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The importance of the Hurst exponent in describing financial time series

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Abstract. Memory in financial time series is a well-known phenomenon. Long-term self-affine memory leads to fractional Brownian function and is characterized by the Hurst exponent *H*. This study aims to clarify the role of the Hurst exponent in describing financial markets. Hurst exponents were found for time series of Baltic and international stock exchange indices using the longest available period, from the beginning of the Baltic markets in 1996 up to August 2002. The Hurst exponents found varied from market to market. No considerable correlation was observed between the markets with similar Hurst exponents, but they did have similarities in economic situation. Therefore, the scaling analysis reveals hidden similarities between the markets, which open new possibilities in securities analysis.

Key words: fractals, econophysics, scaling, Hurst exponent.

1. INTRODUCTION

Regular Brownian motion, having a century-long history of research (cf. [1,2]), is typically considered as random walk in financial industry. It refers to a stochastic process where the increments in time are identically and independently distributed random variables. Regular Brownian motion has the following property of self-affine scaling:

$$\Delta B = kT^H, \qquad H = 1/2. \tag{1}$$

Equation (1) can be tested empirically using simple techniques. It appears that for time series from numerous fields of nature, science, and also financial markets, the exponent is not 0.5. The first to pay attention to the values of this scaling exponent was British hydrologist H. E. Hurst who measured water flows in the Nile River. He studied a 847-year (see [3] and references therein) period and found that larger-than-average water flows were followed by larger larger-than-average water flows, and *vice versa*, lower-than-average flows were followed by lower lower-than-average flows. It seemed to him a cycle, but it was nonperiodic. He developed his own method and found the scaling exponent for the Nile River, $H = 0.91$.

Fractional Brownian motion is a generalization of regular Brownian motion suggested by B. B. Mandelbrot and others (see [4] and references therein). The basic principle is that increments in time depend on previous increments. If the process is turning back to the previous state, then $H < 0.5$; this property is called antipersistance [5]. If the process tends to move away, then $H > 0.5$; this property is called persistance. If $H = 0.5$, the process has no memory; it is a regular Brownian motion. Financial time series have often been described by fractional Brownian motion (for a good review see [6]). Fractional Brownian motion is characterized by the formula

$$\Delta B = kT^{H}, \quad H \neq 0, \tag{2}$$

where H is the Hurst exponent.

In the case of the financial time series, it is convenient to use the term "trend". If investors can see a strong trend (no matter up or down), more likely $H > 0.5$. If the market is "swinging" up and down, then it is likely that $H < 0.5$.

The Hurst exponent can be measured for different periods. If H remains the same for all periods, it is said to have unifractal or simply fractal structure. If H changes intermittently in time, it is called multifractal. Stock markets are believed to obey a multifractal behaviour [4,7,8]. Multifractality and its applications are theoretically covered by many authors [9–11]. However, a proper multifractal analysis needs very long time series. In most cases, the time series are so short that calculation of multifractal spectra is, in effect, meaningless. This is why we have opted for a simplified approach and calculated integral Hurst exponents for fixed periods of time. Although such an integral exponent is far from being a perfect measure for intermittently fluctuating financial data, it still does provide an additional insight, as compared with the traditional "linear" measures (e.g. volatility).

There are two questions to answer:

1. What are the integral Hurst exponents for different stock markets?
2. To what degree do they fluctuate in time?

For this analysis, data of recent 5–6 years were used for different time series. Such a choice was dictated by the length of the history of the Baltic stock exchanges. Also, H was found for the S&P500 index, using 50-year data for determining the long-term Hurst exponent.

2. MEASURING THE HURST EXPONENT

Here the analysis of stock market time series is based on the following rules:

1. Daily closings were used for calculations.

2. Trading days were used as time increments (i.e. weekends were ignored, Friday and Monday closings were considered sequential).

3. Exactly the first 1300 data points were used for each time series when measuring the short-term Hurst exponent, and 12 500 when measuring the long-term Hurst exponent.

We define $V(t)$ as the value of an index at time t; n is the total number of data points. Further, we calculate the quantity $\mathrm{D}B(T) = \langle|V(t+T) - V(t)|\rangle$, for $T = [1, 2, \ldots, (k-1)(1+1\%), \ldots, n/5]$. This algorithm gives us 169 data points while analysing the short-term scaling, and 382 data points for the long-term analysis.

The Hurst exponent was calculated according to the definition (2), using the least-squares fit between $\ln \mathrm{D}B(T)$ and $\ln T$ (see Fig. 1). This method was applied to the following time series:

(1) Morgan Stanley Capital Index (MSCI) World – MSCI leading stock market indicator covering 23 developed markets;

(2) MSCI Europe – key indicator for European stock markets;

(3) MSCI North America (NA) – MSCI American index covering the US and Canada;

(4) Standard & Poors' 500 (S&P500) – key American index and one of the world's most widely used index; covers 70% of US market capitalization;

(5) MSCI Emerging Markets Free (EMF) – MSCI proxy to developing world; the index covers 26 countries all over the Globe;

(6) MSCI EMF Asia – MSCI Asian index covering nine countries;

(7) MSCI Emerging Markets (EM) Eastern Europe (EE) – covers four Eastern European countries, including Russia;

(8) Talse – Tallinn Stock Exchange Index;

(9) Rici – Riga Stock Exchange Index;

(10) Litin – Vilnius Stock Exchange Index.

All the indices are in US dollars except for Talse (in Estonian Kroons), Rici (Latvian Lats), and Litin (Lithuanian Lits).

Financial time series are single realizations of intermittently fluctuacting nonstationary time-series, which makes calculation of exact uncertainties of the scaling exponents impossible. However, rough estimates of the uncertainties have been obtained as follows. The least-squares fitted trendline was found as described above, except that the slope h was not optimized, i.e. it was considered as a fixed parameter. Further, the sum of squared residuals $\mathrm{r}(h)$ was calculated as a function of h. The error estimate was found as $\mathrm{e} = (H' - H)$, where H is the least-squares fitted value of the slope, and H' satisfies the condition $\mathrm{r}(H') = 2\mathrm{r}(H)$. The results of calculations are shown in Table 1.

As can be seen from the table, the long-term behaviour of the S&P500 index corresponds to $H = 0.64$, which is in good agreement with the results of previous studies (cf. [3]). However, for short time periods, all the developed market indices showed almost no persistance (or even weak antipersistance) with $H \sim 0.5$. This fact is not very surprising because $H \sim 0.55$ was previously measured for the

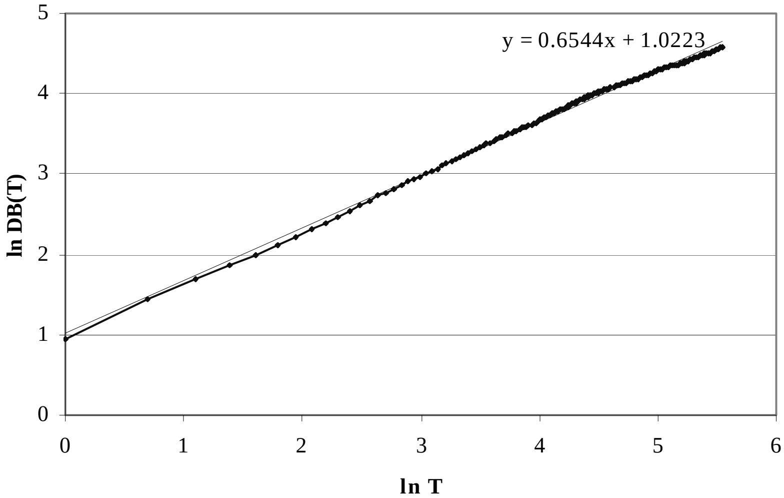

Fig. 1. Calculation of the Hurst exponent: the case of Talse. *H* is found as the slope of the fitted trendline.

30-year period of 1968–98 for the Deutsche Aktienindex (DAX) [12] and the São Paulo Stock Exchange (Bovespa) [13] Index.

Developing markets generally (MSCI EMF), MSCI EM EE, Litin, and specifically MSCI EMF Asia, had the lower-than-average value of $H \sim 0.6$, which is still significantly higher than the results for the developed world indices. The highest persistance was observed for Tallinn and Riga exchanges, the *H*-values of which (0.65–0.68) were even higher than the long-term Hurst exponent of the S&P500 index. Other authors have found for similar periods $H = 0.63$ for Taiwan [14] and $H = 0.91$ for Bombay Stock Exchange Index [15]. The developments of all these markets are presented in Fig. 2.

Table 1. Hurst exponents (*H*) for ten markets. For abbreviations see p. 200

Name of time series	Period measured	*H*	Error
MSCI World	29 Aug 97–02 Sep 02	0.535	0.014
MSCI Europe	29 Aug 97–02 Sep 02	0.484	0.003
MSCI NA	29 Aug 97–03 Sep 02	0.501	0.011
S&P500	03 Jun 96–30 Aug 02	0.542	0.018
MSCI EMF	29 Aug 97–02 Sep 02	0.601	0.007
MSCI EM EE	29 Aug 97–03 Sep 02	0.568	0.008
MSCI EMF Asia	29 Aug 97–03 Sep 02	0.607	0.005
Talse	03 Jun 96–03 Sep 02	0.654	0.010
Rici	02 Apr 96–03 Sep 02	0.687	0.013
Litin	07 Apr 97–02 Sep 02	0.577	0.020
S&P500	03 Sep 52–03 Sep 02	0.643	0.015

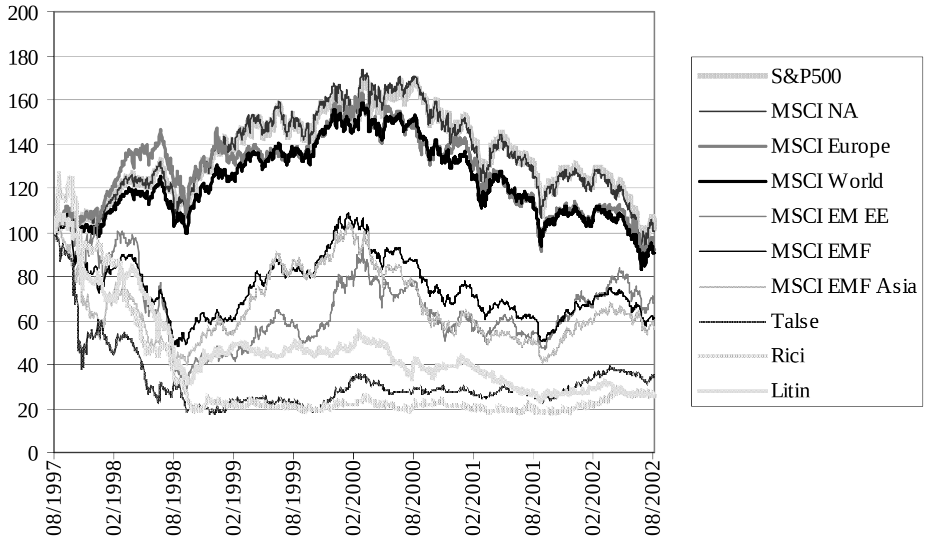

Fig. 2. All ten covered market indices are indexed to 100 as of 29 August 1997. For abbreviations see p. 200.

As can be seen, all the four developed market indices were quite volatile, resembling a regular Brownian motion. Emerging market indices are, despite a serious drop in the beginning, much more persistant in their movements. Their fractional Brownian nature becomes recognizable in a rescaled graph (Fig. 3).

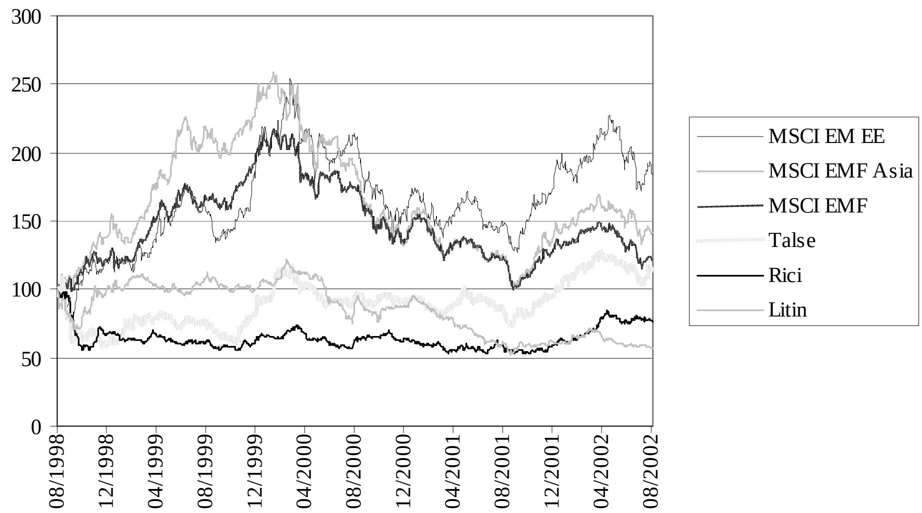

Fig. 3. Emerging market indices are indexed to 100 as of 28 August 1998. For abbreviations see p. 200.

3. HURST EXPONENTS AND STANDARD STATISTICS

The results of the previous section lead to the following questions:

(1) Can we be sure that the similarity of certain Hurst exponent values is not merely related to a high correlation between the indices? Indeed, it could be quite natural to presume that markets that have the same tendency to remember, or the same level of persistance, are moving closely. To answer that question, correlations were calculated for all of the indices mentioned above.

(2) Is it possible that the similarity of certain Hurst exponent values is related to similar volatility? Financial markets have periods of high and low volatility, so standard deviation of price indices is not constant. We already know (see Table 1) that for the recent period of the developed market indices, the value of the exponent H is significantly lower than in the case of long-term S&P500 time series. In order to determine the possible relationship between H and standard deviation, standard deviations of daily returns (in financial texts usually referred to as volatility) were calculated.

As seen from Table 2, the correlations between Tallinn and Riga Stock Exchanges are weak, although the Hurst exponent values are similar. The situation is the same with Litin and MSCI EM EE indices. These results indicate that classical mean–variance security analysis (that is done mostly by using correlations and standard deviations) has to be re-evaluated.

By comparing the standard deviations (volatility) and Hurst exponents we can see that in the developed markets case higher H corresponds to lower volatility. It means that when market is very volatile, then it tends to behave more randomly than in the case of a more stable market.

Table 2. Correlations and standard deviations. All figures are percentages. For abbreviations see p. 200.

	MSCI World	MSCI Europe	S&P500	MSCI NA	MSCI EMF	MSCI EMF Asia	MSCI EM EE	TALSE	Rici	Litin	Standard deviation
MSCI World	100										1.00
MSCI Europe	75	100									1.23
S&P500	88	42	100								1.32*
MSCI NA	89	42	100	100							1.30
MSCI EMF	60	53	41	41	100						1.24
MSCI EMF Asia	27	26	9	9	77	100					1.61
MSCI EM EE	43	50	23	24	62	39	100				2.23
TALSE	11	13	3	4	18	21	20	100			2.23
Rici	−1	−2	−1	0	4	6	4	15	100		1.56
Litin	0	4	−5	−5	9	11	9	14	16	100	1.48

* The value was calculated for the short-term case. Standard deviation for the 50-year time series was 0.90%.

In the case of emerging markets, the situation is opposite: higher volatility corresponds to a higher value of *H*. In other words, higher volatility leads to a stronger persistant memory. Such a behaviour can be explained by the market participants' tendency to overreact to sharp moves.

Why do emerging markets behave unlike developed markets? This phenomenon deserves further studies; a possible explanation is the following. In stock markets, short-term moves are described by psychology of crowd behaviour [16]. In developed markets, investors have seen drops and rises of indices and they behave more rationally: when the index is down, they are entering the market to buy; when the index is up, they are selling. This balances the market, but creates a short-term randomness in prices, which leads to a lower *H*.

In emerging markets, local investors are not experienced with financial markets and they behave more unpredictably. They can buy when prices are rising and create bubbles, or they can sell when prices are dropping, and create crashes. Additional volatility can be ascribed to typical weaknesses of such markets: lack of investors, dependence on foreign interest, etc. From the economic point of view, the high *H* of such markets is achieved by the interplay of two scenarios: on one hand, there is much headroom for economic growth, on the other hand, there is increased risk of major failures (e.g. due to political instability).

4. CONCLUSIONS

Hurst exponents were found for ten financial time series by using the period of 1996–2002 and for the 50-year long time series of the S&P500 index. The results confirmed the presence of long-term memory in this kind of time series. Different periods chosen confirm that *H* is not constant but a changing variable (more detailed multifractal analysis has been rejected due to too short time-series). Surprisingly, in the case of developed markets, short-term *H* results showed almost no persistance in memory.

The Hurst exponent was also found to vary from market to market. Although the markets with a similar Hurst exponent showed no considerable correlation, they did have similarities in the economic situation. This opens new possibilities in securities analysis: similar trends can be found from other characteristics than employed by the classical mean–variance theory.

By comparing Hurst exponents and standard deviations it appeared that for developed markets a higher volatility corresponded to a lower value of *H*. For developing markets the relationship turned to be *vice versa*: a higher volatility meant also a higher memory. This phenomenon can be explained in simple terms. However, to construct a quantitatively motivated model, further analysis is needed.

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Hursti astmenäitaja olulisus finantsaegridade kirjeldamisel

Robert Kitt

Mälu on finantsturgudel teada-tuntud nähtus. Pikaajaline eneseafiinne mälu viitab murrulisele Browni liikumisele, mida kirjeldab Hursti astmenäitaja *H*. Käesoleva artikli eesmärk oli näidata Hursti astmenäitaja olulisust finantsturgude

kirjeldamisel. Eesmärgi saavutamiseks tehti empiiriliselt kindlaks Balti ja teiste finantsturgude Hursti astmenäitaja, kasutades 1996.–2002. aasta aegridu, mis olid ühtlasi pikimad võimalikud aegread. Tulemused näitasid tugeva mälu olemasolu ($H \sim 0{,}66$–$0{,}69$) Tallinna ja Riia turgudel. See oli kooskõlas rahvusvaheliste arenenud turgude pikaajalisema (1952–2002) käitumisega, kuid erines oluliselt lühema perioodi Hursti astmenäitajatest ($H \sim 0{,}5$). Oleks loogiline arvata, et sarnase Hursti astmenäitajaga turud liiguvad tugevas korrelatsioonis. Paraku näitas analüüs, et korrelatsioonikoefitsiendid sarnaste Hursti astmenäitajatega turgude vahel olid tihti nõrgad. Seega annab skaleerimine uusi võimalusi finantsturgude paremaks kirjeldamiseks. Lisaks korrelatsiooni tugevusele mõõdeti ka Hursti astmenäitaja sõltuvust volatiilsusest. Leiti, et arenenud turgudel eksisteerib negatiivne seos: suuremale volatiilsusele vastab juhuslikum käitumine (ehk madalam H). Küll aga ei kehti see tähelepanek arenevate turgude kohta. Seda on võimalik küllalt lihtsalt seletada, kuid kvantitatiivse mudeli jaoks on vaja lisaanalüüsi.