LÜHITEATEID * КРАТКИЕ СООБЩЕНИЯ SHORT COMMUNICATIONS

https://doi.org/10.3176/phys.math.1990.1.12

Proc. Estonian Acad. Sci. Phys. Math., 1990, 39, № 1, 80—81

УДК [539.219.1 : 535.217] : 535.34

Jaak KIKAS

ON SPATIO-TEMPORAL SYMMETRY OF SPECTRAL HOLE-BURNING

Jaak KIKAS. SPEKTRAALSÄLKAMISE AEGRUUMILISEST SÜMMEETRIAST

Яак КИКАС. О ПРОСТРАНСТВЕННО-ВРЕМЕННОЙ СИММЕТРИИ ВЫЖИГАНИЯ СПЕКТРАЛЬНЫХ ПРОВАЛОВ.

(Presented by K. K. Rebane)

In this report, symmetry properties of the equations describing spectral hole-burning (SHB) [1, 2] in an optically dense sample are considered and some consequences are analysed. The consideration of [3] is generalized to non-monochromatic linearly polarized light. SHB process is then governed by the equations

$$\partial \varrho(x, t, \nu_i)/\partial t = -\varrho(x, t, \nu_i) \int \sigma(\nu_i, \nu_p) I(x, t, \nu_p)\, \mathrm{d}\nu_p \qquad \text{(1a)}$$

$$\partial I(x, t, \nu_p)/\partial x = -I(x, t, \nu_p) \int \sigma(\nu_i, \nu_p) \varrho(x, t, \nu_i)\, \mathrm{d}\nu_i. \qquad \text{(1b)}$$

Here ϱ and I are the distributions of impurities and photons, respectively, σ is the absorption cross section ν_i and ν_p (i — impurity; p — photon) are frequencies of impurity transition and photon, resp. The impurity transition dipole and the electric component of the photon are supposed to be colinear. In (1), the SHB quantum yield is supposed to be unity: formally this can be achieved by a simple time scaling. (1) describes SHB in a planar sample at a rectangular incidence of light (planar wave). (1a) governs SHB kinetics at a certain distance x from the front surface and (1b) — the attenuation of light in the sample according to the Beer's law.

System (1) can be represented in an integral form:

$$\varrho(x, t, \nu_i) = \varrho(x, 0, \nu_i) \exp\left\{-\int_0^t \int \sigma(\nu_i, \nu_p) I(x, t', \nu_p)\, \mathrm{d}\nu_p\, \mathrm{d}t'\right\} \qquad \text{(2a)}$$

$$I(x, t, \nu_p) = I(0, t, \nu_p) \exp\left\{-\int_0^x \int \sigma(\nu_i, \nu_p) \varrho(x', t, \nu_i)\, \mathrm{d}\nu_i\, \mathrm{d}x'\right\}. \qquad \text{(2b)}$$

In (2), $I(0, t, \nu_p)$ is the distribution of photons at the front surface of the sample and $\varrho(x, 0, \nu_i)$ is the initial distribution of impurities in the sample. In a general case, (2) can be solved numerically by iterative methods (see [3] for a particular case).

In what follows, a more implicit form of the absorption cross section is assumed:

$$\sigma(\nu_i, \nu_p) = \sigma_0 \varkappa(\nu_i - \nu_p), \qquad (3)$$

where $\varkappa$ is the normalized homogeneous absorption spectrum. $\varkappa$ is supposed to be a symmetric function of its argument: $\varkappa(\nu_i - \nu_p) = \varkappa(\nu_p - \nu_i)$, which holds, e.g. for the Lorentzian $\varkappa$. The spectral inhomogeneity is supposed to be manifested by different shifts of homogeneous spectra without any shape distortion.

It is easy to prove that systems (1, 2) are invariant under the following transformation:

$$\begin{gathered} \varrho \to I, \quad I \to \varrho \\ t \to x, \quad x \to t \\ \nu_i \to \nu_p, \quad \nu_p \to \nu_i . \end{gathered} \tag{4}$$

As a result the following statement holds.

Let $\varrho(x, t, \nu_i)$ and $I(x, t, \nu_p)$ be the solution of system (1) under the initial (boundary) conditions $\varrho(x, 0, \nu_i)$ and $I(0, t, \nu_p)$. Then

$$\begin{gathered} \tilde{\varrho}(x, t, \nu_i) = I(t, x, \nu_i) \\ \tilde{I}(x, t, \nu_p) = \varrho(t, x, \nu_p) \end{gathered} \tag{5}$$

is also the solution of system (1) under initial (boundary) conditions

$$\begin{gathered} \tilde{\varrho}(x, 0, \nu_i) = I(0, x, \nu_i) \\ \tilde{I}(0, t, \nu_p) = \varrho(t, 0, \nu_p). \end{gathered} \tag{6}$$

Note that in the case of non-symmetric $\varkappa$, the above-made statement is modified: $\tilde{\varrho}$ and $\tilde{I}$ are solutions of system (1) with $\varkappa(\nu)$ changed to $\varkappa(-\nu)$.

Whereas for impurity dipoles such constraint does not apply.

Let us now consider some special cases.

A. Monochromatic burning in case of a broad (as compared to the width of $\varkappa$) inhomogeneous spectral distribution of impurities [3]. In the sense stated above this case is equivalent to that of burning with a broadband (white) light in the absence of inhomogeneous broadening, and vice versa.

B. System (1) ((2)) is analytically solvable in case $\varkappa(\nu_i - \nu_p) = \delta(\nu_i - \nu_p)$ (no homogeneous broadening). Following [4], one obtains

$$\begin{gathered} I(x, t, \nu) = I(0, t, \nu)\, e^{Q(t,\nu)} [e^{Q(t,\nu)} + e^{D(x,\nu)} - 1]^{-1} \\ \varrho(x, t, \nu) = \varrho(x, 0, \nu)\, e^{D(x,\nu)} [e^{Q(t,\nu)} + e^{D(x,\nu)} - 1]^{-1}, \end{gathered} \tag{7}$$

where

$$Q(t, \nu) = \sigma_0 \int_0^t I(0, t', \nu)\, dt'$$

$$D(x, \nu) = \sigma_0 \int_0^x \varrho(x', 0, \nu)\, dx'.$$

Symmetry properties of this solution are obvious.

Author is indebted to K. K. Rebane for interest and valuable remarks.

REFERENCES

1. *Гороховский А. А., Каарли Р. К., Ребане Л. А.* // Письма в ЖЭТФ, 1974, 20, вып. 7, 474—479.
2. *Kharlamov, B. M., Personov, R. I., Bykovskaya, L. A.* // Opt. Commun., 1974, 12, № 1, 191—193.
3. *Кикас Я., Малкин Е.* // Изв. АН ЭССР. Физ. Матем., 1987, 36, № 1, 62—65.
4. *Rebane, L. A., Gorokhovskii, A. A., Kikas, J. V.* // Appl. Phys. B, 1982, 29, № 4, 235—250.

Academy of Sciences of the Estonian SSR,
Institute of Physics

Received
July 12, 1989,