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CLASSICAL STRINGS AND BORN-INFELD EQUATIONS

A. AINSAAR, M. KÕIV. KLASSIKALISED STRINGID JA BORN-INFELDI VÕRRANDID
А. АЙНСААР, М. КЫЙВ. КЛАССИЧЕСКИЕ СТРУНЫ И УРАВНЕНИЯ БОРНА—ИНФЕЛЬДА

(*Presented by H. Keres*)

So far two different forms of the string action integral have been proposed [1]:

$$S_N = m \int \sqrt{\sigma^2}\, du^0\, du^1, \tag{1}$$

$$S_S = \int \sigma^2\, du^0\, du^1, \tag{2}$$

where

$$\sigma^2 = -\frac{1}{2}\sigma^{\mu\nu}\sigma_{\mu\nu}, \tag{3}$$

$$\sigma^{\mu\nu} = \partial(x^\mu, x^\nu)/\partial(u^0, u^1). \tag{4}$$

Action (1), called the Nambu action, is a generalization of a free mass point, thus having a strict geometrical meaning. It leads to the following Lagrange-Euler equations:

$$\partial/\partial u^a (\partial \sqrt{\sigma^2}/\partial x_{,a}) = 0, \tag{5}$$

where

$$x^\mu_{,a} = \partial x^\mu/\partial u^a. \tag{6}$$

This string model cannot be consistently quantized in an arbitrary space-time dimension. The Schild action (2) is hoped to be free from such difficulties [2]. As was shown by Nambu [3], action (2) can be obtained by the Hamilton-Jacobi formalism basing on two-form relations. The equations of motion corresponding to (2) are

$$\partial/\partial u^a (-\varepsilon^{ab}\sigma_{\mu\nu} x^\nu_{,b}) = 0. \tag{7}$$

Action (1) is invariant under an arbitrary change of the parameters $u' = u'(u)$. This enables one to choose a part of the space co-ordinates as independent parameters, the rest being treated as fields [4]. As far as the choice is arbitrary, it leads to the equivalence of the co-ordinates and fields.

Let us take the case of two independent parameters of the string u^0, u^1 and an arbitrary number m of space-time dimensions. Then, if one takes the space co-ordinates x^0, x^1 as new parameters with the transition Jacobian

$$D = \partial(x^0, x^1)/\partial(u^0, u^1), \tag{8}$$

then regardless of the action one can write

$$-2\sigma^2=\Phi D^2, \tag{9}$$

where

$$\Phi=\Psi_{fg}\varepsilon^{fh}\Psi_{hi}\varepsilon^{gi}=\det\Psi, \tag{10}$$

$$\Psi_{fg}=\overset{\alpha}{\psi}_{,f}\overset{\alpha}{\psi}_{,g}+g_{fg}, \tag{11}$$

$$\overset{\alpha}{\psi}_{,f}=\partial\psi^\alpha/\partial x^f. \tag{12}$$

Here Latin indices take the values 0 and 1, the other co-ordinates, denoted by ψ^α ($\alpha=1, \ldots, m-1$), are considered as fields.

Equation (9) shows that the Nambu string is a sympleon [4,5], i. e., the Lagrangian admits the factor D at a reparametrization. The Lagrange-Euler equations of the Lagrangian in the new parameters

$$L\sim\sqrt{\Phi}, \tag{13}$$

in accordance with the work by B. M. Barbashov and N. A. Chernikov [6], prove to be the $(m-1)$-dimensional Born-Infeld equation system

$$B^\alpha=0, \tag{14}$$

where

$$B^\alpha\equiv\varepsilon_{ik}\varepsilon_{jl}\overset{\alpha}{\psi}_{,ij}\Psi_{kl}. \tag{15}$$

The Schild string is not a sympleon, for D does not fall out of the action integral. Unlike the Nambu string case, the action integral cannot be written in such a way that independently of the parametrization, the variation of certain co-ordinates, called fields, would lead to field equations devoid of the initial parameters u. Still we pose the question whether the structure keeps the Born-Infeld equation in this case too.

Equations (7) split into two parts, where the first part corresponds to the values $\mu\to\alpha$ and the second one to $\mu=0,1$:

$$\begin{aligned}&[\varepsilon^{if}\varepsilon^{kg}(\overset{\alpha}{\psi}_{,i}\Psi_{fg})_{,k}-\varepsilon^{if}\overset{\alpha}{\psi}_{,i}\Psi_{fg}U_g]D^2=0,\\&[\varepsilon^{kg}\Psi_{fg,k}-\Psi_{fg}U_g]D^2=0,\end{aligned} \tag{16}$$

where

$$U_g=D^{-2}\varepsilon^{ba}\,\partial D/\partial u_a x^{g}_{,b}. \tag{17}$$

Using the immediate consequence of (7)

$$\partial\sigma^2/\partial u^a=0, \tag{18}$$

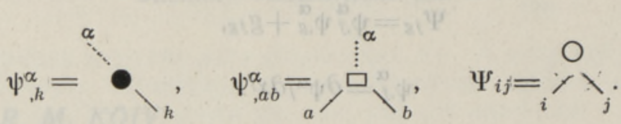
one can give (17) the parametrization-independent form

$$U_g=\varepsilon^{kg}\Phi_{,k}/2\Phi. \tag{19}$$

Thus, in (16) the u-dependence has remained only in the factors D^2. Apart from it the following field equations ensue:

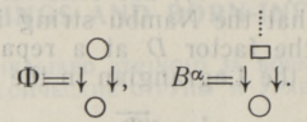
$$\begin{aligned}&\varepsilon^{kg}\varepsilon^{if}\varepsilon^{jm}\varepsilon^{np}(\overset{\alpha}{\psi}_{,ik}\Psi_{fg}\Psi_{jn}\Psi_{mp}+\overset{\alpha}{\psi}_{,i}\Psi_{fg,k}\Psi_{jn}\Psi_{mp}-\overset{\alpha}{\psi}_{,i}\Psi_{fg}\Psi_{jn}\Psi_{mp,k})=0,\\&\varepsilon^{kg}\varepsilon^{jm}\varepsilon^{np}(\Psi_{fg,k}\Psi_{jn}\Psi_{mp}-\Psi_{fg}\Psi_{jn}\Psi_{mp,k})=0.\end{aligned} \tag{20}$$

The following analysis of these equations in a usual way would be quite cumbersome and not straightforward. We worked out a graphical method for such purposes. At that an index corresponds to a graph vertex, 2-dimensional δ symbol is a line and ε is an arrow between two vertices. The summing up over a function index α is denoted by a dotted line. Let us denote our functions as



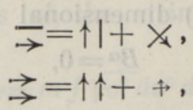
(21)

Then one can obtain immediately



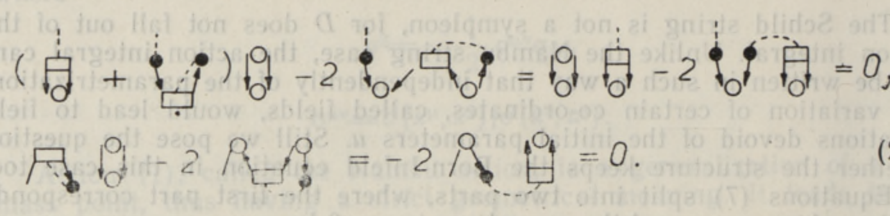
(22)

It is an easy matter to find out general displacement rules for lines and arrows within a formula. All we need for our purpose is



(23)

by which one can straightforwardly change equations (20) in the way that the Born-Infeld expression arises as a factor:



(24)

It can be rewritten at once:

$$\left(2\overset{\alpha}{\psi}_{,i}\varepsilon^{if}\Psi_{fg}\varepsilon^{jd}\overset{\beta}{\psi}_{,d}+\Phi\delta^{\alpha\beta}\right)B^{\beta}=0,$$

$$2\Psi_{fj}\varepsilon^{jd}\overset{\beta}{\psi}_{,d}B^{\beta}=0, \qquad (25)$$

the nontrivial consequence of which is the corresponding Born-Infeld system of equations (14). In order to get a complete system corresponding to action (2) one can add to it equation (19).

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