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COVARIANT FORMULATION OF THE $N=1$, $D=10$ SUPERPARTICLE IN A CURVED SUPERSPACE

(Presented by H. Keres)

The action for the Brink-Schwarz superparticle together with the Kallosh-Rahmanov action for auxiliary (harmonic) variables is considered. A generalized action in a curved superspace background in proposed and the constraints on the background geometry are derived.

1. Recently R. Kallosh and M. Rahmanov [1] proposed a spacetime and world-sheet covariant quantization of the Green-Schwarz heterotic superstring in a flat superspace background (for alternative approaches see [2]). The main problem they had to overcome was how to fix Lorentz-covariantly the Siegel fermionic symmetry. From the geometrical point of view, their solution is equivalent to the introducing of a local (spinorial) frame of reference $E^{\hat{\alpha}}{}_{\alpha}$, $\hat{\alpha}=\left\{+\frac{1}{2}L, -\frac{1}{2}L\right\}$, $L=1, \ldots, 8$, $\alpha=1, \ldots, 16$, and to the considering of eight frame components of the fermionic coordinates of the string, $\theta^{-\frac{1}{2}L}=E^{-\frac{1}{2}L}{}_{\alpha}\theta^{\alpha}$, as physical degrees of freedom. The local frame of reference is constructed from auxiliary (harmonic) variables [3] that are pure gauge.

For several reasons it is desirable to generalize the action proposed in refs. [1] to the case of nontrivial background superfields. Some work has already been done in this direction. In refs. [4], the σ-model approach has been developed and β-functions have been calculated by means of the background field method with expansion in normal coordinates. In [5], the structure of ϰ-anomalies for the heterotic Green-Schwarz superstring in the Yang-Mills-Einstein background has been investigated. From the point of view of the GIKOS-type harmonic superspace [6], the curved background is briefly discussed also by R. Kallosh [7].

In the present paper the point-particle limit of the Green-Schwarz heterotic superstring, the Brink-Schwarz superparticle [8, 9], is considered. The covariant action containing harmonic variables in a curved background is proposed. The constraints on the background geometry are derived. Our main idea is to leave the auxiliary frame of reference its pure gauge nature. The background geometry will be determined by another local basis $\Pi^{A}{}_{M}(z)$ and connection one-forms $\omega_{A}{}^{B}(z)$. We also try to maintain the important property of the ordinary (reducible) theory — the independence of the action from the background connection $\omega_{A}{}^{B}$ [10]. The same ideas can be applied in case of the covariant formulation of the Green-Schwarz heterotic superstring [11].

Our sign convention for the symmetric $D=10$ Dirac matrices is as follows:

$$\gamma_{a\alpha\beta}\gamma_{b}{}^{\beta\gamma}=-\eta_{ab}\delta^{\gamma}_{\alpha}+(\gamma_{ab})_{\alpha}{}^{\gamma}. \tag{1}$$

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We shall also use the $D=6$ Dirac matrices [3]

$$\Gamma^{(6)}_I=\begin{pmatrix} 0 & (\varrho_I)_{k\dot{l}} \\ (\tilde{\varrho}_I)^{\dot{k}l} & 0 \end{pmatrix} \quad I=1,\ldots,6, \quad k,\dot{k},l,\dot{l}=1,\ldots,4.$$

and $D=6$ charge conjugation matrix

$$C^{(6)}=\begin{pmatrix} 0 & C^{k\dot{l}} \\ C^{\dot{k}l} & 0 \end{pmatrix} \quad C^T=C^{-1}=C.$$

2. Following the ideas of [1,3], let us introduce auxiliary (harmonic) variables — two commuting local Majorana-Weyl spinors $v_\alpha^{+\frac{1}{2}}(\tau)$, $v_\alpha^{-\frac{1}{2}}(\tau)$ and eight local vectors $u^L{}_a(\tau)$, $L=1,\ldots,8$. From these variables a local frame of reference in the tangent space of the world line $z^M(\tau)=\{x^m(\tau),\theta^\mu(\tau)\}$ can be constructed, separately in the bosonic part,

$$u^{\hat{a}}{}_a=\left\{u^+{}_a=v^{+\frac{1}{2}}\gamma_a v^{+\frac{1}{2}},\ u^-{}_a=v^{-\frac{1}{2}}\gamma_a v^{-\frac{1}{2}},\ u^L{}_a\right\} \tag{2}$$

and in the fermionic part $(\gamma^{\hat{a}}=u^{\hat{a}}{}_a\gamma^a)$

$$E^{\hat{\alpha}}{}_\alpha=\left\{E^{+\frac{1}{2}L}=\frac{1}{\sqrt{-G^{+-}}}v^{-\frac{1}{2}}\gamma^L\gamma^+,\ E^{-\frac{1}{2}L}=\frac{1}{\sqrt{-G^{+-}}}v^{+\frac{1}{2}}\gamma^L\gamma^-\right\}. \tag{3}$$

The duality relations

$$\begin{gathered} u^{\hat{a}}{}_a\tilde{u}_{\hat{a}}{}^b=\delta^b_a, \quad E^{\hat{\alpha}}{}_\alpha\tilde{E}_{\hat{\alpha}}{}^\beta=\delta^\beta_\alpha, \\ u^{\hat{a}}{}_a\tilde{u}_{\hat{b}}{}^a=\delta^{\hat{a}}_{\hat{b}}, \quad E^{\hat{\alpha}}{}_\alpha\tilde{E}_{\hat{\beta}}{}^\alpha=\delta^{\hat{\alpha}}_{\hat{\beta}} \end{gathered} \tag{4}$$

determine the dual spinor frame as follows:

$$\tilde{E}_{+\frac{1}{2}L}=-\gamma_L v^{-\frac{1}{2}}, \quad \tilde{E}_{-\frac{1}{2}L}=-\gamma_L v^{+\frac{1}{2}}. \tag{5}$$

In the bosonic part a metric tensor $G^{\hat{a}\hat{b}}(\tau)$ can be introduced:

$$G^{\hat{a}\hat{b}}(\tau)=u^{\hat{a}}{}_a u^{\hat{b}}{}_b\eta^{ab}, \quad G^{\hat{a}\hat{b}}G_{\hat{b}\hat{d}}=\delta^{\hat{a}}_{\hat{d}}, \quad \tilde{u}_{\hat{b}}=G_{\hat{b}\hat{a}}u^{\hat{a}}. \tag{6}$$

Two of its components $G^{++}(\tau)$ and $G^{--}(\tau)$ vanish identically as a consequence of the $D=10$ Fiertz identity:

$$G^{\pm\pm}(\tau)=\left(v^{\pm\frac{1}{2}}\gamma_a v^{\pm\frac{1}{2}}\right)\left(v^{\pm\frac{1}{2}}\gamma_b v^{\pm\frac{1}{2}}\right)\eta^{ab}\equiv 0. \tag{7}$$

This means that two basis vectors $u^+{}_a(\tau)$ and $u^-{}_a(\tau)$ are null vectors by the construction.

We propose the following generalization to the curved background for the action of harmonic variables given in [1]:

$$S_H=\int d\tau\left\{p_i\left[\dot{q}^i-\lambda^{\mathfrak{A}}R^i{}_{\mathfrak{A}}(q)\right]-\mu^{\tilde{\mathfrak{A}}}h_{\tilde{\mathfrak{A}}}(q)-\Gamma_a{}^b M^a{}_b(p,q)\right\}. \tag{8}$$

Here $q^i(\tau)$ denote the harmonic variables $\left(v_\alpha^{\pm\frac{1}{2}},\ u^L{}_a\right)$ and $p_i=$

$=\left(p^{\alpha}_{\pm\frac{1}{2}},\ p_{L}{}^{a}\right)$ are the corresponding conjugate momenta. $\lambda^{\mathfrak{A}}$, $\mu^{\tilde{\mathfrak{A}}}$ and $\Gamma_a{}^b$ are the Lagrange multipliers at the constraints $p_iR^i{}_{\mathfrak{A}}(q)\approx 0$, $\mathfrak{A}=1, \ldots, 59$; $h_{\tilde{\mathfrak{A}}}(q)\approx 0$, $\mathfrak{A}=1, \ldots, 53$ and $M^a{}_b(p, q)\approx 0$. The precise form of the constraints is as follows.

Harmonic constraints $h_{\tilde{\mathfrak{A}}}(q)\approx 0$ convert the bosonic metric $G^{\hat{a}\hat{b}}(\tau)$ into a constant [1]:

$$h_{\tilde{\mathfrak{A}}}(q)=G^{\hat{a}\hat{b}}(\tau)-\mathring{G}^{\hat{a}\hat{b}}\approx 0,$$

$$\mathring{G}^{\hat{a}\hat{b}}=\left(\begin{array}{cc|cc} 0 & -1 & \multicolumn{2}{c}{\multirow{2}{*}{0}} \\ -1 & 0 & & \\ \hline \multicolumn{2}{c|}{\multirow{2}{*}{0}} & 0 & C^{k\dot{l}} \\ & & C^{\dot{k}l} & 0 \end{array}\right)\qquad \begin{array}{l} K=(k, \dot{k}), \\ \\ L=(l, \dot{l}), \end{array} \tag{9}$$

where C^{KL} is the $D=6$ charge conjugation matrix.

Constraints $p_iR^i{}_{\mathfrak{A}}(q)\approx 0$ are [1]

$$p_iR^i{}_{\mathfrak{A}}(q)=\{H_{+-},\ H_{IJ},\ H_0,\ F_{-K},\ F_I,\ K_{KL}\}, \tag{10}$$

$$H_{+-}=-\frac{1}{2}v^{-\frac{1}{2}}\gamma_{+-}p_{-\frac{1}{2}}-\frac{1}{2}v^{+\frac{1}{2}}\gamma_{+-}p_{+\frac{1}{2}},$$

$$H_{IJ}=\tilde{u}_{\dot{k}a}C^{\dot{h}k}(\varrho_{IJ})_{k}{}^{l}p_{l}{}^{a}+\tilde{u}_{ka}(\bar{\varrho}_{IJ})_{\dot{k}}{}^{\dot{l}}p_{\dot{l}}{}^{a},$$

$$H_0=\frac{1}{2}\tilde{u}_{\dot{k}a}C^{\dot{k}k}p_{k}{}^{a}-\frac{1}{2}\tilde{u}_{ka}C^{k\dot{k}}p_{\dot{k}}{}^{a},$$

$$F_{-K}=-\tilde{u}_{-a}p_{K}{}^{a}-\frac{1}{2}v^{-\frac{1}{2}}\gamma_{-K}p_{-\frac{1}{2}}-\frac{1}{2}v^{+\frac{1}{2}}\gamma_{-K}p_{+\frac{1}{2}},$$

$$F_I=\tilde{u}_{\dot{k}a}C^{\dot{h}k}(\varrho_I)_{k}{}^{\dot{l}}p_{\dot{l}}{}^{a},$$

$$K_{KL}=-\frac{1}{2}v^{+\frac{1}{2}}\gamma_{KL}p_{+\frac{1}{2}}-\frac{1}{2}v^{-\frac{1}{2}}\gamma_{KL}p_{-\frac{1}{2}}.$$

Constraints $M^a{}_b(p, q)\approx 0$ generate the local Lorentz transformations of the auxiliary variables p, q:

$$M^a{}_b=\frac{1}{2}(p_L{}^a u^L{}_b-u^{La}p_{Lb})-\frac{1}{4}(\gamma^a{}_b)_\alpha{}^\beta\left(p^\alpha_{+\frac{1}{2}}v_\beta^{+\frac{1}{2}}+p^\alpha_{-\frac{1}{2}}v_\beta^{-\frac{1}{2}}\right). \tag{11}$$

Let us calculate the algebra of these constraints. We get

$$[p_iR^i{}_{\mathfrak{A}},\ p_jR^j{}_{\mathfrak{B}}]=f_{\mathfrak{A}\mathfrak{B}}{}^{\mathfrak{D}}p_iR^i{}_{\mathfrak{D}}+\eta^i{}_{\mathfrak{A}\mathfrak{B}}{}^{\tilde{\mathfrak{A}}}h_{\tilde{\mathfrak{A}}}p_i,$$

$$[p_iR^i{}_{\mathfrak{A}},\ h_{\tilde{\mathfrak{A}}}]=0,$$

$$[h_{\tilde{\mathfrak{A}}},\ h_{\tilde{\mathfrak{B}}}]=0,$$

$$[p_i R^i_{\mathfrak{A}}, M^a{}_b] = \tilde{\eta}_{\mathfrak{A}}{}^a{}_b{}^{\tilde{\mathfrak{A}}} h_{\tilde{\mathfrak{A}}},$$

$$[M^a{}_b, h_{\tilde{\mathfrak{A}}}] = 0,$$

$$[M^a{}_b, M^c{}_d] = f^a{}_b{}^c{}_{de}{}^f M^e{}_f. \tag{12}$$

Here $f^a{}_b{}^c{}_{de}{}^f$ are the structure constants of the Lorentz group and $f_{\mathfrak{A}\mathfrak{B}}{}^{\mathfrak{D}}$ are the structure functions of the 59-parameter gauge group; the latter ones are constant only on the harmonic constraints [1]. $p_i \eta^i{}_{\mathfrak{A}\mathfrak{B}}{}^{\tilde{\mathfrak{A}}}$ and $\tilde{\eta}_{\mathfrak{A}}{}^a{}_b{}^{\tilde{\mathfrak{A}}}$ denote the nonclosure functions for the algebra of constraints $p_i R^i_{\mathfrak{A}} \approx 0$ and $M^a{}_b \approx 0$ if considered off the harmonic constraints.

To clarify the geometrical meaning of the 59-parameter group generated by the constraints $p_i R^i_{\mathfrak{A}}(q) \approx 0$ let us consider the corresponding transformations for the harmonic variables $q^i(\tau)$:

$$\delta v^{\pm\frac{1}{2}} = -\frac{1}{4} \xi^{AB} v^{\pm\frac{1}{2}} \gamma_{AB}, \quad AB = (+-, L-, KL),$$

$$\delta u^L{}_a = \xi^{CD} I_{CD}{}^{L\bar{b}} \tilde{u}_{\bar{b}a} \equiv A^{L\bar{b}} \tilde{u}_{\bar{b}a}, \tag{13}$$

$$\bar{b} = (-, K), \quad CD = (L-, IJ, I, 0).$$

Instead of explicit expressions for matrices $I_{CD}{}^{L\bar{b}}$ and $A^{L\bar{b}}$ we give the transformation rules for the whole frame of reference $u^{\hat{a}}{}_a(\tau)$ (2):

$$\delta u^+{}_a = \xi^{+-} \tilde{u}_{-a},$$

$$\delta u^-{}_a = \xi^{-+} \tilde{u}_{+a} + \xi^{-L} \tilde{u}_{La},$$

$$\delta u^k{}_a = \xi^{IJ} (\varrho_{IJ})_l{}^k C^{ll} \tilde{u}_{\dot{l}a} + \frac{1}{2} \xi^0 C^{k\dot{l}} \tilde{u}_{\dot{l}a} + \xi^{k-} \tilde{u}_{-a}, \tag{14}$$

$$\delta u^{\dot{k}}{}_a = \xi^{IJ} (\bar{\varrho}_{IJ})_{\dot{l}}{}^{\dot{k}} C^{\dot{l}l} \tilde{u}_{la} - \frac{1}{2} \xi^0 C^{\dot{k}l} \tilde{u}_{la} + \xi^{\dot{k}-} \tilde{u}_{-a} + \xi^I (\varrho_I)_k{}^{\dot{k}} C^{k\dot{l}} \tilde{u}_{\dot{l}a}.$$

The fermionic frame of reference (3) transforms as follows:

$$\delta E^{\pm\frac{1}{2}K} = -\frac{1}{4} \xi^{AB} (\gamma_{AB})^{\pm\frac{1}{2}K}{}_{\hat{\alpha}} E^{\hat{\alpha}} + (A^{KN} - \xi^{KN}) G_{NL} E^{\pm\frac{1}{2}L}, \tag{15}$$

$$\delta \tilde{E}_{\pm\frac{1}{2}K} = -\frac{1}{4} \xi^{AB} (\gamma_{AB})_{\pm\frac{1}{2}K}{}^{\hat{\alpha}} \tilde{E}_{\hat{\alpha}} + (A^{NL} - \xi^{NL}) G_{NK} \tilde{E}_{\pm\frac{1}{2}L} \tag{16}$$

and the metric tensor (6) does not transform at all:

$$\delta G^{\hat{a}\hat{b}}(\tau) = 0. \tag{17}$$

The last condition means that $A^{\hat{a}\hat{b}} = -A^{\hat{b}\hat{a}}$ in $\delta u^{\hat{a}} = A^{\hat{a}\hat{b}} \tilde{u}_{\hat{b}}$ and that on the harmonic constraints transformations (14) constitute a subgroup of a (local) Lorentz group acting on the frame indices $\hat{a}$.

The spinor frame components have the following transformation properties under rotations (15), (16):

$$\delta\varkappa_{+\frac{1}{2}K} = V_K{}^L \varkappa_{+\frac{1}{2}L} + \frac{1}{G^{+-}} \xi^{\hat{a}-} v^{-\frac{1}{2}} \gamma_K \gamma_{\hat{a}} \gamma^L v^{+\frac{1}{2}} \varkappa_{-\frac{1}{2}L},$$
$$\delta\varkappa_{-\frac{1}{2}K} = W_K{}^L \varkappa_{-\frac{1}{2}L} + \frac{1}{G^{+-}} \xi^{-+} v^{+\frac{1}{2}} \gamma_K \gamma_- \gamma^L v^{-\frac{1}{2}} \varkappa_{+\frac{1}{2}L}, \quad (18)$$

$$\delta\theta^{+\frac{1}{2}K} = D^K{}_L \theta^{+\frac{1}{2}L} - \frac{1}{G^{+-}} \xi^{-+} v^{-\frac{1}{2}} \gamma^K \gamma_- \gamma_L v^{+\frac{1}{2}} \theta^{-\frac{1}{2}L},$$
$$\delta\theta^{-\frac{1}{2}K} = B^K{}_L \theta^{-\frac{1}{2}L} - \frac{1}{G^{+-}} \xi^{\hat{b}-} v^{+\frac{1}{2}} \gamma^K \gamma_{\hat{b}} \gamma_L v^{-\frac{1}{2}} \theta^{+\frac{1}{2}L}, \quad (19)$$

where $V_K{}^L$, $W_K{}^L$, $D^K{}_L$, $B^K{}_L$ are certain matrices. We can see that on the harmonic constraints the conditions $\theta^{+\frac{1}{2}K} \equiv E^{+\frac{1}{2}K}{}_\alpha \theta^\alpha = 0$ and $\varkappa_{-\frac{1}{2}K} \equiv \equiv \tilde{E}_{-\frac{1}{2}K}{}^\alpha \varkappa_\alpha = 0$ are invariant under the 59-parameter transformation group.

3. The total action for the Brink-Schwarz superparticle in a curved background can now be considered as a sum of the ordinary action [8] and S_H (8):

$$S = \int d\tau \left\{ -\frac{1}{2V} \omega^a \omega^b \eta_{ab} + p_i [\dot{q}^i - \lambda^{\mathfrak{A}} R^i{}_{\mathfrak{A}}(q)] - \mu^{\tilde{\mathfrak{A}}} h_{\tilde{\mathfrak{A}}}(q) - \Gamma_a{}^b M^a{}_b \right\}. \quad (20)$$

Here $\omega^A(\tau) = \dot{z}^M(\tau) \Pi^A{}_M(z(\tau))$ is the vector tangent to the world line of the particle $z^M(\tau)$, $\Pi^A{}_M(z)$ are the basis vectors of the curved background superspace and V^{-1} is the Lagrange multiplier at the mass shell constraint. The action has five local symmetry groups.

a. The general coordinate transformations of the $D=10$ curved superspace, $z^{M'} = z^{M'}(z^N)$.

b. The local Lorentz transformations of the $D=10$ superspace basis one-forms $\Pi^A(z)$ and connection one-forms $\omega_A{}^B(z)$. To achieve the invariance of S_H at the transformations

$$\delta u^L{}_a = L_a{}^b(z(\tau)) u^L{}_b(\tau), \qquad \delta v_\alpha^{\pm\frac{1}{2}} = -\frac{1}{4} L_a{}^b (\gamma^a{}_b)_\alpha{}^\beta v_\beta^{\pm\frac{1}{2}} \quad (21)$$

the Lagrange multiplier $\Gamma_a{}^b$ must transform as follows:

$$\delta\Gamma_a{}^b = \dot{L}_a{}^b - \Gamma_e{}^g f^e{}_g{}^c{}_{da}{}^b L_c{}^d. \quad (22)$$

c. The reparametrization of the world line $z^M(\tau)$:

$$\delta\tau = \varrho(\tau), \qquad \delta\lambda^{\mathfrak{A}} = \frac{d}{d\tau}(\lambda^{\mathfrak{A}} \varrho),$$
$$\delta(V^{-1}) = \frac{d}{d\tau}(V^{-1}\varrho), \qquad \delta\mu^{\tilde{\mathfrak{A}}} = \frac{d}{d\tau}(\mu^{\tilde{\mathfrak{A}}} \varrho), \quad (23)$$
$$\delta\Gamma_a{}^b = \frac{d}{d\tau}(\Gamma_a{}^b \varrho).$$

The reparametrization transformation for S_H can be considered as generated by the constraints and trivial transformations.

d. The constraints $p_i R^i_{\mathfrak{A}}(q) \approx 0$, $h_{\tilde{\mathfrak{A}}}(q) \approx 0$, $M^a{}_b(p,q) \approx 0$ generate the following transformations with the local parameters $\xi(\tau)$:

$$\delta q^i = R^i_{\mathfrak{A}}(q)\xi^{\mathfrak{A}} + \frac{\partial}{\partial p^i}(M^a{}_b)\xi_a{}^b,$$

$$\delta p_i = -p_j \frac{\partial}{\partial q^i}(R^j_{\mathfrak{A}})\xi^{\mathfrak{A}} - \frac{\partial}{\partial q^i}(h_{\tilde{\mathfrak{A}}})\xi^{\tilde{\mathfrak{A}}} - \frac{\partial}{\partial q^i}(M^a{}_b)\xi_a{}^b$$

$$\delta\lambda^{\mathfrak{A}} = \dot{\xi}^{\mathfrak{A}} - \lambda^{\mathfrak{B}}\xi^{\mathfrak{D}} f^{\mathfrak{A}}_{\mathfrak{B}\mathfrak{D}}, \tag{24}$$

$$\delta\mu^{\tilde{\mathfrak{A}}} = \dot{\xi}^{\tilde{\mathfrak{A}}} - \lambda^{\mathfrak{B}}\xi^{\mathfrak{D}} p_i \eta^i{}^{\tilde{\mathfrak{A}}}_{\mathfrak{B}\mathfrak{C}} - \lambda^{\mathfrak{A}}\xi_a{}^b \tilde{\eta}_{\mathfrak{A}}{}^a{}_b{}^{\tilde{\mathfrak{A}}},$$

$$\delta\Gamma_a{}^b = \dot{\xi}_a{}^b - \Gamma_c{}^d \xi_e{}^g f^c{}_d{}^e{}_g{}_a{}^b.$$

Here the parameters $\xi_a{}^b(\tau)$ determine a local Lorentz transformation that seems to transform only the harmonic variables and the Lagrange multiplier $\Gamma_a{}^b$. However, since the superspace where a superparticle can live, allows an arbitrary redefinition of the connection [10], the background connection along the world line can be considered to coincide with the Lagrange multiplier $\Gamma_a{}^b$ and so the $\xi_a{}^b$-transformations are, in fact, a part of the local Lorentz transformations:

$$\dot{z}^M(\tau)\omega_{Ma}{}^b(z(\tau)) = \Gamma_a{}^b, \quad \xi_a{}^b(\tau) = L_a{}^b(\tau). \tag{25}$$

e. The local fermionic Siegel symmetry has now eight Lorentz-invariant parameters $\varkappa_{+\frac{1}{2}L}(\tau) = E_{+\frac{1}{2}L}{}^\alpha \varkappa_\alpha$, $\varkappa_{-\frac{1}{2}L} = 0$:

$$\delta z^M = \omega^a \gamma_a{}^{\alpha\beta} E^{+\frac{1}{2}L}{}_\alpha \varkappa_{+\frac{1}{2}L} \Pi_\beta{}^M,$$

$$\delta q^i = 0, \quad \delta p_i = 0, \tag{26}$$

$$\delta\lambda^{\mathfrak{A}} = 0, \quad \delta\mu^{\tilde{\mathfrak{A}}} = 0, \quad \delta\Gamma_a{}^b = 0.$$

It holds, if the background torsion tensor $T_{AB}{}^D$ satisfies the same constraints as in the usual (reducible) formulation [8],

$$T_{\beta\delta}{}^a = T\gamma^a_{\beta\delta}, \tag{27}$$

$$T_{\beta(a}{}^d \eta_{b)d} = \gamma_{(a|\beta\delta|} T^\delta{}_{b)} \tag{28}$$

and the transformation rule for V^{-1} is

$$\delta(V^{-1}) = 2V^{-1}(T\omega^\alpha + \omega^a T^\alpha_a) E^{+\frac{1}{2}L}{}_\alpha \varkappa_{+\frac{1}{2}L}. \tag{29}$$

Finally, the closure properties of the eight-parameter group of transformations (26) must be investigated. A direct calculation (using constraint (27)) gives the following result:

$$[\delta_{\underset{2}{\varkappa}}, \delta_{\underset{1}{\varkappa}}] z^M = \omega^a \gamma_a{}^{\alpha\beta} E^{+\frac{1}{2}L}{}_\alpha \varkappa_{3+\frac{1}{2}L} \Pi_\beta{}^M + \varrho \frac{dz^M}{d\tau} +$$

$$+ \zeta^M(\tau) - T(\omega^a\omega^b\eta_{ab}) \varkappa_{[1+\frac{1}{2}L} \varkappa_{2]+\frac{1}{2}K} \gamma^{d+\frac{1}{2}L+\frac{1}{2}K} \Pi_d{}^M. \tag{30}$$

The first term is a new $\varkappa$-transformation with the parameter $\varkappa_{3+\frac{1}{2}L}(\tau)$,

$$\varkappa_{3+\frac{1}{2}L} = 4T\omega^{\delta} E_{\delta}^{+\frac{1}{2}K} \varkappa_{[1+\frac{1}{2}K\,2]} \varkappa_{+\frac{1}{2}L} -$$

$$- T\omega^{\delta} E_{\delta}^{+\frac{1}{2}K} \varkappa_{[1+\frac{1}{2}M\,2]} \varkappa_{+\frac{1}{2}N} \gamma_{+}^{+\frac{1}{2}M+\frac{1}{2}N} G_{KL}, \tag{31}$$

the second term is a reparametrization of τ with the parameter $\varrho(\tau)$,

$$\varrho(\tau) = 2T\omega^{b} \varkappa_{[1+\frac{1}{2}K\,2]} \varkappa_{+\frac{1}{2}L} \gamma_{b}^{+\frac{1}{2}K+\frac{1}{2}L} \tag{32}$$

and the last term vanishes on the mass shell. The remaining term ζ^{M} can be written as follows:

$$\zeta^{M} = 2\omega^{d}\omega^{b} \varkappa_{[1+\frac{1}{2}K\,2]} \varkappa_{+\frac{1}{2}L} \gamma_{b}^{+\frac{1}{2}L\rho} \Big[T_{\rho d}{}^{a} \gamma_{a}^{+\frac{1}{2}K\delta} -$$

$$- \frac{1}{2} T_{\rho\alpha}{}^{\delta} \gamma_{d}^{+\frac{1}{2}K\alpha} + \frac{1}{4} \omega_{\rho e}{}^{f} (\gamma^{e}{}_{f})^{+\frac{1}{2}K}{}_{\alpha} \gamma_{d}^{\alpha\delta} \Big] \Pi_{\delta}{}^{M}. \tag{33}$$

For further investigation of ζ^{M} let us use the following background field redefinitions [12]:

$$\Pi'_{a} = \Pi_{a} + H_{a}{}^{\beta}\Pi_{\beta}, \quad \Pi'_{\beta} = \Pi_{\beta}; \tag{34}$$

$$\omega'_{A}{}^{B} = \omega_{A}{}^{B} + X_{A}{}^{B}. \tag{35}$$

They do not change action (20) and constraints (27), (28) and they can be used to derive from eq. (27), (28) all known full sets of constraints on the torsion tensor. A direct calculation confirms that on harmonic constraints transformations (34), (35) do not change the general structure of commutator (30). In particular, ζ^{M} retains its form (33) at transformation (34) and does not change at all at the redefinition of connection (35).

In case of the simplest full set of constraints on the torsion tensor [13]

$$T_{\alpha b}{}^{d} = \mathrm{i}\,\delta^{d}_{b} T_{\alpha}, \quad T_{\alpha\beta}{}^{\varepsilon} = \mathrm{i}\,\delta^{\varepsilon}_{(\alpha} T_{\beta)}, \quad T = \mathrm{i} \tag{36}$$

we have

$$\zeta^{M} = D^{M}\omega^{b}\omega^{d}\eta_{bd} + \omega^{d}\gamma_{d}^{\alpha\delta} \left(E_{\alpha}^{+\frac{1}{2}L} \varkappa_{4+\frac{1}{2}L} + E_{\alpha}^{-\frac{1}{2}L} \varkappa_{4-\frac{1}{2}L} \right) \Pi_{\delta}{}^{M} \tag{37}$$

with

$$\varkappa_{4-\frac{1}{2}L} = \frac{1}{2} \omega^{b}\gamma_{b}^{+\frac{1}{2}K\rho} \varkappa_{[1+\frac{1}{2}P\,2]} \varkappa_{+\frac{1}{2}K} \omega_{\delta e}{}^{f} (\gamma^{e}{}_{f})^{+\frac{1}{2}P}{}_{-\frac{1}{2}L}. \tag{38}$$

The algebra closes on shell, if $\varkappa_{4-\frac{1}{2}L}$ vanishes on harmonic constraints.

The only nonvanishing nondiagonal frame components of $\gamma^{e}{}_{f}$ are these of $\gamma^{-}{}_{N}$, so the condition for the closure of the algebra is now ($[\Pi_{A}, \Pi_{B}] = C_{AB}{}^{D}\Pi_{D}$)

$$\delta_{\varkappa} z^{M} \Pi_{M}{}^{\rho} C_{\rho e}{}^{f}(z(\tau))\, \tilde{u}_{-}{}^{e}(\tau)\, u^{N}{}_{f}(\tau) \approx 0. \tag{39}$$

It does not change at transformations (14), (15) of $\hat{u}^a{}_a$, $\hat{E}^\alpha{}_\alpha$ on the harmonic constraints and imposes an additional condition for the choice of the basis vectors of the background superspace.

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Piret KUUSK

KOVARIANTNE TEOORIA $N=1$, $D=10$ SUPEROSAKESE KOHTA KÕVERAS SUPERRUUMIS

On uuritud Brinki—Schwarzi superosakese mõjufunktsiooni koos Kalloshi—Rahmanovi harmooniliste abimuutujate mõjufunktsiooniga, konstrueeritud analoogiline mõjufunktsioon kõvera superruumi foonil ja tuletatud seosed, mis kitsendavad fooniruumi geomeetriat.

Пирет КУУСК

КОВАРИАНТНОЕ ОПИСАНИЕ $N=1$, $D=10$ СУПЕРЧАСТИЦЫ В ИСКРИВЛЕННОМ СУПЕРПРОСТРАНСТВЕ

Рассматриваются свойства симметрии действия суперчастицы Бринка—Шварца вместе с действием Каллош—Рахманова для гармонических переменных. Предлагается обобщенное действие, описывающее суперчастицу на фоне искривленного суперпространства, и выводятся связи на фоновую геометрию.