Proc. Estonian Acad. Sci. Phys. Math., 2005, **54**, 3, 181–192

About the complexity evaluation of large structured objects

Tõnu Lausmaa

Department of Computing Engineering, Tallinn University of Technology, Raja 15, 12618 Tallinn, Estonia; tonu.lausmaa@mail.ee

Received 8 February 2005, in revised form 20 June 2005

Abstract. An algebraic informational measure is presented for evaluating the intrinsic complexity of large structured objects on the basis of the distribution of the basic property of their elements. The method is based on the notion of partition, formed by classifying the object elements according to their property values. Partition is evaluated by the notion of extropy, which characterizes its information content. The discrete property function is extrapolated into a continuous property curve and an extropic measure called the extropy index is calculated for the object on the basis of this curve. To test the proposed method, it was applied to evaluate the structural complexity of state economies on the basis of the population income distribution. The comparison of the extropy index with the GDP index and the human development index gave the correlation coefficients of 0.77 and 0.86, respectively.

Key words: extropy, partition, structural complexity, informational measure, economy evaluation.

1. INTRODUCTION

We share A. N. Kolmogorov's opinion that the foundations of the information theory have a finite combinatorial character. As an initial step of our approach, we start with the simplest informational model for structural complexity – a partition on a finite set. Already von Neumann and Morgenstern [[1]] pointed out that the notion of partition on a finite set can be interpreted as information. This point of view on the partition as an algebraic equivalent for the notion of information was developed further by Hartmanis and Stearns [[2]] in the theory of finite state machines. The extropy[1] of the partition gives us the initial quantitative

[1] Instead of the rather widespread notion of *negentropy* we use the term *extropy*, as *neg en* means in Greek *not into*, which is equivalent to *out*, or *ex* in Greek.

measure for evaluating the structural complexity of finite objects represented by this partition, assuming that the information content characterized by the partition extropy is proportional to the structural complexity. Unfortunately, in practice the formation of a classification partition is seldom a well-defined process. More often than not, and especially with a large set of elements, this process is a rather voluntary one, depending strongly on the fineness of the classification for, as a rule, there does not exist a clear-cut property-based natural classification pattern. Therefore, to get a strictly determined complexity evaluation outcome, we have to find such a complexity measure which does not depend on this classification process. It makes us to base the complexity measure on a limit value of the classification process while enhancing steadily the classification fineness by increasing the partition rank. Although the extropy value for a partition while increasing its rank is not limited, the difference between extropies of two partitions with their ranks simultaneously approaching infinity through the one and the same partitioning process, is limited. This fact gives us a chance to measure the extropy value for a property curve in relation to an ideal property curve with the maximum extropy value, which is characterized by an exponential function. An exponential function $f(x)$ has the wonderful property $f(x_2') - f(x_1') = f(x_2'') - f(x_1'') \Rightarrow \int_{x_1'}^{x_2'} f(x)\,\mathrm{d}x = \int_{x_1''}^{x_2''} f(x)\,\mathrm{d}x$, from which it follows that all blocks of the partition contribute to the final result to the same extent, i.e. the higher the rank of a block in the partition, the smaller its size in it. As all elements contribute equally to the outcome of the partitioning process, the solution is the best and has therefore the maximum value compared to all other choices. Thus, to find the limit value for this extropy difference, we will extrapolate the discrete property function of an object into a continuous differentiable property curve. To this curve we will juxtapose an exponential curve in such a way that the end points of both curves coincide. Next we will divide both curves into n cuts, which have all equal projections to the ordinate axes, reflecting the principle that all properties should be equally represented by the partition-forming process. The partitions for both curves are now formed by cutting the surface area between the curves and the argument axis into blocks according to these n cuts of the curve. Now we will find the limit value for the extropy difference of these two partitions while their rank n approaches infinity. This limit gives us a foundation for an extropic measure, which is invariant to the partitioning process and is solely characterized by the given property curve of the object under consideration. On the basis of this limit value we define the notion of the extropy index as its normalized complementary which evaluates in informational terms the vicinity of the property curve to the ideal exponential one and thus measures the structural complexity for objects characterized by a property distribution which can be extrapolated into this curve.

To test the proposed method, it was applied to evaluate the structural complexity of a state economy on the basis of the population income distribution. It should be emphasized that this attempt was not meant to break new ground in economics. We fully admit that the result of the application of our approach in economy is a preliminary one as there is too much uncertainty about the initial data gathered by different organizations under different conditions and background. It is but a humble attempt to introduce our method and call attention to this application in the hope that some in-depth study would follow.

2. EXTROPY OF THE PARTITION AS A MEASURE OF ITS INFORMATION CONTENT

It goes without saying that one can use many different models for representing the structural complexity of a large object, a contemporary economy being a good example of it. On the other hand, dealing with such a difficult problem as measuring structural complexity, one should start with the simplest strategy to elaborate the model in the course of the study. And the partition is just the very thing satisfying this strategy. One cannot find a simpler informational model than a partition and from a partition it is easy to move on to more complicated models. We are not going to assume a priory that an extropy function on partitions is just the parameter that informs us about structural complexity, taking mechanically over the extropy interpretation as a quality parameter for objects it is applied to. Instead we are going to use an axiomatic approach, defining first an information-related evaluation function on partitions and only after this reaching the so-called Shannon entropy formula; we accept this as the searched parameter of structural complexity. As in our context this formula is linked to complexity, it is natural to call this measure an extropic one (this choice has a strong justification from the ecological point of view, as by consuming energy quality (extropy), the complexity of organic life would increase, which means that the extropy of energy is converted into the extropy of organic life). But the main difference between our approach and that of Claude Shannon is that while the latter is a probabilistic approach to information, the former is an axiomatic algebraic one.

In this chapter we will find an extropic measure for partitions defined on a finite set which satisfies the basic intuitive properties of information. This axiomatic approach is based on the fundamental information-related properties of the partition lattice.

Let us define a partition $\pi_i(X)$ on a finite set $X = \{x_1, x_2, ..., x_m\}$ as a class of its subsets (blocks of the partition) $\{B_i^{(1)}, B_i^{(2)}, ..., B_i^{(\alpha)}, ..., B_i^{(m_i)}\}$, satisfying the following conditions:

(a) $\bigcup_{\alpha=1}^{m_i} B_i^{(\alpha)} = X$;

(b) for any arbitrary $B_i^{(\alpha)}, B_i^{(\beta)} \in \pi_i(X)$ we have $B_i^{(\alpha)} \bigcap B_i^{(\beta)} = \varnothing$.

A block $B^{(\alpha)} \in \pi(X)$ consisting of elements $x_{\alpha 1}, x_{\alpha 2}, ..., x_{\alpha m} \in X$ will be denoted by $\overline{x_{\alpha 1}, x_{\alpha 2}, ..., x_{\alpha m}}$. Extreme partitions are a zero partition (denoted by 0_X), having in each block no more than one element, and a unit partition (denoted by 1_X), having only one block. If for any arbitrary $B_i^{(\alpha)} \in \pi_i$ there exists $B_j^{(\beta)} \in \pi_j$ such that $B_i^{(\alpha)} \subseteq B_j^{(\beta)}$, then we will denote it by $\pi_i(X) \leq \pi_j(X)$. It is not hard to show that if $\pi_i(X) \leq \pi_j(X)$ and $\pi_j(X) \leq \pi_i(X)$, then

$\pi_i(X)=\pi_j(X)$. We define for any arbitrary $\pi_i(X)$ and $\pi_j(X)$ operations $\pi_i\cdot\pi_j \underset{\mathrm{Df}}{=} \{B_i^{(\alpha)}\cap B_j^{(\beta)} \mid B_i^{(\alpha)}\in\pi_i \wedge B_j^{(\beta)}\in\pi_j\}$ and $\pi_i+\pi_j=\bigcap\{\pi_k \mid \pi_k\geq\pi_i,\pi_j\}$. The restriction of a partition $\pi(X)$ onto $X'\subset X$ will be denoted by $\overline{\pi}(X)\underset{\mathrm{Df}}{=}\{B^{(\alpha)}\cap X' \mid B^{(\alpha)}\in\pi\}$. With respect to the above-defined multiplication "$\cdot$" and addition "+" operations all possible partitions on X will build up a semimodular lattice [3], which will be denoted by $\mathcal{L}(X)$. For any subset $X'\subseteq X$ we will define its weight $q_X(X')$ as a ratio $q_X(X')\underset{\mathrm{Df}}{=}\|X'\|/\|X\|$ (as a rule, the subscript by q will be omitted). Partitions $\pi_i(X')$ and $\pi_j(X'')$ will be called equivalent (the corresponding denotation is $\pi_i(X')\equiv\pi_j(X'')$) iff there exists a bijection $\varphi:\pi_i\to\pi_j$ such that for any $B_i^{(\alpha)}\in\pi_i$ we have $q_{X'}(B_i^{(\alpha)})=q_{X''}(\varphi(B_i^{(\alpha)}))$.

Let us call $\pi_i(X)$ quasi-independent with respect to $\pi_j(X)$ (denoted by $\pi_i\top\pi_j$) iff for any $B\in\pi_i+\pi_j$ and $B_j^{(\alpha)}\in\pi_j$ with $B_j^{(\alpha)}\subset B$ the condition $\overline{\pi}_i(B)\equiv\overline{\pi}_i(B_j^{(\alpha)})$ is satisfied. The reflectivity of quasi-independence follows directly from its definition (i.e. $(\forall\pi_i)(\pi_i\top\pi_i)$). Iff partitions π_i and π_j satisfy the condition $\pi_i(X)+\pi_j(X)=1_X$, they will be called orthogonal. Quasi-independent and orthogonal partitions will be called independent (denoted by $\pi_i\dagger\pi_j$). It is easy to see that the independent relation $\dagger$ is anti-reflexive and symmetric.

Definition 1. *A real-value domain function $G(\pi_i)$ defined for the aggregate of all partitions $\pi_i(X)$ on all possible finite sets X is called extropy of the partition $\pi_i(X)$ if it satisfies the following axioms corresponding to the intuitive properties of the object-related information content*:

(i) *positivity*: $G(\pi_i)\geq 0$;
(ii) *invariability*: $\pi_i(X')\equiv\pi_j(X'')\Rightarrow G(\pi_i)=G(\pi_j)$;
(iii) *monotony*: $\pi_i(X)\leq\pi_j(X)\Rightarrow G(\pi_i)\geq G(\pi_j)$;
(iv) *subadditivity*: $G(\pi_i(X))+G(\pi_j(X))\geq G(\pi_i\cdot\pi_j)+G(\pi_i+\pi_j)$.

Theorem 1 [4]. *For any arbitrary partition $\pi_i(X)$ its extropy $G(\pi_i)$ equals $aH(\pi_i)+b$, where $H(\pi_i)\underset{\mathrm{Df}}{=}-\Sigma_{\alpha=1}^{m_i}q(B_i^{(\alpha)})\ln q(B_i^{(\alpha)})$ and a, b are some positive constants*.

Theorem 1 gives us the justification to use the partition extropy as the only quantitative partition evaluation that satisfies the basic intuitive properties of information.

It is not hard to prove that for any $\pi_i(X)$ and $\pi_j(X)$ the following conditions are equivalent:

(i) $\pi_i\top\pi_j$;
(ii) $\pi_j\top\pi_i$;
(iii) $H(\pi_i)+H(\pi_j) = H(\pi_i\cdot\pi_j)+H(\pi_i+\pi_j)$;
(iv) for any $B\in\pi_i+\pi_j$, $B_i^{(\alpha)}\in\pi_i$, and $B_j^{(\beta)}\in\pi_j$ with $B_i^{(\alpha)}$, $B_j^{(\beta)}\subset B$ we have $q_B(B_i^{(\alpha)}\cap B_j^{(\beta)})=q_B(B_i^{(\alpha)})q_B(B_j^{(\beta)})$.

One can show that the relation $\pi_i(X)\dagger\pi_j(X)$ is equivalent to the following two conditions:

(i) $H(\pi_i) + H(\pi_j) = H(\pi_i \cdot \pi_j)$;

(ii) for any $B_i^{(\alpha)} \in \pi_i$ and $B_j^{(\beta)} \in \pi_j$ the equality $q(B_i^{(\alpha)} \bigcap B_j^{(\beta)}) = q(B_i^{(\alpha)})\, q(B_j^{(\beta)})$ holds.

3. STRUCTURAL COMPLEXITY OF OBJECTS REPRESENTED BY THEIR PROPERTY DISTRIBUTION

In this chapter we are going to extend the partition-based extropy evaluation defined on a finite set onto a continuous real-value function. The approach applied enables us to evaluate large complicated objects that can be characterized by a basic property function on their elements. The property function of these objects can be extrapolated into a continuous real-valued curve which will be submitted to extropic evaluation. The remarkable feature of our approach is that the derived extropy measure does not depend on the partitioning process of the property curve. This enables us to get a unique quantitative complexity measure reflecting the most fundamental properties of the algebraic concept of information.

We will denote by R^+ the set of positive real numbers and define $X \underset{\mathrm{Df}}{=} [\alpha, \omega] \subset R^+$ as a segment on this set. Now we are going to define an object $\mathfrak{O}$ as a triple $\mathfrak{O} \underset{\mathrm{Df}}{=} \langle Z, P, \delta \rangle$, where Z is a finite set of its elements, $P \subset R^+$ is the set of basic properties of the elements and the function $\delta : Z \to P$ gives us the element-related basic property distribution for the object. Next we will index Z by $M = \{1, 2, ..., m\}$ with $m = |Z|$ in such a way that for each $i, k \in M$ we have $i \le k \Rightarrow \delta(z_i) \le \delta(z_k)$. Thus δ induces a real-number function $\overline{\delta} : M \to P$. Then we are going to extrapolate $\overline{\delta}$ into a continuous differentiable function $f : X \to Q$, i.e. $\delta \subset f$ with $M \subset X$ and $P \subset Q \subset R^+$. Next we are going to find for $f(x)$ an exponential approximation as a function $f_0(x) \underset{\mathrm{Df}}{=} Ae^{Bx}$ with the constants A and B chosen in such a way that $f_0(\alpha) = f(\alpha)$ and $f_0(\omega) = f(\omega)$. If we denote $F_\alpha^\omega \underset{\mathrm{Df}}{=} f(\alpha) - f(\omega)$, $\Delta y \underset{\mathrm{Df}}{=} f(\omega - \alpha)$, and $\Delta x \underset{\mathrm{Df}}{=} \omega - \alpha$, we get $A = \Delta y \big/ (e^{\ln F_\alpha^\omega \cdot \omega / \Delta x} - e^{\ln F_\alpha^\omega \cdot \alpha / \Delta x})$ and $B = \ln F_\alpha^\omega \big/ \Delta x$. It is easy to prove that for any $\Delta^{(1)} y \underset{\mathrm{Df}}{=} f_0(x_1'') - f_0(x_1')$ and $\Delta^{(2)} y \underset{\mathrm{Df}}{=} f_0(x_2'') - f_0(x_2')$ from $\Delta^{(1)} y = \Delta^{(2)} y$ it follows that $\int_{x=x_1'}^{x_1''} f_0(x)\,\mathrm{d}x = \int_{x=x_2'}^{x_2''} f_0(x)\,\mathrm{d}x$. In the following we are going to rely on the definitions

(i) $\Delta_n y \underset{\mathrm{Df}}{=} (f(\omega) - f(\alpha))/n$;

(ii) $F_\tau(x) \underset{\mathrm{Df}}{=} f_\tau(x) / f_\tau'(x)$;

(iii) $S^{(n)}(f_\tau) \underset{\mathrm{Df}}{=} \Sigma_{i=1}^n ((f_\tau(x_{i+1}) + f_\tau(x_i))/2) \cdot (x_{i+1} - x_i)$ and $H^{(n)}(f_\tau) \underset{\mathrm{Df}}{=} \Sigma_{i=1}^n (((f_\tau(x_{i+1}) + f_\tau(x_i))/2) \cdot (x_{i+1} - x_i)) \cdot \ln(((f_\tau(x_{i+1}) + f_\tau(x))/2) \cdot (x_{i+1} - x_i))$ with $(\forall i \in \{1, ..., n+1\})((x_i \underset{\mathrm{Df}}{=} \alpha + (\omega - \alpha) \cdot (i-1) \big/ n) \wedge (f_\tau(x_{i+1}) - f_\tau(x_i) = \Delta_n y))$;

(iv) $S(f_\tau) \underset{\mathrm{Df}}{=} \int_{x=\alpha}^{\omega} f_\tau(x)\,\mathrm{d}x$.

It is easy to see that $F_0(x) = \text{const}$ and $S(f_0) = A/B(e^{B\cdot\omega} - e^{B\cdot\alpha}) = \Delta y \cdot \Delta x / \ln F_\alpha^\omega$. Now we are prepared to prove the main theorem concerning the extropic evaluation of large structured objects characterized by the property function based on their basic elements.

Theorem 2. *For any real-value continuous differentiable function* $f(x)$ *we have*

$$\lim_{n\to\infty} (H^{(n)}(f_0(x)) - H^{(n)}(f(x))) =$$

$$-\ln(F_0/S(f_0)) + (1/S(f)) \cdot \int_{x=\alpha}^{\omega} f(x) \cdot \ln(F(x)/S(f))\,\mathrm{d}x.$$

Proof. Indeed,

$$\begin{aligned}
&\lim_{n\to\infty}(H^{(n)}(f_0(x)) - H^{(n)}(f(x)))\\
&= -\lim_{n\to\infty}\sum_{i=1}^{n}(((F_0\cdot\Delta_n y/S^{(n)}(f_0))\cdot\ln(F_0\cdot\Delta_n y/S^{(n)}(f_0)))\\
&\quad -((F(x_i)\cdot\Delta_n y/S^{(n)}(f))\cdot\ln(F(x_i)\cdot\Delta_n y/S^{(n)}(f)))\\
&= -\lim_{n\to\infty}\sum_{i=1}^{n}((F_0/S^{(n)}(f_0))\cdot\ln(F_0/S^{(n)}(f_0)) - (F(x_i)/S^{(n)})\cdot\ln F(x_i)/S^{(n)}(f))\Delta_n y\\
&\quad -\lim_{n\to\infty}\ln\Delta_n y\cdot\sum_{i=1}^{n}(F_0/S^{(n)}(f_0) - F(x_i)/S^{(n)}(f))\cdot\Delta_n y\\
&= -\lim_{n\to\infty}(1/S^{(n)}(f_0))\cdot\sum_{i=1}^{n}(F_0\cdot\ln(F_0/S(f_0))\cdot\Delta_n y\\
&\quad +\lim_{n\to\infty}(1/S^{(n)}(f))\cdot\sum_{j=1}^{n}(F(x_j)\cdot\ln(F(x_j)/S(f))\cdot\Delta_n y\\
&= -\ln(F_0/S(f_0)) + \lim_{n\to\infty}(1/S^{(n)}(f))\cdot\sum_{j=1}^{n}(F(x_j)\cdot\ln F(x_j))\cdot\Delta_n y\\
&\quad +\lim_{n\to\infty}\ln S^{(n)}(f)\cdot\sum_{j=1}^{n}(F(x_j)\cdot\Delta_n y)\\
&= -\ln(F_0/S(f_0)) + (1/S(f))\cdot\int_{x=\alpha}^{\omega} f_1(x)\cdot\ln(F(x)/S(f))\,\mathrm{d}x\,. \qquad \square
\end{aligned}$$

Now we will define an extropy index C_H for an object $\mathfrak{O}_\tau = \langle Z_\tau, P_\tau, \delta_\tau\rangle$ as $C_H(\mathfrak{O}_\tau) \underset{\mathrm{Df}}{=} (1 - \Delta H(f_\tau(x)))\cdot 100$, where $\Delta H(f_\tau(x)) \underset{\mathrm{Df}}{=} \lim_{n\to\infty}(H^{(n)}(f_0(x)) - H^{(n)}(f_\tau(x)))$ and $f_\tau(x)$ is an extrapolation of the property function $\delta_\tau : Z_\tau \to P_\tau$. The extropy index $C_H(\mathfrak{O}_\tau)$ reflects the structural complexity of the object $\mathfrak{O}_\tau$, whose property function δ_τ can be extrapolated into a continuous differentiable function $f_\tau(x)$.

4. STRUCTURAL COMPLEXITY OF A STATE ECONOMY

Talking about sustainable development, one cannot ignore economy. If we cannot find a way to follow sustainable economy, all our efforts in other walks of activity would end up in a ditch as well. Unfortunately, economy is a very complicated process and it is extremely difficult to get hold of reliable data about its functioning. The best available data are linked to the income distribution of the population for the country under scrutiny. This income distribution determines inequity which is one of the vital parameters of placing a judgement about the welfare of an economy. There are quite a few papers on finding such an inequity factor which would best characterize the well-being of an economy [5–7]. Unfortunately, all these approaches to determine the prosperity level of an economy on the basis of an equity factor lack a tangible logical background linking this factor to the structural complexity of an economy, but are only good mathematical studies into the subtle properties of the income distribution curve.

To evaluate the structural complexity of an economy, we are going to use the extrapolation of the population income distribution as initial data for the income inequity curve. As this cursory example is not to establish final truth about which economy would be preferred in absolute terms, we have not taken pains to work hard on the initial income distribution data resorting to the raw data flows but have been satisfied with the data already worked up in [8], presented in Table 1.

Based on the income distribution given in Table 1 we have found the income inequity (Table 2) for the countries under consideration, i.e. we have found the derivatives for the extrapolated income distribution curves.

Table 1. Income distribution

Country	Income distribution							Gini index
	Lowest 10%	Lowest 20%	Second 20%	Third 20%	Fourth 20%	Highest 20%	Highest 10%	
Australia	2.0	5.9	12.0	17.2	23.6	41.3	25.4	35.2
Belarus	5.1	11.4	15.2	18.2	21.9	33.3	20.0	21.7
Belgium	3.7	9.5	14.6	18.4	23.0	34.5	20.2	25.0
Finland	4.2	10.0	14.2	17.6	22.3	35.8	21.6	25.6
France	2.8	7.2	12.6	17.2	22.8	40.2	25.1	32.7
Germany	3.3	8.2	13.2	17.5	22.7	38.5	23.7	30.0
Hungary	3.9	8.8	12.5	16.6	22.3	39.9	24.8	30.8
India	3.5	8.1	11.6	15.0	19.3	46.1	33.5	37.8
Japan	4.8	10.6	14.2	17.6	22.0	35.7	21.7	24.9
Mexico	1.4	3.6	7.2	11.8	19.2	58.2	42.8	53.7
Netherlands	2.8	7.3	12.7	17.2	22.8	40.1	25.1	32.6
Russia	1.7	4.4	8.6	13.3	20.1	53.7	38.7	48.7
Slovak Republic	5.1	11.9	15.8	18.8	22.2	31.4	18.2	19.5
South Africa	1.1	2.9	5.5	9.2	17.7	64.8	45.9	59.3
Sweden	3.7	9.6	14.5	18.1	23.2	34.5	20.1	25.0
Switzerland	2.6	6.9	12.7	17.3	22.9	40.3	25.2	33.1
Ukraine	3.9	8.6	12.0	16.2	22.0	41.2	26.4	32.5
United Kingdom	2.6	6.6	11.5	16.3	22.7	43.0	27.3	36.1
United States	1.8	5.2	10.5	15.6	22.4	46.4	30.5	40.8
Vietnam	3.6	8.0	11.4	15.2	20.9	44.5	29.9	36.1

Table 2. Income inequity

Country	Income inequity																			
	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100
Australia	0.8	1.3	1.7	2.1	2.5	2.9	3.2	3.5	3.8	4.2	4.5	4.9	5.3	5.7	6.2	6.7	7.4	8.6	11.2	14.0
Belarus	2.5	2.8	3.1	3.4	3.6	3.8	4.0	4.1	4.3	4.4	4.6	4.8	5.0	5.3	5.5	5.8	6.2	7.1	9.0	11.0
Belgium	1.5	2.2	2.7	3.1	3.4	3.6	3.8	4.0	4.2	4.5	4.7	5.0	5.3	5.7	6.0	6.4	6.9	7.5	8.8	11.4
Finland	1.9	2.4	2.8	3.1	3.3	3.5	3.7	3.9	4.1	4.3	4.6	4.8	5.1	5.4	5.7	6.1	6.6	7.4	9.2	12.4
France	1.3	1.7	2.1	2.4	2.7	3.0	3.3	3.6	3.9	4.2	4.5	4.8	5.1	5.4	5.8	6.3	6.9	8.0	10.9	14.0
Germany	1.5	1.9	2.3	2.7	3.0	3.3	3.5	3.8	4.0	4.2	4.5	4.7	5.0	5.4	5.8	6.2	6.8	8.0	10.2	13.5
Hungary	1.8	2.2	2.5	2.8	3.0	3.2	3.4	3.6	3.8	4.0	4.2	4.5	4.8	5.2	5.6	6.1	6.9	8.2	10.7	14.1
India	1.5	1.9	2.2	2.4	2.6	2.8	3.0	3.1	3.3	3.5	3.7	4.0	4.3	4.6	4.9	5.3	5.8	6.9	11.4	22.1
Japan	2.3	2.6	2.9	3.1	3.3	3.4	3.6	3.8	4.0	4.2	4.5	4.7	5.0	5.3	5.6	5.9	6.4	7.4	9.6	12.1
Mexico	0.6	0.8	1.0	1.2	1.4	1.6	1.9	2.2	2.5	2.8	3.1	3.5	3.9	4.4	5.0	5.8	6.7	8.7	14.8	28.0
Netherlands	1.3	1.7	2.1	2.4	2.7	3.0	3.3	3.6	3.9	4.2	4.4	4.7	5.0	5.3	5.7	6.3	7.0	8.2	10.8	14.1
Russia	0.8	1.0	1.2	1.4	1.6	1.9	2.1	2.4	2.7	3.0	3.3	3.7	4.1	4.5	5.0	5.6	6.5	8.6	14.6	24.1
Slovak Republic	2.5	2.9	3.2	3.5	3.8	4.0	4.2	4.4	4.6	4.8	5.0	5.1	5.2	5.3	5.5	5.8	6.3	7.1	8.3	9.9
South Africa	0.5	0.6	0.8	0.9	1.1	1.3	1.5	1.7	1.9	2.2	2.5	2.9	3.3	3.9	4.7	5.7	7.7	11.8	17.8	28.1
Sweden	1.5	2.2	2.7	3.1	3.4	3.6	3.8	4.0	4.2	4.4	4.6	4.8	5.1	5.5	5.9	6.3	6.9	7.8	9.2	10.9
Switzerland	1.2	1.6	2.0	2.4	2.8	3.1	3.4	3.7	4.0	4.2	4.5	4.8	5.1	5.5	5.9	6.4	7.0	8.1	10.1	14.1
Ukraine	1.9	2.0	2.2	2.4	2.6	2.8	3.0	3.3	3.6	3.9	4.2	4.5	4.8	5.1	5.5	6.1	6.8	8.0	10.9	15.5
United Kingdom	1.1	1.5	1.9	2.2	2.5	2.8	3.1	3.4	3.7	4.0	4.3	4.6	4.9	5.3	5.8	6.4	7.2	8.6	11.2	16.1
United States	0.8	1.2	1.5	1.8	2.1	2.4	2.7	3.0	3.3	3.7	4.1	4.5	4.9	5.2	5.5	6.0	6.9	9.0	12.4	18.1
Vietnam	1.7	1.9	2.1	2.3	2.5	2.7	2.9	3.1	3.4	3.6	3.9	4.2	4.6	5.0	5.4	5.9	6.6	8.1	11.9	18.0

Next we extrapolated the income inequity into a continuous curve for the countries under consideration and found for each of these curves a corresponding exponential curve. Figures 1 and 2 show these curves for Australia and India as the extreme cases for the set of countries observed.

Next we evaluated an economy on the basis of the extrapolated continuous property curve by the extropy index. In the teeth of presupposed inaccuracy, the result shows a strong correlation between the extropy index and the development level of an economy, represented by the GDP index and the human development index. Table 3 represents a ranking list of countries under consideration according to their extropy index, together with the comparison with the Gini index, GDP index, and human development index.

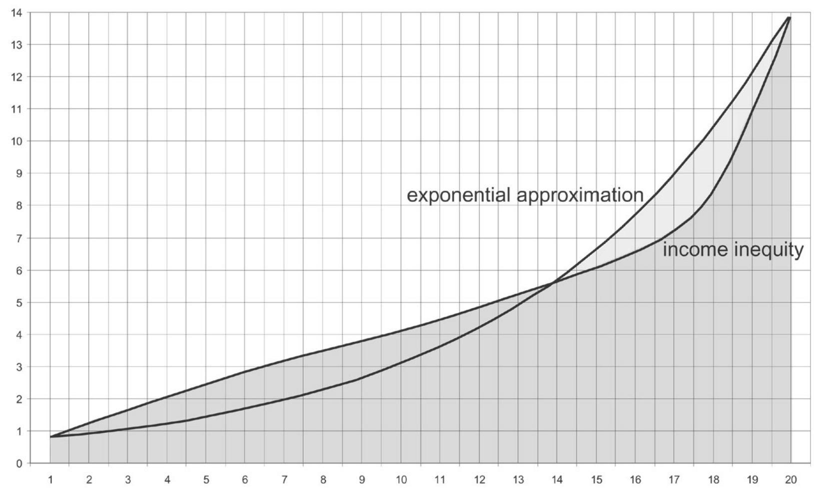

Fig. 1. Income inequity in Australia.

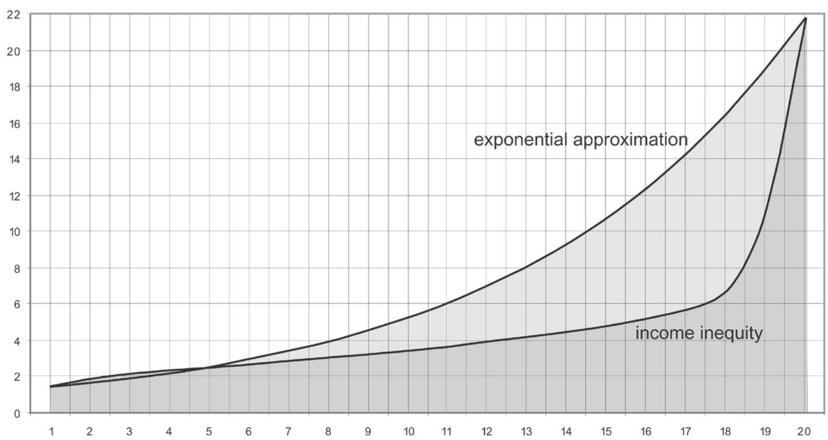

Fig. 2. Income inequity in India.

Table 3. Economy ranking list according to the extropy index

Country	Survey year	Gini index	Extropy index	GDP index (1998)	Human development index (1998)
Australia	1994	35.2	86.1	0.986	0.932
Sweden	1992	25.0	85.5	0.986	0.936
Netherlands	1994	32.6	84.0	0.986	0.941
France	1995	32.7	82.2	0.987	0.946
United Kingdom	1991	36.1	81.8	0.986	0.932
Belgium	1992	25.0	81.6	0.987	0.933
United States	1997	40.8	81.2	0.992	0.943
Germany	1994	30.0	80.0	0.986	0.925
Switzerland	1992	33.1	79.0	0.991	0.930
Finland	1991	25.6	79.0	0.985	0.942
Japan	1993	24.9	77.7	0.987	0.940
Hungary	1996	30.8	76.8	0.957	0.857
Russia	1998	48.7	76.7	0.713	0.769
South Africa	1993–1994	59.3	75.9	0.682	0.717
Mexico	1995	53.7	75.5	0.957	0.855
Slovak Republic	1992	19.5	74.4	0.960	0.875
Belarus	1998	21.7	73.2	0.692	0.783
Vietnam	1998	36.1	72.9	0.183	0.560
Ukraine	1996	32.5	71.4	0.364	0.665
India	1997	37.8	51.5	0.213	0.451

The comparison of the extropy index with the GDP index and the human development index gave the correlation coefficients of 0.77 and 0.86, respectively. This surprisingly good result, considering all the inaccuracies of the broad-scale input data, shows that the extropy index points to some fundamental law of Nature that governs the structures of large complicated objects.

5. CONCLUSIONS

The extropy index proposed as a complexity measure for large-scale structured objects, derived through an axiomatic approach from the basic information-related properties of a partition lattice, reflects the intrinsic structural complexity of objects and gives us a tool for evaluating various complicated structured objects on the basis of the distribution of the basic property of their elements. Though the presented approach is based on a single property distribution, in principle, it can be easily generalized on objects with multiple properties. As one can witness from the results of testing this measure on state economies, the ranking list of various countries ordered according to their extropy index is very promising. It shows clearly that the exponential property curve as the best choice for structured objects reflects some fundamental law of Nature, which means that the best structure for an object is one with all its elements contributing equally to the structure formation. Considering the inaccuracy of the initial data and the fact that not the raw data but the data already modified for other purposes were used, the outcome is even better than could have been expected. It is obvious that further work is needed to elaborate this approach for measuring the complexity of large structured objects whose elements are characterized by multiple properties and test it on evaluating various economic and other complicated situations.

REFERENCES

1. von Neumann, J. and Morgenstern, O. *Theory of Games and Economic Behaviour*. Princeton University Press, 1953.
2. Hartmanis, J. and Stearns, R. E. *Algebraic Structure Theory of Sequential Machines*. Prentice-Hall, Englewood Cliffs, New York, 1966.
3. Birkhoff, G. *Lattice Theory*. American Mathematical Society, New York City, 1948.
4. Lausmaa, T. On the algebraic foundations of the concept of entropy. *Eesti NSV Tead. Akad. Toim. Füüs. Matem.*, 1983, **32**, 128–134 (in Russian).
5. Kämpke, T., Pestel, R. and Radermacher, F. J. A computational concept for normative equity. *European J. Law Economics*, 2003, **15**, 129–163.
6. Kämpke, T. and Stark, M. *The world Income Distribution and its Equity*. Technical Report, FAW-TR-02004, 2002.
7. Radermacher, F. J. *Balance oder Zestörung: Ökosoziale Marktwirtschaft als Schlüssel zu einer weltweiten nachhaltigen Entwicklung*, August 2002, Ökosoziales Forum Europa (ed.), Franz-Josefs-Kai 13, A-1010 Wien, Austria.
8. *World Development Indicators 2000*. Copyright 2000 by the International Bank for Reconstruction and Development/The World Bank, 1818 H Street, NW, Washington DC 20433, USA.

Suurte struktuursete objektide keerukushinnangust

Tõnu Lausmaa

On esitatud struktuursete objektide algebraline infopõhine keerukushinnang antud objekti elementide põhiomaduse jaotuskõvera alusel. Kasutatav meetod baseerub tükeldusel, mis moodustatakse vaadeldava objekti elementide klassifitseerimisel nende põhiomaduse alusel blokkideks. Eeldatakse, et objekti struktuurne keerukus on võrdeline info hulgaga, mida väljendab objekti iseloomustav tükeldus, mille infosisaldus on võrdeline tükelduse ekstroopiaga. Objekti elementide põhiomaduse diskreetne jaotuskõver ekstrapoleeritakse pidevaks funktsiooniks ja selle kõvera alusel arvutatakse antud objekti jaoks ekstroopne keerukusmõõt, mida nimetatakse ekstroopia indeksiks. Ekstroopia indeks väljendab teatud klassi maksimaalse keerukusega objekti ekstroopia ja sellesse klassi kuuluva antud objekti ekstroopia vahe piirväärtust, kui pidevalt suurendada objekti elementide hulgal moodustatud tükelduse blokkide arvu elementide klassijaotuse täpsustamise kaudu. Graafiliselt mõõdab ekstroopia indeks ideaalsele juhule vastava eksponentsiaalse kõvera ja vaadeldava objekti elementide põhiomaduse

jaotuskõvera erinevust. Saadud tulemus annab sobiva vahendi keeruliste struktuursete objektide hindamiseks nende elementide põhiomaduse jaotuskõvera alusel. Kuigi töös esitatud keerukusmõõt on leitud vaid ühe põhiomaduse kaudu iseloomustatud objekti jaoks, on seda kerge üldistada juhule, kui objekti elemendid on iseloomustatud mitme sõltumatu põhiomaduse kaudu. Antud meetodi testimiseks katsetati seda riikide majanduse edukuse hindamisel eeldusel, et majanduse edukus on võrdeline selle ekstroopse keerukushinnanguga, mis on määratud elanikkonna sissetulekute jaotuskõvera alusel. Võrreldes arvutatud ekstroopia indeksit SKP indeksi ja inimarengu indeksiga juhuslikult valitud 20 riigi puhul, saadi võrdlusel korrelatsiooni koefitsientideks vastavalt 0,77 ja 0,86, mis on üllatavalt hea tagajärg, arvestades kõrget täpsusnõuet algandmetele usaldusväärsete tulemuste saavutamiseks ekstroopia indeksi arvutamisel. Saadud kõrge korrelatsioon annab tunnistust sellest, et eksponentsiaalne objekti elementide põhiomaduse jaotuskõver kui parim valik struktuursele objektile peegeldab fundamentaalset loodusseadust, et objekti parim struktuur on selline, kus kõik objekti elemendid annavad võrdse panuse antud objekti struktuuri moodustamisse.