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ENERGY BALANCE AND VOLTAGE-CURRENT RELATION FOR LINEAR INDUCTORS WITH TIME-VARYING TURNS
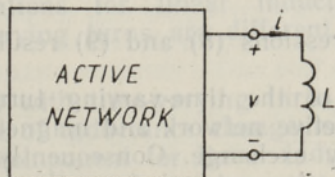
(Presented by I. Öpik)

Introduction

Inductors can perform their functions in electric circuits due to their energy-storing ability. Therefore the conscious control of energy storing and restoring processes allows a more efficient and versatile use of inductors. For that purpose the variation of the inductance may be used.

A time-varying inductor can be implemented by controlling either the permeance or the turns of the inductor, but it should be pointed out that both the energy flow and the voltage-current relation are different in these two cases. Until now only the voltage-current relation of time-varying inductors with time-varying permeance and time-invariant turns has been considered in the literature. On the other hand, contemporary static switches enable to implement time-varying inductors using step-wise variation of the turns [1-14].

In this paper the voltage-current relation for a linear inductor with smooth variation of turns is derived and discussed. The new relation may be useful for better understanding the performance characteristics of the inductors with time-varying turns as well as an approximate design equation in case of step-wise variation of the turns.

Voltage-current relations

Let us consider a linear inductor with the time-varying turns N, time-invariant permeance P, magnetic flux Φ, current i, and voltage v connected to an active network as shown in the Figure.

Its time-varying inductance L may be expressed in the form

$$L = PN^2 = N\Phi/i. \tag{1}$$

The voltage across the inductor is given by Faraday's law as

$$v = N\frac{d\Phi}{dt} = N\dot\Phi. \tag{2}$$

In accordance with Eq. (1), the magnetic flux

$$\Phi = PNi. \tag{3}$$

Since both the turns N and the current i are functions of time and the permeance P is assumed to be a constant, the derivative of the magnetic flux

$$\dot\Phi = P\left(N\frac{di}{dt} + i\frac{dN}{dt}\right). \tag{4}$$

From Eqs. (2) and (4) we obtain

$$v = PN\left(N\frac{di}{dt} + i\frac{dN}{dt}\right) = L\frac{di}{dt} + PNi\frac{dN}{dt}. \tag{5}$$

Since from Eq. (1)

$$N = (L/P)^{1/2}, \tag{6}$$

we get

$$\frac{dN}{dt} = \frac{1}{2(PL)^{1/2}}\cdot\frac{dL}{dt} = \frac{1}{2NP}\cdot\frac{dL}{dt}. \tag{7}$$

Finally, from Eqs. (5) and (7), the voltage across a linear inductor with time-varying turns

$$v = L\frac{di}{dt} + \frac{1}{2}i\frac{dL}{dt}. \tag{8}$$

As we can see, the new voltage-current relation (8) for linear time-varying inductors differs from the well-known classical expression

$$v = L\frac{di}{dt} + i\frac{dL}{dt} \tag{9}$$

by the factor 1/2 in the second term. However, there is no contradiction between these two different expressions (8) and (9). They merely correspond to two different modes of energy flow possible in time-varying inductors.

Energy balance

Physically different expressions (8) and (9) result from different energy conversion processes.

In the above case of the time-varying turns N and unchanging permeance P only the active network and magnetic field of the inductor take part in the energy exchange. Consequently, the energy delivered to the inductor by the active network from time t_0 to t is

$$W(t_0, t) = E[\Phi(t), t] - E_0[\Phi(t_0), t_0] = [i^2(t)L(t) - i^2(t_0)L(t_0)]/2, \tag{10}$$

where in the right-hand side of the equation we can see the difference between the energy stored in the magnetic field at time t and at time t_0, respectively.

Since the instantaneous power of the inductor

$$p_L(t) = v(t)i(t) = \frac{dW(t_0,\, t)}{dt} = i(t)L(t)\frac{di(t)}{dt} + \frac{1}{2}i^2(t)\frac{dL(t)}{dt}, \quad (11)$$

we get from energetic considerations again

$$v(t) = \frac{p_L(t)}{i(t)} = L(t)\frac{di(t)}{dt} + \frac{1}{2}i(t)\frac{dL(t)}{dt}, \quad (12)$$

i. e. an expression identical with (8).

On the other hand, the voltage-current relation (9) corresponds to a time-varying inductor characterized by time-varying permeance and constant turns. In this case not only the magnetic field of the inductor and active network, but also the agent which changes the permeance and via that the characteristic of the inductor, take part in the energy exchange [15]. Therefore, the energy delivered to the inductor by the active network in the time interval from t_0 to t is

$$W(t_0,\, t) = \frac{1}{2}L(t)i^2(t) - \frac{1}{2}L(t_0)i^2(t_0) + \int_{t_0}^{t}\frac{1}{2}L(t')i^2(t')\,dt', \quad (13)$$

where the third term is the energy delivered by the active network to the permeance changing agent [15].

From Eq. (13), the instantaneous power of a linear time-varying inductor with time-varying permeance and time-invariant turns equals

$$p_L(t) = v(t)i(t) = \frac{dW(t_0,\, t)}{dt} =$$

$$= i(t)L(t)\frac{di(t)}{dt} + \frac{1}{2}i^2(t)\frac{dL(t)}{dt} + \frac{1}{2}i^2(t)\frac{dL(t)}{dt} = \quad (14)$$

$$= i(t)L(t)\frac{di(t)}{dt} + i^2(t)\frac{dL(t)}{dt}.$$

Hence the voltage across the inductor

$$v(t) = L(t)\frac{di(t)}{dt} + i(t)\frac{dL(t)}{dt}, \quad (15)$$

i. e. an expression identical with (9).

Conclusions

1. A practical linear time-varying inductor can be implemented changing either the permeance or the turns of the inductor.

2. Voltage-current relations for linear inductors with time-varying permeance and time-varying turns are different due to various energy conversion processes.

3. For linear inductors with time-varying turns the voltage component proportional to the rate of inductance change is twice smaller than the corresponding voltage component for linear inductors with time-varying permeance.

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V. SARV

MUUTUVA KEERDUDE ARVUGA LINEAARSE PAISPOOLI ENERGIABILANSS NING PINGE JA VOOLU SEOS

On näidatud, et ajas muutuva induktiivsusega paispooli energiavahetus ümbritseva keskkonnaga võib toimuda põhiliselt kahel moel ning et sellele vastab kaks erinevat pinge ja voolu seost. Senituntud seos (9) vastab induktiivsuse muutmisele paispooli magnetilise juhtivuse muutmise teel. Sel puhul osalevad energiavahetuses paispooli klemmidega ühendatud elektriahel, paispooli magnetväli ning magnetilist juhtivust muutev tegur. Kui aga induktiivsust muudetakse voolustatud keerdude arvu muutes, nagu see on osutunud otstarbekaks mitmetes uutes voolusilumisskeemides [1–14], siis toimub energiavahetus ainult paispooli klemmidega ühendatud elektriahela ja paispooli magnetvälja vahel. Artiklis esitatud uus pinge ja voolu seos (8), mis vastab niisugusele juhule, on tuletatud nii Faraday elektromagnetilise induktsiooni seaduse kui ka energiavahetuse iseloomule vastava paispooli hetkvõimsuse avaldise alusel.

В. САРВ

БАЛАНС ЭНЕРГИИ И СООТНОШЕНИЕ МЕЖДУ НАПРЯЖЕНИЕМ И ТОКОМ В ЛИНЕЙНОМ ДРОССЕЛЕ С ПЕРЕМЕННЫМ ВО ВРЕМЕНИ ЧИСЛОМ ВИТКОВ

Показано, что между дросселем с переменной во времени индуктивностью и окружающей средой существуют два вида энергообмена, которые и описываются, соответственно, двумя различными соотношениями между напряжением и током дросселя. Известное соотношение (9) соответствует изменению индуктивности с изменением магнитной проводимости дросселя. В этом случае в энергообмене участвуют электрическая цепь, к которой присоединен дроссель, магнитное поле дросселя и фактор, обусловливающий изменение магнитной проводимости. Когда же индуктивность меняют изменением числа участвующих в работе витков (напр., в схемах сглаживания тока [1–14]), энергообмен происходит только между электрической цепью и магнитным полем. Новое соотношение (8) выведено в статье двояко — исходя из закона электромагнитной индукции Фарадея и исходя из выражения мгновенной мощности дросселя с соответствующим видом энергообмена.