*y^*\|,\quad x^*, y^*\in X^*.$$

P r o o f. Let $\varepsilon \in (0, 1)$. Pick $x = Px$ and $y = Qy$ so that $\|x\|, \|y\| \leqslant 1$ and $(P^*x^*)(x) \geqslant (1-\varepsilon)\|P^*x^*\|$, $(Q^*y^*)(y) \geqslant (1-\varepsilon)\|Q^*y^*\|$. Then

$$\|P^*x^*+Q^*y^*\| \geqslant \sup\{|(P^*x^*+Q^*y^*)(ax+by)|: a, b \geqslant 0, \|ax+by\| \leqslant 1\} \geqslant$$
$$\geqslant \sup\{a(P^*x^*)(x)+b(Q^*y^*)(y): N(a, b) \leqslant 1\} \geqslant$$
$$\geqslant (1-\varepsilon)N^*(\|P^*x^*\|, \|Q^*y^*\|).$$

R e m a r k. The special case of Proposition 3 that $K(E, F)$ is an M-ideal in $L(E, F)$ for real space $E = d(v, p)^*$ and $F = d(w, q)$ was proved in [14] by Banach lattices methods.

Let $p > (p-1)q$ now. We have two alternative cases: $d(v, p)^* \subset \subset d(w, q)$, and $d(v, p)^* \not\subset d(w, q)$. Let us see now that they both may occur.

Denote by (e_k), (e_k^*) and (f_k) the unit vector bases in $d(v, p)$, $d(v, p)^*$ and $d(w, q)$, respectively. For any choice of scalars $b_1, \ldots, b_n$ and permutation $\pi: \mathbf{N} \to \mathbf{N}$, we get by Hölder's inequality

$$\|\sum_{i=1}^{n} |b_i|^{1/q} v_i^{1/p} e^*_{\pi(i)}\|^{p'} \leqslant \sum_{i=1}^{n} |b_i|^{p'/q}. \tag{3}$$

If $w \in l_{(p'/q)'}$, then similarly to Section 2 we have for some $c > 0$ and every choice of scalars $a_1, \ldots, a_n$

$$c(\sum_{i=1}^{n} w_i |a_i|^q)^{1/q} \leqslant (\sum_{i=1}^{n} |a_i|^{p'})^{1/p'} \leqslant$$
$$\leqslant \sup\{|(\sum_{i=1}^{n} a_i e_i^*)(\sum_{j=1}^{n} \lambda_j e_j)|: \|\sum_{j=1}^{n} \lambda_j e_j\| \leqslant 1\} \leqslant$$
$$\leqslant \|\sum_{i=1}^{n} a_i e_i^*\|.$$

Consequently, $d(v, p)^* \subset d(w, q)$.

If $(v_i^{q/p} w_i) \notin l_{(p'/q)'}$, then there exists a sequence $(b_i) \in l_{p'/q}$ such that $b_1 \geqslant b_2 \geqslant \ldots > 0$ and $\sum b_i v_i^{q/p} w_i = \infty$. Therefore $x = (b_i^{1/q} v_i^{1/p}) \notin \notin d(w, q)$. But $x \in d(v, p)^*$ according to (3). Thus, $d(v, p)^* \not\subset d(w, q)$.

Let us deduce from Proposition 1

C o r o l l a r y 5. *Let E and F be closed subspaces of $d(v, p)^*$ and $d(w, q)$, respectively. If $p > (p-1)q$ and $d(v, p)^* \not\subset d(w, q)$, then $K(E, F) = L(E, F)$.*

P r o o f. Since every subsequence of (e_k) is equivalent to (e_k), every subsequence of (e_k^*) is equivalent to (e_k^*). By hypothesis, (f_k) is not dominated by (e_k^*). And in the proof of Corollary 2 we have seen that the space F may be taken instead of Y in Proposition 1.

Consider $(x_n) \subset E$ such that $x_n \to 0$ weakly and $\|x_n\| \not\to 0$. By passing to a subsequence we may assume that (x_n) is equivalent to a normalized block basis (u_n^*) of (e_k^*). Let (u_n) be a normalized block basis of (e_k) such that $1 \geqslant u_n^*(u_n) \geqslant 1/2$, and $u_n^*(u_m) = 0$ for $n \neq m$. Substituting (u_n^*) by the equivalent sequence $(u_n^*/(u_n^*(u_n)))$ we may assume that $u_n^*(u_m) = \delta_{nm}$. As in the proof of Corollary 2, we have $(u_{n_k}) > (e_k)$ for some subsequence (u_{n_k}). But then $(u_{n_k}^*) < (e_k^*)$ and therefore $(x_{n_k}) < < (e_k^*)$.

We conclude by applying Proposition 1.

C o r o l l a r y 6. *If $p > (p-1)q$ and $d(v, p)^* \subset d(w, q)$, then $K(d(v, p)^*, d(w, q))$ is not a semi M-ideal in $L(d(v, p)^*, d(w, q))$;*

$K(d(v,p)^*, d(w,q))$ *is an uncomplemented HB-subspace of* $L(d(v,p)^*, d(w,q))$.

Proof. Since $d(v,p)^* \subset d(w,q)$, by the closed graph theorem, the formal identity map $T: d(v,p)^* \to d(w,q)$ is continuous. Therefore, the second statement may be proved similarly to that of Corollary 4. Let us now prove the first statement using Proposition 2.

Define $D \in K(d(v,p)^*, d(w,q))$ by $De_1^* = \|T\| f_1$, $De_2^* = De_3^* = \ldots = 0$. Then $\|D\| = \|T\|$. Denote $B = B(0,1)$ of $l_{p'/q}$. For all $n \in \mathbf{N}$ and $b = (b_i) \in B$, we put

$$x_{nb} = (\underbrace{|b_1|^{1/q}, 0, \ldots, 0}_{n}, |b_2|^{1/q} v_2^{1/p}, |b_3|^{1/q} v_3^{1/p}, \ldots).$$

By (3), we have $x_{nb} \in B(0,1)$ of $d(v,p)^*$ and therefore

$$\|P^n T + D\| \geqslant \sup_{b \in B} \|(P^n T + D) x_{nb}\| \geqslant$$

$$\geqslant \sup_{b \in B} (\|T\|^q |b_1| + \sum_{i=2}^{\infty} w_i |b_i| v_i^{q/p})^{1/q} =$$

$$= (\|T\|^{qr} + \sum_{i=2}^{\infty} v_i^{qr/p} w_i^r)^{1/(qr)} = \delta, \quad n \in \mathbf{N},$$

where $r = (p'/q)'$. Since $\delta > \|T\|$, it suffices to apply Proposition 2.

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KOMPAKTSETE OPERAATORITE *M*-IDEAALIDEST JA LORENTZI JADARUUMIDEST

On kindlaks tehtud, millal kompaktsete operaatorite alamruumid $K(l_p(\Gamma), d(w,q))$ ja $K(d(v,p)^*, d(w,q))$ kujutavad endast *M*-ideaale vastavates kõigi pidevate lineaarsete operaatorite ruumides ning millal mitte.

Эве ОЯ

ОБ *M*-ИДЕАЛАХ КОМПАКТНЫХ ОПЕРАТОРОВ И ЛОРЕНЦОВЫХ ПРОСТРАНСТВАХ ПОСЛЕДОВАТЕЛЬНОСТЕЙ

Выясняют, когда подпространства $K(l_p(\Gamma), d(w,q))$ и $K(d(v,p)^*, d(w,q))$ компактных операторов являются или не являются *M*-идеалами в соответствующих пространствах всех непрерывных линейных операторов.