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Some new stability margins for discrete-time systems

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Abstract. Some new stability margins for discrete-time systems are proposed in the system characteristic polynomial coefficient space by making use of the so-called reflection vectors of monic Schur polynomials. Reflection vector margins give the distance (not minimal) to the stability boundary in directions of $2n$ reflection vectors of an nth-degree polynomial. The relations between the reflection vectors and the roots of a polynomial on the unit circle are given.

Key words: stability, polynomials, discrete-time systems.

1. INTRODUCTION

The stability of linear dynamic systems is a well-studied topic of linear differential equations. However, some serious problems of so-called robust stability arise when the parameters of systems are not exactly known [1,2]. That is why several stability margins are defined in different domains: the gain and phase margin in the frequency domain, minimal distance from the imaginary axis in the pole domain, the stability radius in the system parameter domain.

For robust pole placement the domain of characteristic polynomial coefficients is of interest [1]. Here, a stability margin can be obtained by the Kharitonov theorem [3] or edge theorem [4]. Unfortunately, the first one does not hold for discrete-time systems.

An alternative approach is to use the boundary crossing theorem to define the stability radius in the polynomial coefficient space [2]. One needs to determine the distances to the real pole boundary and to the complex poles boundary, and select the minimal of them. The first task is simple but the second one is quite complicated for high-order systems, because the sweeping over the complex poles phase domain is needed.

In this paper the reflection coefficient stability criterion [5] is used to define a Schur stability margin in the polynomial coefficient space. The reflection vectors of an nth-order system will be introduced as $2n$ specific points on the stability boundary. The line segments between an arbitrary Schur polynomial (a point in coefficient space) and its reflection vectors will be Schur stable. So the minimal distance between a polynomial and its reflection vectors can be used as a stability margin for linear discrete-time systems. The crucial reflection vector gives us the critical direction also in the polynomial coefficient space.

The paper is organized as follows. In Section 2 we recall the stability condition via reflection coefficients and introduce reflection vectors of a monic Schur polynomial. Section 3 is devoted to the boundary surfaces of the stability region. It turns out that most of the faces of the stability hypercube of reflection coefficients will be transformed into the $(n-1)$-dimensional hyperplanes of polynomial coefficients and only some of the crucial faces will be transformed into the n-dimensional surfaces. In Section 4 the placement of reflection vectors is studied and the reflection vector stability margin introduced.

2. REFLECTION COEFFICIENTS OF SCHUR POLYNOMIALS

A polynomial $a(z)$ of degree n with real coefficients $a_i \in \mathcal{R}$, $i = 0, ..., n$,

$$a(z) = a_n z^n + ... + a_1 z + a_0,$$

is said to be a Schur polynomial if all its roots are placed inside the unit circle. A linear discrete-time dynamical system is stable if its characteristic polynomial is a Schur polynomial, i.e. if all its poles lie inside the unit circle.

Besides the unit circle criterion some other criteria are known for checking the stability of a linear system. It is interesting to mention that the well-known Jury's stability test leads precisely to the stability hypercube of reflection coefficients. In the following we use the stability criterion via reflection coefficients.

Let us recall the recursive definition of reflection coefficients $k_i \in \mathcal{R}$ of a polynomial $a(z)$ [5]:

$$k_i = -a_i^{(i)}, \tag{1}$$

$$a_i^{(n)} = \frac{a_{n-i}}{a_n}, \qquad i = 1, ..., n, \tag{2}$$

$$a_j^{(i-1)} = \frac{a_j^{(i)} + k_i a_{i-j}^{(i)}}{1 - k_i^2}, \qquad j = 1, ..., i-1. \tag{3}$$

Reflection coefficients are well known in signal processing and digital filters. They are called also PARCOR coefficients and k-coefficients [6]. The stability criterion via reflection coefficients is as follows [5].

Lemma 1. *A polynomial $a(z)$ will be a Schur polynomial if and only if its reflection coefficients $k_i,\ i = 1, ..., n$, lie within the interval $-1 < k_i < 1$.*

A polynomial $a(z)$ lies on the stability boundary if some $k_i = \pm 1, i = 1, ..., n$. For monic Schur polynomials, $a_n = 1$, there is one-to-one correspondence between the vectors $\tilde{a} = (a_0, ..., a_{n-1})^T$ and $k = (k_1, ..., k_n)^T$.

The transformation from reflection coefficients k_i to polynomial coefficients $a_{i-1},\ i = 1, ..., n$, is multilinear. For monic polynomials we obtain from (1)–(3)

$$\begin{aligned} a_i &= a_{n-i}^{(n)}, \\ a_i^{(i)} &= -k_i, \\ a_j^{(i)} &= a_j^{(i-1)} - k_i a_{i-j}^{(i-1)}, \quad i = 1, ..., n,\ \ j = 1, ..., i-1, \end{aligned}$$

or, in the matrix form,

$$a = R(k)a^{(r)}, \qquad r = 1, ..., n-1, \tag{4}$$

$$a^{(r)} = \begin{bmatrix} 0^T \\ R_r(k_r) \end{bmatrix} a^{(r-1)},$$

where

$$a = [a_0, ..., a_{n-1}, 1]^T,$$

$$a^{(r)} = [0, a_r^{(r)}, ..., a_1^{(r)}, 1]^T,$$

$$a^{(0)} = [0, 1]^T,$$

$$R(k) = R_n(k_n) \begin{bmatrix} 0^T \\ R_{n-1}(k_{n-1}) \end{bmatrix} \cdots \begin{bmatrix} 0^T \\ R_r(k_r) \end{bmatrix},$$

$$R_j(k_j) = I_{j+1} - k_j E_{j+1},$$

I_n is an $n \times n$ unit matrix and E_n is a mirror image of I_n, i.e.,

$$E_n = \begin{bmatrix} 0 & . & . & 1 \\ . & . & . & . \\ 1 & . & . & 0 \end{bmatrix}.$$

Lemma 2. [[7]] *Through an arbitrary stable point $a = [a_0, a_1, ..., a_{n-1}]$, with reflection coefficients $k_i^a \in (-1, 1)$, $i = 1, ..., n$, you can put n stable line segments*

$$a^i(\pm 1) = \operatorname{conv}\{a | k_i^a = \pm 1\},$$

where $\operatorname{conv}\{a | k_i^a = \pm 1\}$ denotes the convex hull obtained by varying the reflection coefficient k_i^a between -1 and 1.

Now let us introduce the reflection vectors of a monic polynomial $a(z)$. They will be useful for introducing some new stability margins in the polynomial coefficient space.

Definition 1. *Let us call the vectors*

$$a^i(1) = (a|k_i = 1), \;\; i = 1, ..., n,$$

positive reflection vectors *and*

$$a^i(-1) = (a|k_i = -1), \;\; i = 1, ..., n,$$

negative reflection vectors *of a monic polynomial* $a(z)$.

It means that reflection vectors are the extreme points of the Schur stable line segment $a^i(\pm 1)$ through the point a defined by Lemma 2. Due to the definition and Lemmas 1 and 2 the following assertions hold:

(1) every Schur polynomial has $2n$ reflection vectors $a^i(1)$ and $a^i(-1)$, $i = 1, ..., n$;

(2) all the reflection vectors lie on the stability boundary ($k_i = \pm 1$);

(3) the line segments between reflection vectors $a^i(1)$ and $a^i(-1)$ are Schur stable.

3. STABILITY REGION AND THE UNIT HYPERCUBE OF REFLECTION COEFFICIENTS

The stability region in the reflection coefficient space is simply the n-dimensional unit hypercube $\mathcal{K} = \{k_i \in (-1, 1), i = 1, ..., n\}$. Because the mapping (4) is continuous, we can find the boundary surfaces of the stability region in the polynomial coefficient space starting from the faces $k_i = \pm 1, \;\; i \in \{1, ..., n\}$; $k_j \in (-1, 1)$, $j = 1, ..., n$; $j \neq i$. The mapping (4) is one-to-one for monic Schur polynomials. However, for the stability boundary this transformation is not one-to-one because the matrix $R(k_j)$ will be singular if $k_j = \pm 1$.

Theorem 1. *The stability boundary in the reflection coefficient space, which is composed of* $2n$ *faces* $k_i = \pm 1, \; i = 1, ..., n$*, of the hypercube* $\mathcal{K}$*, is transformed to the following boundary surfaces in the polynomial coefficient space*:

(1) $(n + 1)/2$*-dimensional hyperplane*

$$\begin{cases} a_0 = -1 \\ a_j = -a_{n-j}, \qquad\quad j = 1, ..., (n - 1)/2, \end{cases} \tag{5}$$

for $k_n = 1$ *and* n *odd*;

(2) $n/2$*-dimensional hyperplane*

$$\begin{cases} a_0 = -1 \\ a_{n/2} = 0 \\ a_j = -a_{n-j}, \qquad j = 1, ..., n/2-1, \end{cases} \tag{6}$$

for $k_n = 1$ *and* n *even;*

(3) $(n+1)/2$ (*or* $n/2+1$)*-dimensional hyperplane*

$$\begin{cases} a_0 = 1 \\ a_j = a_{n-j}, \end{cases} \tag{7}$$

where $j = 1, ..., (n-1)/2$ *when* n *is odd or* $j = 1, ..., n/2-1$ *when* n *is even for* $k_n = -1$;

(4) $(n-2)$*-dimensional hyperplane*

$$\begin{cases} a_0 + a_1 + ... + a_{n-1} + 1 = 0 \\ a_0 + ... + a_{n-i/2-1} + a_{n-i/2+1} + ... a_{n-1} + 1 = 0 \end{cases} \tag{8}$$

for $k_i = 1$ *and* i *even;*

(5) $(n-1)$*-dimensional hyperplane*

$$a_0 + a_1 + ... + a_{n-1} + 1 = 0 \tag{9}$$

for $k_i = 1$ *and* i *odd;*

(6) $(n-1)$*-dimensional hyperplane*

$$a_0 - a_1 + ... + (-1)^{n-1} a_{n-1} + 1 = 0 \tag{10}$$

for $k_i = -1$ *and* i *odd;*

(7) n*-dimensional hypersurface*

$$a = R_n(k_n) ... \begin{bmatrix} 0^T \\ R_i(-1) \end{bmatrix} ... \begin{bmatrix} 0^T \\ R_1(k_1) \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \tag{11}$$

for $k_i = -1$, i *even, and* $i < n$.

The proof of the theorem is given in [8].

Corollary 1.1. *All the faces* $k_i = 1$, $i = 1, ..., n$, *of the stability hypercube* $\mathcal{K}$ *are transformed to the positive real root boundary hyperplane* (9).

Corollary 1.2. *All the faces* $k_i = -1$, i *odd and* $k_n = -1$, *are transformed to the negative real root boundary hyperplane* (10).

4. STABILITY MARGIN VIA REFLECTION VECTORS

In this section we study the placement of reflection vectors on the stability boundary.

Lemma 3. *The reflection vectors $a^n(\pm 1)$ of an nth-order monic Schur polynomial $a(z)$ have n roots on the unit circle.*

Proof. First, let us mention that according to Theorem 1 the reflection vectors $a^n(\pm 1)$ have symmetric coefficient values. For $a^n(1)$ we have from (5) and (6)

$$a_j = -a_{n-j}, \qquad j = 1, ..., n/2(\text{or}(n-1)/2),$$

and for $a^n(-1)$ from (7)

$$a_j = a_{n-j}, \qquad j = 1, ..., n/2(\text{or}(n-1)/2).$$

Second, for monic polynomials with symmetric coefficient values it is well known that the number of roots inside the unit circle equals the number of roots outside the unit circle. But all the reflection vectors of Schur polynomials are by definition placed on the stability boundary. Hence $a^n(\pm 1)$ has no root outside the unit circle, and so all the n roots of it are placed on the unit circle. □

Lemma 4. *Let us consider monic Schur polynomials $a(z)$ and $b(z)$ with reflection coefficients $k_j^a = k_j^b$, $j = 1, ..., i-1$. Then the reflection vectors $a^i(\pm 1)$ and $b^i(\pm 1)$ have the same i roots on the unit circle.*

Proof. Let us start from an auxiliary ith-order polynomial $\bar{a}^i(z)$ with reflection coefficients $k_j^{\bar{a}} = k_j^a = k_j^b$, $j = 1, ..., i$. According to Lemma 3, the polynomial $\bar{a}^i(z)$ has i roots on the unit circle and its coefficients have symmetric values. For $k_j^a = k_j^b = -1$ we have

$$\bar{a}^i(z) = z^i + a_1 z^{i-1} + a_2 z^{i-2} + ... + a_2 z^2 + a_1 z + 1$$

and for $k_j^a = k_j^b = 1$

$$\bar{a}^i(z) = z^i + a_1 z^{i-1} + a_2 z^{i-2} + ... - a_2 z^2 - a_1 z - 1.$$

Our aim is to show that the polynomial $\bar{a}^i(z)$ is a common divisor of both $a(z)$ and $b(z)$. This can be easily done by increasing the order of polynomials $a(z)$ and $b(z)$ by transformation (4) and taking into account the symmetric coefficient values of $\bar{a}^i(z)$.

For example, let $k_i^a = -1$, i even, and $n(a) = i + 1$. Then

$$a = \begin{bmatrix} 1 & 0 & \dots & 0 & -k_{i+1} \\ . & . & . & . & . \\ 0 & \dots & 1-k_{i+1} & \dots & 0 \\ . & . & . & . & . \\ -k_{i+1} & 0 & \dots & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1-a_1k_i \\ a_1-a_2k_i \\ \vdots \\ a_2-a_1k_i \\ a_1-k_i \\ 1 \end{bmatrix}$$

and

$$a(z) = (z-k_i)\bar{a}^i(z).$$

For $n(a) = i+2$ we obtain

$$a(z) = [z^2 + k_{i+1}(k_{i+2}-1)z - k_{i+2}]\bar{a}^i(z).$$

In a similar way we can find for arbitrary $n(a)$ and $|k_j| < 1$, $j = i+1, ..., n(a)$,

$$a(z) = a^{n-i}(z)\bar{a}^i(z),$$

i.e., $\bar{a}^i(z)$ is a common divisor for all $a(z)$ (and also for $b(z)$) with $n(a) > i$. □

Lemma 5. *Let a monic polynomial $a(z)$ of order n have a real root of multiplicity i on the Schur stability boundary $r_1 = r_2 = ... = r_i$, $|r_i| = 1$, and all the other roots be placed inside the unit circle $|r_j| < 1$, $j = i+1, ..., n$. Then the first i reflection coefficients of $a(z)$ are placed on the stability boundary $|k_j| = 1$, $j = 1, ..., i$, and all the other reflection coefficients inside the unit hypercube $|k_j| < 1$, $j = i+1, ..., n$.*

Proof. By Lemma 4 the ith reflection coefficient of $a(z)$ must be of modulus equal to one, $|k_i| = 1$, and $|k_j| < 1$, $j = i+1, ..., n$, because $a(z)$ has only i roots on the stability boundary. Now let us rewrite $a(z)$ as

$$a(z) = a^{n-i}(z)\bar{a}^i(z) = a^{n-i}(z)(z \pm 1)^i.$$

Because $\bar{a}^i(z)$ has symmetric coefficient values, we can claim that the first i reflection coefficients of $a(z)$ are determined by $\bar{a}^i(z)$, i.e. $k_j^{\bar{a}} = k_j^a$, $j = 1, ..., i$. By formulas (1)–(3) we can easily find that $|k_j^{\bar{a}}| = 1$, $j = 1, ..., i$. □

Theorem 2. *Reflection vectors $a^i(\pm 1)$, $i = 1, ..., n$, of a monic Schur polynomial $a(z)$ have the following i roots $r(j)$, $j = 1, ..., i$, on the stability boundary*:

(1) *the reflection vector $a^i(1)$ has*
- *for i even $r(1) = 1$, $r(2) = -1$, and $(i-2)/2$ pairs of complex roots on the unit circle,*
- *for i odd $r(1) = 1$ and $(i-1)/2$ pairs of complex roots on the unit circle,*

(2) *the reflection vector* $a^i(-1)$ *has*

– *for* i *even* $i/2$ *pairs of complex roots on the unit circle,*

– *for* i *odd* $r(1) = -1$ *and* $(i-1)/2$ *pairs of complex roots on the unit circle*.

Proof. The assertion that a reflection vector $a^i(\pm 1)$ has i roots on the unit circle follows immediately from Lemmas 3 and 4.

Now the question is: which of the reflection vectors has a real root on the stability boundary and of what sign is it? By Theorem 1 the hyperplanes (9) and (10) are the real root boundaries for $r = 1$ and $r = -1$, respectively, and (8) for both $r = \pm 1$. Because all the reflection vectors of a Schur polynomial have by definition only one reflection coefficient on the stability boundary, they do not have by Lemma 5 any real multiple roots of modulus equal to one. Hence, all the rest of the stability boundary roots must be complex roots on the unit circle. □

Now we can introduce some stability margins via reflection vectors of a Schur polynomial.

Definition 2. *Let us call the distance between a Schur stable polynomial* $a(z)$ *and its reflection vector* $a^i(\pm 1),\ i = 1, ..., n,$ *the stability margin in coefficient space in the direction of the* i*th reflection vector, or simply the i*th reflection vector margin, *and denote it by* $d_i(\pm 1)$.

The reflection vector margins are useful for robust controller design by pole (or characteristic polynomial coefficient) placement [7]. Then we are looking for the minimal reflection vector margin

$$d_{\min} = \min_{i=1}^{n} d_i(\pm 1)$$

in order to maximize it by a proper robust controller.

Taking the background of reflection vectors into account (according to Theorem 2), we can claim that in practice the first three of the reflection vectors are the most attractive. Indeed,

– the first positive reflection vector margin $d_1(1)$ gives us the distance to the real positive root boundary,

– the first negative reflection vector margin $d_1(-1)$ gives us the distance to the real negative root boundary,

– the second negative reflection vector margin $d_2(-1)$ gives us the distance to the complex root boundary,

– the second positive reflection vector margin $d_2(1)$ gives us the distance to two different real root boundaries ($r_1 = 1$, $r_2 = -1$),

– the third positive reflection vector margin $d_3(1)$ gives us the distance to the real positive and complex root boundaries ($r_1 = 1$, $r_{2,3} = \alpha \pm 1\beta\mathbf{i},\ \alpha^2 + \beta^2 = 1$),

– the third negative reflection vector margin $d_3(-1)$ gives us the distance to the real negative and complex root boundaries ($r_1 = -1$, $r_{2,3} = \alpha + \beta\mathbf{i},\ \alpha^2 \pm 1\beta^2 = 1$),

– higher reflection vector margins give us the distance to several complex root boundaries.

As a matter of fact, the reflection vector margins do not give the minimal distances to real and complex root boundaries, i.e.,

$$d_{\min} \geq \rho,$$

$$d^1(1) \geq \rho_{+1},$$
$$d^1(-1) \geq \rho_{-1},$$

where ρ, ρ_{+1}, and ρ_{-1} are the stability radius and the minimal distances to the positive and negative real root boundaries of $a(z)$. However, the minimal distances to real and complex root boundaries can be easily found by a simple search procedure in the directions of reflection vectors. For example, the minimal distance to the positive real root boundary can be found via the first positive reflection vector as follows:

1. For a given Schur polynomial $a(z)$ find the starting-point $b(0) = a^1(1)$.

2. Put $n-1$ line segments $\operatorname{conv}\{[b(0)]^i(\pm 1)\}$, $i = 2, ..., n$, through the point $b(0)$.

3. Find $b(1)$ as the nearest point of all line segments $\operatorname{conv}\{[b(0)]^i(\pm 1)\}$, $i = 2, ..., n$, to the point a.

4. If $|b(i) - b(i-1)| > \epsilon$ for some given small $\epsilon > 0$, return to step 2.

Similarly, we can find the negative real root margin via the first negative reflection vector and the complex root margin via the second negative reflection vector.

Example 1. Let $n = 2$. Then the stability region in the polynomial coefficient space $a = (a_1, a_0)$ is the triangle AGH (Fig. 1). Let us find the stability margins for the polynomial $a(z) = z^2 + 0.75z + 0.5$ (point F in Fig. 1). According to Lemma 2, we can put two stable line segments through the point F. By varying the first reflection coefficient k_1, $-1 < k_1 < 1$, we get the line segment AB and by varying the second reflection coefficient k_2, $-1 < k_2 < 1$, we get the line segment CD. By definition the second-order polynomial $a(z)$ has four reflection vectors:

$$\begin{array}{lclr} a^1(1) & = & [\; -1.5 \quad 0.5 \;], & \text{(point C)}, \\ a^1(-1) & = & [\; 1.5 \quad 0.5 \;], & \text{(point D)}, \\ a^2(1) & = & [\; 0 \quad -1 \;], & \text{(point A)}, \\ a^2(-1) & = & [\; 1 \quad 1 \;], & \text{(point B)}, \end{array}$$

and the stability margins in the directions of reflection vectors are determined by the line segments FC, FD, FA, and FB, respectively.

To find the minimal distance to the negative real root boundary, we start from the first negative reflection vector (point D), $b(0) = [1.5 \quad 0.5]$. By varying the second reflection coefficient k_2, $-1 < k_2 < 1$, we get the line segment AH. The point K = [1.125 0.125], with reflection coefficients $k^{\mathrm{K}} = [-1 \quad -0.125]$, is the nearest point on the negative real root boundary, and the distance to the negative real root boundary is $\rho_- = 0.53$.

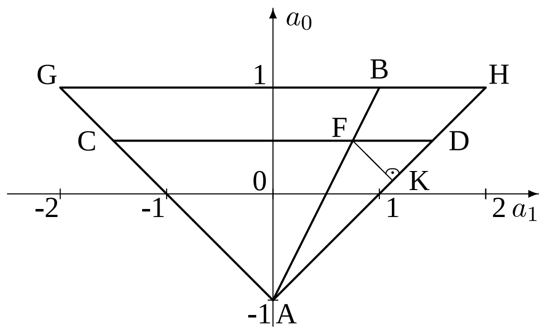

Fig. 1. The stability region and stability margins in the directions of reflection vectors ($n = 2$).

Similarly, starting from the first positive reflection vector (point C), we can find the minimal distance to the positive real root boundary (line segment AG) $\rho_+ = 1.591$.

Starting from the second negative reflection vector (point B), we get the minimal distance to the complex root boundary (line segment GH) $\rho_c = 0.5$. So the stability radius is

$$\rho = \min(\rho_+, \rho_-, \rho_c) = 0.5.$$

Example 2. Let us now consider the example of Bhattacharyya [2, pp. 136–138] for $n = 4$:

$$a(z) = z^4 + 0.3z^3 + 0.4z^2 + 0.2z + 0.1.$$

The reflection coefficients of $a(z)$ are $k^a = [-0.1714 - 0.3246 - 0.1717 - 0.1]$. Because $|k_i^a| < 1$, $i = 1, ..., 4$, $a(z)$ is a Schur polynomial and we can find its reflection vectors $a^i(\pm 1)$ and reflection vector margins $d_i(\pm 1)$ as follows:

$$\begin{array}{lclccccclll}
a^1(1) & = & [& -1.2516 & 0.1069 & 0.0448 & 0.1 &], & d_1(1) & = 1.5866, \\
a^1(-1) & = & [& 1.3974 & 0.6073 & 0.3097 & 0.1 &], & d_1(-1) & = 1.1222, \\
a^2(1) & = & [& -1.1545 & -1.0999 & 0.1545 & 0.1 &], & d_2(1) & = 1.5679, \\
a^2(-1) & = & [& 0.5317 & 1.1646 & 0.2232 & 0.1 &], & d_2(-1) & = 0.7993, \\
a^3(1) & = & [& -0.1975 & 0.1073 & -1.0097 & 0.1 &], & d_3(1) & = 1.3403, \\
a^3(-1) & = & [& 0.6517 & 0.6069 & 1.0551 & 0.1 &], & d_3(-1) & = 0.9474, \\
a^4(1) & = & [& 0.1111 & 0 & -0.1111 & -1 &], & d_4(1) & = 1.2256, \\
a^4(-1) & = & [& 0.4545 & 0.7272 & 0.4545 & 1 &], & d_4(-1) & = 1.0028.
\end{array}$$

Starting from the reflection vectors $a^1(1)$, $a^1(-1)$, and $a^2(-1)$, the following minimal distances to real positive, real negative, and complex pole boundaries have been found, respectively, $\rho_{+1} = 1.0$, $\rho_{-1} = 0.5$, $\rho_c = 0.4987$. This confirms the result given in [2]. The stability radius is

$$\rho = 0.4987$$

and the critical point b on the stability boundary is

$$b \;=\; [\;\; 0.2335 \quad 0.746 \quad 0.2818 \quad -0.2434 \;\;].$$

The reflection coefficients of $b(z)$ are

$$k^b \;=\; [\;\; 0.0194 \quad -1.0 \quad -0.36 \quad 0.2434 \;\;].$$

By Theorem 3, $b(z)$ has a pair of complex roots on the unit circle. Indeed, the roots of $b(z)$ are

$$r_1 = 0.3756,$$
$$r_2 = -0.648,$$
$$r_{3,4} = 0.0194 \pm 0.9998\mathbf{i}.$$

5. CONCLUSIONS

A new kind of stability margin for discrete-time systems is proposed in the system characteristic polynomial coefficient space by making use of the so-called reflection vectors of monic Schur polynomials. It is shown that (1) reflection vectors are placed on the stability boundary, with specific roots placement depending on the reflection vector number and the argument sign, and (2) the line segments between an arbitrary Schur polynomial and its reflection vectors are Schur stable.

Even though the reflection vector margins do not give the minimal distance to the stability boundary, they are nevertheless quite informative: in addition to distances, they give also the directions of crucial points. The reflection vector margins can be used for robust controller design in the system characteristic polynomial coefficient domain.

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Diskreetaja süsteemide stabiilsusvarust

Ülo Nurges

Lähtudes süsteemi karakteristliku polünoomi peegelduskoefitsientidest, on defineeritud süsteemi stabiilsusvaru peegeldusvektorite suunas. On selgitatud seosed süsteemi peegeldusvektorite ja stabiilsuspiiril asuvate pooluste vahel: esimesele positiivsele (negatiivsele) peegeldusvektorile vastab positiivne (negatiivne) reaalne poolus ühikringil, teisele negatiivsele peegeldusvektorile vastab komplekssete pooluste paar ühikringil jne. Sobiva peegeldusvektori abil on suhteliselt lihtne leida süsteemi stabiilsusraadius.