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COMPUTABILITY IN HOMOGENEOUS STRUCTURES

In this paper the general results of the work [1,2] are presented. The formal definitions for our paper are presented in [3–8].

Key Words and Phrases. Erasable configuration (EC), inside block (IB) [5].

Türing machine with s symbols and q states (MT_q^s); TAG-system has alphabet $C = \{c_1, \ldots, c_m\}$, deletion number w and m elementary transformations

$$c_i \overset{w}{\Rightarrow} b_i \qquad (i = \overline{1, m}),$$

where b_i — words in C [3].

Post-productions system (PPS) has alphabet C and d basis productions

$$a_i W \Rightarrow W b_i \qquad (i = \overline{1, d}),$$

where a_i, b_i, W — words in C; semi-Thue system [3, 7].

Büchi regular system (BRS) has alphabet C and d basis transformations

$$a_i W \Rightarrow b_i W \qquad (i = \overline{1, d}),$$

where a_i, b_i, W — words in C [4].

SS-machine [6]; $\mathrm{ST}(Z^1\ S^p,\ L^v_{(n)})$ — homogeneous structures [8].

Let M_1 be an algorithm whose alphabet is A, and M_2 be an algorithm whose alphabet is $A'(A \subseteq A')$. Let $M_1^i s = s^*$ $(M_1^0 s = s)$ be the result of the i-fold rewriting of word s in A by M_1. For an arbitrary word s in A the algorithm M_1 generates a sequence of the words.

$$M_1^0 s,\ M_1^1 s,\ \ldots,\ M_1^i s,\ \ldots . \qquad (\alpha)$$

Let s' be any word in A', word s' in A is equal to s, and M_2 generates a sequence of the words for word s'

$$M_2^0 s',\ M_2^1 s',\ \ldots,\ M_2^i s',\ \ldots . \qquad (\beta)$$

Then M_2 T-models M_1 if there exists some procedure which allows for any word s to evolve from (β) such a subsequence

$$M_2^0 s',\ M_2^{j_1} s',\ M_2^{j_2} s',\ \ldots,\ M_2^{j_i} s',\ \ldots$$

that in A $(\forall i \in N)\,(M_2^{j_i}s = M_1^i s)$ and $j_i = T_i$. If j_i depends on the length of the rewritten word s, then M_2 slightly T-models M_1.

Theorem 1. For an arbitrary MT_q^s there exists $ST(Z^1, S^p, L_{(3)}^{v_1})$ $(p = s + 3q)$ which 2-models it.

Theorem 1.1. For an arbitrary MT_q^S [7] there exists $ST(Z^1, S^p, L_{(3)}^{v_1})$ $(p = s + q + 9)$ which 8-models it.

As consequences of Theorem 1.1 we have very interesting results on the algorithmic unsolvability of some problems concerning the behaviour of the finite configurations in homogeneous structures.

Theorem 1.2. For an arbitrary SS-machine there exists $ST(Z^1, S^p, L_{(3)}^{v_1})$ $(p = 2n_1 + n_2 + 4)$ which slightly T-models it, where $T = 2l(s^*) + 2$ and n_1, n_2 are numbers of the instructions $\{P_0, P_1\}$ and SD(k), respectively.

Theorem 2. For an arbitrary MT_q^s there exists $ST(Z^1, S^p, L_{(4)}^{v_2})$ $(p = s + q)$ which 1-models it.

As an immediate consequence of Theorem 2 and Tritter's results [3] we have

Corollary 1. There is $ST(Z^1, S^{11}, L_{(4)}^{v_2})$, which 1-models Universal Türing machine.

Theorem 3. For an arbitrary TAG-system there exists $ST(Z^1, S^p, L_{(3)}^{v_1})$ which slightly T-models it, where $p = w + m + \sum_1^m l(b_i) + 3$ and $T = l(s^*) + l(b_i)$, where rewritten word s^* is $s^* = c_i s$.

Another natural class is the LAG-systems. A LAG-system is a set of $\leqslant m^\beta$ translations

$$c_{i_1}c_{i_2}\ldots c_{i_\beta}W \Rightarrow Wb_i \;\; (i = \overline{1, d})\;(d \leqslant m^\beta),$$

where b_i, W — words in C, such that if the first β symbols of a word s are $c_{i_1}\ldots c_{i_\beta}$, the first symbol, viz., c_{i_1}, is deleted and word b_i is appended at the end of the word s. Obviously when $\beta = 1$, TAG-systems and LAG-systems coincide.

Theorem 3.1. For an arbitrary $ST(Z^1, S^p, L_{(n)}^{\bar{n}})$ there exists a LAG-system with $C = S^p \cup \overline{0}$, $\beta = n$ and $d = 2\sum_1^n p^i - p^n$, which slightly T-models it, where $T = l(s^*) + n - 1$.

Theorem 3.2. For an arbitrary LAG-system there exists $ST(Z^1, S^p, L_{(\beta+1)}^{v_1})$ which slightly T-models it, where $p = m + \sum_1^d l(b_i) + 3$ and $T = l(s^*) + l(b_i)$ if $s^* = c_{i_1}\ldots c_{i_\beta}W$.

Theorem 4. For an arbitrary TAG-system there exists $ST(Z^1, S^p, L_{(2)}^v)$ $(v = \{-1, 0\})$ which slightly T-models it, where $p = (w + 1)m + m^2 + \sum_1^m l(b_i) + 2$ and $T = l(s^*) + l(b_i) + 1$.

Theorem 5. For an arbitrary PPS there exists $ST(Z^1, S^p, L_{(3)}^{v_1})$ which slightly T-models it, where $p = 3\sum_1^d l(a_i) + l(b_i) + 3d + m + 10$ and $T = 4l(a_i) + 2l(s^*) + 2l(b_i)$.

Theorem 6. For an arbitrary BRS there exists $ST(Z^1, S^p, L_{(3)}^{v_1})$ which slightly T-models it, where $p = 3\sum_1^d \{l(a_i) + l(b_i)\} + m - 6d + 10$ and $T = l(a_i) + \max\{l(a_i), l(b_i)\}$.

Lemma 1. For $ST(Z^1, S^p, L^v_{(n)})$ there exists $ST(Z^1, S^{p'}, L^v_{(k)})$ $(p' = \sum_0^{T-1} p^{k^i})$ which T-models it, where $n > k$ and

$$T = \left[\frac{n-k}{k-1}\right] + \operatorname{sg}\left(\frac{n-k}{k-1} - \left[\frac{n-k}{k-1}\right]\right) + 1.$$

This lemma allows us to expand results of the theorems 1—6 for the case $ST(Z^1, S^p, L^v_{(2)})$.

Theorem 7. For an arbitrary ST $(Z^1, S^p, L^v_{(2)})$ there exists MT^S_q which slightly T-models it, where $s = p + 1$, $q = p + 4$ and $T = 3l(s^*) + 2$.

Theorem 8. For an arbitrary $ST(Z^1, S^p, L^v_{(n)})$ there exists a semi-Thue system which slightly T-models it.

Theorem 9. For an arbitrary $ST(Z^1, S^p, L^v_{(2)})$ there exists PPS which slightly T-models it, where $C = 3p + 5$ and

$$T = \left]\frac{l+4}{2}\right[+ 2\left[\frac{l+3}{2}\right] \quad (l = l(s^*)).$$

Theorem 10. For an arbitrary Markov normal algorithm there exists $ST(Z^1, S^p, L^v_{(2)})$ which slightly T-models it.

Theorem 11. For an arbitrary $ST(Z^1, S^p, L^v_{(2)})$ there exists Markov normal algorithm which slightly T-models it, where $\#A = 2p + 5$ and $T = 3l(s^*) + 2$.

Let $\hat{L}_{(n)} = \{L^1_{(n)}, \ldots, L^d_{(n)}\}$ be set of different functions $L^v_{(n)}$. Then, $d = p^{l^n - 1}$. With each function $L^{(\Omega)}_{(n)}$ we associate the sequence of functions from $\hat{L}_{(n)}$.

$$\Omega : X_1, X_2, \ldots, X_m, \ldots \quad (X_i \in \hat{L}_{(n)}),$$

$$L^\Omega_{(n)} = X_1 X_2 \ldots X_m \ldots .$$

This remark leads us to two theorems.

Theorem 12. For an arbitrary periodical Ω and $ST(Z^1, S^p, L^\Omega_{(n)})$ there exists $ST(Z^1, S^{p'}, L^v_{(2)})$ $(p' > p)$ which slightly T-models it.

Theorem 13. Let $L^\Omega_{(2)}$ be associated with the sequence

$$\Omega : \sigma_{i_1}, \sigma_{i_2}, \ldots, \sigma_{i_m}, \ldots,$$

where $\sigma_{i_j} \in \hat{L}_{(2)}$ and function $G(n) = i_n$ $(n \in N)$ is partial recursive. Then there exists $ST(Z^1, S^{p'}, L^{v_2}_{(4)})$ which can generate the same sequences of configurations in S^p, and $ST(Z^1, S^p, L^\Omega_{(2)})$, where $p' = p^{p^2} + 2p^{p^2-1} + p + 68$.

Corollary 2. Let $L^\alpha_{(n)}$ be associated with the sequence

$$\alpha : \sigma_{i_1}, \sigma_{i_2}, \ldots, \sigma_{i_m}, \ldots,$$

where $\sigma_{i_j} \in \hat{L}_{(n)}$. Then there exists the function $L^v_{(q)}$ $\{q = 2^m(n-1) + 1\}$, which for each finite configuration C in alphabet S^p $CL^\alpha_{(n)} = CL^v_{(q)}$ and vice versa.

Theorem 14. For an arbitrary Boolean function $f(x_1, \ldots, x_n)$ there exists $ST(Z^1, S^4, L^v_{(n)})$ which 1-models it. There exists $ST(Z^1, S^p, L^{v_1}_{(n+1)})$ $(p = 2^{2^n} + 2^2)$ which 1-models each Boolean function $f(x_1, \ldots, x_n)$.

Theorem 15. (Moore's Problem [5]). For an arbitrary integer $p > 1$ there exists at least

$$M(p) = \begin{cases} (p!)^{p^2-2}/p^2, & \text{if } p = 2, 3; \\ (p!)^{p^2-2}\left(\sum_{1}^{m-1} i! + (m!)^2\right)/p^2 \quad \left(m = \left[\frac{p}{2}\right]\right), & \text{if } p > 3 \end{cases}$$

such an $ST(Z^1, S^p, L^{v_1}_{(3)})$ that the least size of IB of EC is two. For each $n \geqslant 2$ and $p \geqslant 3$ there exists such $ST(Z^1, S^p, L^v_{(n)})$ that the least size of IB of EC is n.

Let Z^N be homogeneous space and $\sum_x$ be the maximal template of X-automata from Z^N whose neighbourhood index $v = \{(0, \ldots, 0), v_1^2, \ldots \ldots, v_N^2), \ldots, (v_1^n, \ldots, v_N^n)\}$ satisfies the consequence condition

$$(\forall X \in Z^N), \ (\forall i) \left(\sqrt{\sum_{j=2}^{N} (v_j^i)^2} \leqslant \varrho \right).$$

Suppose that template $\sum_x$ consists of $m = [\varphi(\varrho)]$ automata. Now suppose that template of X-automata from Z^N in time T is a subtemplate of $\sum_x$ and is a function of the configuration of the template of X-automata in time $T-1$. Then, the local transformation in the structure $\overline{ST}(Z^N, S^p, \hat{L}^v_{(m)})$ is a function $\hat{L}^v_{(m)}$.

Let $\tilde{L}$ be a set of such different structures. It can be shown that $\#(L) = [p(2^m - 1)]^{(p+1)^m - 2^m}$.

Lemma 2. For an arbitrary $\overline{ST}(Z^N, S^p, \hat{L}^v_{(m)})$ there exists $ST(Z^{N+1}, S^{p'}, L^v_{(2m+1)})$ $(p' = p + 2^m - 1)$ which 1-models it.

Lemma 3. For appropriate coding and any $\overline{ST}(Z^N, S^p, \hat{L}^v_{(m)})$ there exists $ST(Z^N, S^{p'}, L^v_{(m)})$ $(p' = p^{2m-1})$ which 1-models it.

Lemmas 2 and 3 are useful for modelling, by computers, of the structures $\overline{ST}(Z^N, S^p, \hat{L}^v_{(m)})$.

Let us view Türing machine $(MT_q^{s(m)})$ as programmed computers as follows: A $MT_q^{s(m)}$ is a device equipped with a container for cards, a tape scanner-printer-mover, and a tape that is infinite in both directions. The tape is divided into squares along its length, and the scanner can look at one square at a time. The device can print the blank, or one of a finite set S of symbols. On the square it is examining and shifting the tape l squares to right or left $(l = \pm i; i = \overline{0, m})$. The container can hold an arbitrarily large but finite number of cards, together called the program. On each card is printed a single 5-tuple $q_i S_i S'_i l q'_i$. The q_i denotes internal states of the device; the S_i are tape symbols; and l is a move. When the device is in state q_i and scans the symbol S_i, it prints the symbol S'_i, moves the tape l and changes its internal state to q'_i. If there is no card starting with $q_i S_i$ in the container when the machine is in state q_i and scanning the symbol S_i, then the machine stops. Any program is allowed, subject to the condition that any two cards must differ in the initial pair $q_i S_i$.

We have

Lemma 4. For an arbitrary $MT_q^{s(m)}$ there exists $ST(Z^1, S^p, L^v_{(2m+1)})$ $(p = s(q+1))$ which 1-models it.

From lemmas 1 and 4 we immediately have

Theorem 16. For an arbitrary $MT_q^{s(m)}$ there exists $ST(Z^1, S^p, L^v_{(2)})$ $(p = \sum_{i=0}^{2m-1} [s(q+1)]^{2^i})$ which $2m$-models it.

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REFERENCES

1. Аладьев В. З., Вычислимость в однородных структурах, I, ВИНИТИ, № 2850-71, М., 1971.
2. Аладьев В. З., Вычислимость в однородных структурах, II, ВИНИТИ, № 3023-71, М., 1971.
3. Мальцев А. И., Алгоритмы и рекурсивные функции, М., 1965.
4. Büchi J. R., Archiv für Math. Logik und Grundlagenf., Nr. 3—4 (1964).
5. Moore E. F., Proc. Symp. Appl. Math., 14, 17 (1962).
6. Wan Hao, Math. Ann., 152, No. 1 (1965).
7. Fischer P., JACM, No. 4 (965).
8. Аладьев В., Изв. АН ЭССР, Биол., 19, 266 (1970).

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HOMOGEENSETE STRUKTUURIDE ARVUTATAVUSEST

Tõestatakse, et ühedimensioonilised homogeensed struktuurid (HS) võivad modelleerida Posti produktsiooni süsteemi (PPS), TAG-süsteemi, Büchi regulaarset süsteemi, Türingi masinat (TM) ja Markovi normaalset algoritmi (MNA). Ja ka vastupidi — TM, PPS, MNA ja semi-Thue süsteem võivad modelleerida ühedimensioonilisi HS-e.

Modelleerivate algoritmide põhiparameetrid antakse modelleeruvate algoritmide kaudu. Peale selle vaadeldakse mitmeid küsimusi, mis puudutavad HS arvutatavust.

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ВЫЧИСЛИМОСТЬ В ОДНОРОДНЫХ СТРУКТУРАХ

Приводятся некоторые результаты вычислимости в однородных структурах (ОС). Доказывается, что одномерные ОС могут моделировать систему продукций Поста (СПП), ТАГ-систему, регулярную систему Бюхи, машину Тьюринга (МТ) и нормальный алгоритм Маркова (НАМ). И наоборот, одномерные ОС могут моделироваться МТ, СПП, НАМ и системой полу-Туэ.

Даются значения основных параметров моделирующих алгоритмов через основные параметры моделируемых алгоритмов.

Рассмотрены также некоторые другие вопросы, связанные с вычислимостью в однородных структурах.