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INFLUENCE OF THE MAGNETIC FIELD ON THE LATTICE DYNAMICS OF AN IONIC CRYSTAL IN THE LONG-WAVE LIMIT

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Т. ЭРД. ВЛИЯНИЕ МАГНИТНОГО ПОЛЯ НА ДИНАМИКУ РЕШЕТКИ ИОННОГО КРИСТАЛЛА В ДЛИННОВОЛНОВОМ ПРЕДЕЛЕ

(Presented by V. Hizhnyakov)

In this report, the influence of a homogeneous magnetic field (induction $\vec{B}$) on the long-wave lattice vibrations of a cubic ionic crystal have been considered in the continuous approximation with the allowance made for the splitting of the unperturbed frequencies into longitudinal and transversal components. The general formalism for describing the ionic crystal vibrations in the presence of the magnetic field has been developed in [1], where a concrete two-dimensional lattice model has also been studied. Earlier, in [2], some aspects of the problem were analysed, proceeding from symmetry considerations.

Let the unit cell of a crystal consist of two ions with effective charges $\pm\bar{e}$ and masses $m_\pm$. The equations of motion for long-wave lattice vibrations with $\vec{q}=0$ in the continuous approximation [3] for $B\neq 0$ have the form

$$\ddot{w}_j=-\omega_j^2 w_j+\sum_{j'j''}\varepsilon_{jj'j''}\left[\omega_{c1}\dot{w}_{j'}+\omega_{c2}\dot{c}_{j'}\right]b_{j''}; \tag{1}$$

$$\ddot{c}_j=\omega_{c3}\sum_{j'j''}\varepsilon_{jj'j''}\dot{w}_{j'}b_{j''}, \quad j=1,2,3. \tag{2}$$

Here

$$\omega_{1,2}=\omega_t, \qquad \omega_3=\omega_l, \tag{3}$$

$$\vec{w}=\vec{u}_+-\vec{u}_-, \qquad \vec{c}=\frac{\vec{u}_+m_++\vec{u}_-m_-}{m_++m_-}, \qquad \vec{b}=\vec{B}/B, \tag{4}$$

$$\omega_{c\mu}=\bar{e}B/M_\mu, \qquad M_{1,2}^{-1}=m_+^{-1}\mp m_-^{-1}, \qquad M_3=m_++m_-; \tag{5}$$

$\vec{u}_\pm$ are the shifts of positive and negative ions; $\omega_{t,l}$ are the limiting frequencies of transversal and longitudinal optic vibrations; $\varepsilon_{jj'j''}$ are the components of the Levi-Civita tensor. Vibrational motion of the centre of gravity of ions, described by Eqs (2), appears due to the interaction of optic modes with the magnetic field.

Assuming

$$w_j = a_j \exp(i\tilde{\omega}t), \quad c_j = \alpha_j \exp(i\tilde{\omega}t); \tag{6}$$

the system of differential equations (1), (2) leads to

$$\sum_{j'} \{[\tilde{\omega}^2 - \omega_j^2]\delta_{jj'} + i\omega_{c1}\tilde{\omega} \sum_{j''} \varepsilon_{jj'j''} b_{j''} - \omega_{c2}\omega_{c3}[\delta_{jj'} - b_j b_{j'}]\} a_{j'} = 0, \tag{7}$$

$$-\tilde{\omega}\alpha_j = i\omega_{c3} \sum_{j'j''} \varepsilon_{jj'j''} a_{j'} b_{j''}. \tag{8}$$

For $\omega_{c1}/\omega_t \ll 1$, $\omega_{c1}/\sqrt{\omega_l^2 - \omega_t^2} \ll 1$, $\sqrt{\omega_{c2}\omega_{c3}}/\omega_{t,l} \ll 1$ from the condition of the existence of a nontrivial solution for the system of equations (7) by the iterative method we find for the eigenfrequencies in the first approximation of the magnetic field*

$$\tilde{\omega}_{1,2}^2 = \omega_t^2 \pm \omega_t |\omega_{c1} \cos\varphi|, \tag{9}$$

$$\tilde{\omega}_3^2 = \omega_l^2, \tag{10}$$

and in the second approximation

$$\tilde{\omega}_{1,2}^2 = \omega_t^2 - \frac{1}{2}\left[\frac{\omega_{c1}^2\omega_t^2 \sin^2\varphi}{\omega_l^2 - \omega_t^2} - \omega_{c2}\omega_{c3}(1 + \cos^2\varphi)\right] \pm$$
$$\pm \frac{1}{2}\left\{\left[\frac{\omega_{c1}^2\omega_t^2}{\omega_l^2 - \omega_t^2} - \omega_{c2}\omega_{c3}\right]^2 \sin^4\varphi + 4\omega_{c1}^2\left[\omega_t \pm \frac{1}{2}|\omega_{c1}\cos\varphi|\right]^2 \cos^2\varphi\right\}^{1/2}, \tag{11}$$

$$\tilde{\omega}_3^2 = \omega_l^2 + \left[\frac{\omega_{c1}^2\omega_l^2}{\omega_l^2 - \omega_t^2} + \omega_{c2}\omega_{c3}\right] \sin^2\varphi, \tag{12}$$

where

$$\cos\varphi = b_3 = \vec{b}\cdot\vec{o}, \quad \vec{o} = \frac{\vec{q}}{q}, \quad 0 \leqslant \varphi \leqslant \pi/2. \tag{13}$$

On the basis of Eqs (2) and (6) it follows that

$$\tilde{\omega}_{4,5,6} = 0. \tag{14}$$

According to Eqs (7)—(12), (14) the vibrational motions of the centre of gravity of the unit cell, induced by the magnetic field, occur on optic frequencies. It means that the crystal serves as a hypersound generator for $B \neq 0$.

It can be seen from Eqs (9)—(12) that the magnetic field mixes the transversal optic vibrations in the first order of B already, the transversal and longitudinal optic vibrations, however, do it in the second order. The induced motion of the centre of gravity of the unit cell gives a contribution to $\tilde{\omega}_{1,2,3}$ in the second order of the magnetic field. Eigenfrequencies $\tilde{\omega}_{1,2,3}$ form one-dimensional bands in the φ-space, as the angle

* In the cases $b_3 = 0$ and $b_3 = \pm 1$ the problem has a simple precise solution.

φ passes through all the values in the interval from 0 to $\pi/2$. The dependence $\tilde{\omega}_{1,2,3}$ on φ is caused by the unequality of the frequencies ω_l and ω_t.

Now, on the basis of Eqs (9)—(12) we estimate the shifts of the eigenfrequencies $D_j = \tilde{\omega}_j - \omega_j$ for LiH ($\omega_l = 2.1 \cdot 10^{14}$ s^{-1}, $\omega_t = 1.1 \cdot 10^{14}$ s^{-1} [4]; $m_+ = 1.15 \cdot 10^{-23}$ g, $m_- = 0.17 \cdot 10^{-23}$ g, $\bar{e} = e$). For $B = 20$ T the maximum values of D_j equal: $D_{1,2}(\varphi = 0) \approx \pm 8 \cdot 10^8$ s^{-1}; $D_3(\varphi = \pi/2) \approx 10^4$ s^{-1}. Such value of D_3 is negligible, but in agreement with [1], the shifts $D_{1,2}$ should be experimentally observable.

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