Gravity and magnetic modelling of anomalous sources of complex configuration

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Abstract. The automated fitting method was applied to the modelling of gravity and magnetic anomalies. This technique allows us to use for interpretation the observed components of gravity and magnetic fields and their linear transformations, the geoid anomalies, and the course gradient data of magnetic fields. The software is based on the optimization of the sum of square differences between measured and modelled field components. The universal approximation procedure for parametrization of anomalous sources allows us to describe effectively different elements of the modelled space. The optimization procedure for the minimization of objective functions includes gradient-based algorithms and singular value decomposition technique. The technology is tested on practical geological problems.

Key words: gravity, magnetic, modelling, optimization, automated fitting.

INTRODUCTION

Various universal techniques of the modelling of geological objects based both on trial and error methods in general, and on their automated computer modification in particular, are currently widely used in practice. At the same time it is necessary to admit that the informative value of the results received during each modelling procedure are determined not only by the used computer fitting techniques, but also by the outlook, experience, intuition, and knowledge of the interpreter; by the volume of the used geological, geophysical, and experimental data; by the interpreter's understanding of the nonuniqueness, practical equi-

valence, and instability of the inverse problem. Various computer techniques of the automated fitting can only, to some extent, accelerate the modelling process, increase the number of the alternative versions considered, and increase the accuracy of the adjusted parameters. The resources of modern PC give an opportunity to implement effectively the computer techniques of interpretation using multi-variant and multi-stage strategies of the automated adjustment. Computer implementation of such techniques with the use of the intuitively comprehensive graphic interface for manipulation by simulated geological and geophysical objects and models make them popular for a wide range of specialists. One of the techniques of the automated fitting of gravity and magnetic anomalies with the use of multi-stage strategy has been developed on the basis of different approaches to the approximation of the anomalous sources of complex configuration (Bulach & Mikheeva 1993; Kozlenko et al. 1997; Gross et al. 1998).

CRITERION OF APPROXIMATION BETWEEN THE OBSERVED AND MODELLED FIELDS

The adjustment software is based on the optimization of the sum of square differences between the observed and modelled components of the fields. For modelling with a few fields as well as with linear transforms of the type of sliding window

$$U_i^T = \delta U_i = \sum_{j=-n}^{n} c_j U^{i+j}, \qquad j = -n, \ldots, 0, \ldots, n, \tag{1}$$

where δU_i is the value of the transform at point i; $\{c_j\}_1^q$ is the subset of weighting multipliers of transformation operators; $q = 2n + 1$ denotes the number of values of the field component U used for the calculation of a transform at the point. The objective function FC^T is used in software as follows:

$$FC^T = \sum_{m=1}^{M} \alpha_m \sum_{i=1}^{N_m} \frac{a_{im}}{U_m^{cp}} \left\{ \sum_{j=-n_m}^{n_m} c_{jm} \left[U_{R,m}^{i+j} - U_{T,m}^{i+j}(x^m) \right] \right\}^2 ; \tag{2}$$

$$m = 1, \ldots, M; \quad i = 1, \ldots, N.$$

Here

$$U_m^{cp} = \sum_{i=1}^{N_m} U_m^i \Big/ N_m ; \; U_{T,m}^i = \sum_{j=1}^{K_m} U_{j,m}^i + A^m x^i + B^m y^i + C^m z^i + D^m ;$$

$$x^m = (x_1^m, x_2^m, \ldots, x_{P_m}^m);$$

M is the number of the field components used in the adjustment process; m is the index of a field component; α_m is the weighting multiplier of a field component in the value FC^T; N_m is the number of points at which the component is set up; a_{im} is the weight of point i; $U_{R,m}^i$, $U_{T,m}^i$ are the observed and modelled

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values of components; $U^i_{j,m}$ is the contribution of the jth approximation cell in field value; K_m is the number of cells; P_m is the total number of parameters; A^m, B^m, C^m, D^m are coefficients of the linear background of the component m.

FUNCTIONAL PECULIARITIES OF THE FITTING SOFTWARE

The software (Kozlenko et al. 1997; Gross et al. 1998) has a universal character and gives the user wide opportunities for modelling specific geological objects. It allows: (a) adjustment of the parameters of the anomalous sources using the observed components of the gravitational (Δg) and magnetic ($\Delta Z, \Delta T$) fields and the geoid anomalies (h) separately and simultaneously (Δg and ΔZ, Δg and ΔT, Δg and h, ΔZ and ΔT, h and ΔZ, h and ΔT), the calculated course gradient value of a magnetic field, second derivatives of a gravitational potential, and linear transformations of simulated components, set by the operators of weighting summation in a sliding window (1); (b) modelling by solving linear, nonlinear, and combined (linear and nonlinear) inverse problems of the gravity and magnetics; (c) using during adjustment the elements of the modelled components, given along profiles, on a plane, in space, and in the wells; (d) application of various fitting strategies at different stages of modelling (simulation of isolated objects or the whole geological sequence; use of two-dimensional (2D) or 3D objects, etc.).

PARAMETRIZATION OF GEOLOGICAL OBJECTS

The approximation procedure developed in the fitting software is multi-elemental. It includes more than ten 2D and $2\frac{1}{2}$D elementary approximating cells of a simple geometric form (from a dot source to cylindrical bodies with polygonal section). It is open, i.e. presupposes expansion of the approximating opportunities by other elementary approximation elements. It is universal as it allows in an optimum way to carry out parametrization of geological bodies, objects, and sections of different forms (structural, ore, and structural–ore). It is adaptive and controlled because it gives the user an opportunity to perform many operation modes of the fitting programs by fixing separate parameters and whole objects, by using the restrictions in fitting parameters, by preserving the configuration forms of the fitting objects, etc. The vector of the fitting parameters of a separate cell of an approximation procedure is

$$x = x_g \bigcup x_z = (\sigma, J_x, J_y, J_z, y_1, y_2, x_1, z_1, x_2, z_2, ..., x_N, z_N), \quad (3)$$

where σ is surplus density; J_x, J_y, J_z are the components of a magnetization intensity vector; y_1, y_2 are parameters of extension; x_k, z_k are coordinates of angle points of the approximating polygon; N is the number of sides (angle

points). For vertical cylindrical cells it is necessary to change places of the parameters y and z.

The efficient technique of the approximation of star contours of sources by a radius vector which is set by the truncated trigonometric series has been used for cylindrical bodies with polygonal sections (Bulach & Mikheeva 1993)

$$R(\varphi_i) = R_0 + \sum_{j=1}^{N} (A_j \cos j\varphi_i + B_j \sin j\varphi_i),$$ (4)

where R_0 is the initial value of a radius vector; A_j, B_j are coefficients of the members of the approximating series; N is the number of series members; φ_i is the angle of the current radius. This technique allows us to describe sources of complex configuration by a small number of parameters. Including it into the software (Kozlenko et al. 1997; Gross et al. 1998) we can set and perform a multi-stage fitting.

To adapt the approximation technique of star bodies in a software, the approximation procedure was complemented by several approximating cells, which in general are described by a vector of parameters

$$x = x_g \bigcup x_z = (\sigma, J_x, J_y, J_z, y_1, y_2, x_0, z_0,$$
$$mi, R_0, A_1, B_1, ..., A_N, B_N),$$ (5)

where x_0, z_0 are coordinates of the gravity centre of a star body; mi is the dimension of the radius vector describing the contour of the body (number of points). The representation of parameters of additional cells in the form (5) enables their use at modelling simultaneously with all other approximating cells. In a vector (5) a priori the fixed parameters are x_0, z_0, and mi, any other parameter can either be selected or fixed. Using these cells during the modelling, the computation in program modules of solving the direct problem and calculation of a vector gradient of the objective function on the current fitting iteration are performed on analytical expressions for horizontal (2D or $2\frac{1}{2}$D) and vertical cylindrical cells with polygonal sections.

Adaptation of the technique of the star bodies approximation by contours of complex configuration (Bulach & Mikheeva 1993) extends the functional capacities of a software (Kozlenko et al. 1997; Gross et al. 1998), as it allows modelling of geological objects using a multi-stage strategy of fitting. Let us point out some versions of a multi-stage strategy of the automated fitting method.

1. At the first step of interpretation the automated fitting of the anomalous sources is performed by their approximation by polygonal contours of small dimensions (rectangular or trapezoid section). After optimization of the anomalous sources, the received optimized configurations are approximated using the technique of the description of star bodies. At the second step the optimum values of the radius vectors, the number of members of the truncated trigonometric series, and initial values of the coefficients of series are calculated. At the third step the automated adjustment of optimum values of the parameters, received

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as a result of the executed transitions which define complex configurations of anomalous sources, is performed. The transition from step to step can be done automatically, without interpreter, or interactively.

2. At the first step of the above-described interpretation not cylindrical cells with polygonal contours, but singular approximating cells can be used: strings of poles (2D and $2\frac{1}{2}$D), inclined strips, and planes.

OPTIMIZATION PROCEDURE

The automated fitting of parameters of the specific approximation procedure is performed by minimization of the objective function (2) using a special optimization procedure (Korchagin 1993). It includes some modifications of gradient optimization methods (methods of the steepest descent with normalization of the descent direction and acceleration of convergence, gradient, and conjugated gradients), and also the optimization based on the known singular value decomposition (SVD) algorithm (Johansen 1977; Enmark 1981; Lines & Treitel 1984). The computing experiments have shown that far from the minimum of the objective function the steepest descent method is rather effective, whereas in the vicinity of the minimum it is characterized by slow convergence. At the same time, in the vicinity of the minimum the SVD algorithm has doubtless advantages in relation to the steepest descent. The speed of the convergence of the iterative process and also the accuracy of the parameter restoration when the iterative process gets into a global minimum are much higher than those at using the steepest descent. Owing to essential differences between the direction of the steepest descent and the direction determined by the SVD algorithm, very frequently iterative processes from identical initial approximations result in different local minima of the objective function. The developed optimization procedure allows of using the steepest descent at initial iterations of the adjustment and algorithm SVD in the vicinity of the minimum (using in that way the advantages of both methods). The model calculations have shown that while solving a fitting problem with a large number of freedom degrees the stability of the iteration process can be achieved by introducing rather rigid restrictions, both on possible variation limits of fitting parameters and on the changes of parameters on separate fitting iterations. The principle of monotonous decrease of the optimized objective function from iteration to iteration, used in an optimization procedure, is rather rigid. On one hand, it does not allow getting out of a local minimum into which the iterative process gets; on the other hand, it results in the fact that the fitting parameters experience maximum changes (which can lead to "jumping" of an iterative process into another minimum of the objective function, rather far from the initial approach) at initial fitting iterations. Hence, it is sufficient to use the restrictions on the parameter changes (and also regularization of an iterative process) at the initial fitting iterations.

THE LINEAR TRANSFORMATIONS
IN THE FITTING ALGORITHMS

One of the possible approaches to using the linear transformations of potential fields, set by the operators of weighting summation in the sliding window (1) (Bulach et al. 1987), is also applied in the software. The peculiarity of this approach is the following. In the fitting algorithms the same approximate operator of transformation influences the modelled and measured sets of the field components. In this situation the accuracy of the transform calculations does not much influence the adjustment results, only the properties of the appropriate transforms are used. The model examples have shown that the fitting results on the transformed anomalies are practically similar to the results of fitting on the exact analytical anomalies of the corresponding type. The described method of the automated fitting on transforms can be included in procedures and flow charts for finding many solutions to specific model and practical problems.

PRACTICAL APPLICATION OF THE SOFTWARE

The recommendations for using the automated fitting algorithms and software are developed on examples of modelling of (a) a gravitational field along regional seismic profiles, (b) configurations of intrusive massifs and ore objects, (c) airborne magnetic measurements along regional geotransects at different altitudes and recalculated gravitational anomalies on different levels, (d) separate ore bodies in an iron-ore deposit, and (e) gravitational and magnetic fields along geological cross-sections during large-scale geological mapping.

For the practical interpretation of anomalies it is advisable (a) to model complicated objects and cross-sections in parts, (b) to get different solutions of the same problem (with various initial approximations, at different operation modes of the fitting software, for different ways of fixing some of model parameters, with linear transformations and without them, with restrictions for selected parameters and without them), (c) analyse the accuracy of solutions on a special problem constructed for those aims.

Some examples of practical interpretation are given below.

The Ukrainian shield–Dnepr-Donetsk basin

The 400 km long deep seismic sounding profile Putivl–Krivoj Rog (Fig. 1) passes through Krivorozhskaya and Dnepr-Donetsk superdeep wells. The 2D cells with polygonal section were used for the parametrization of the Earth's crust and upper mantle. The vertical and horizontal contacts between the blocks were set in view of the faults and borders, determined by the seismic data. Two versions of section approximation (65 and 57 polygonal blocks) were considered, differing in details of the upper mantle approximation. The density within the limits of
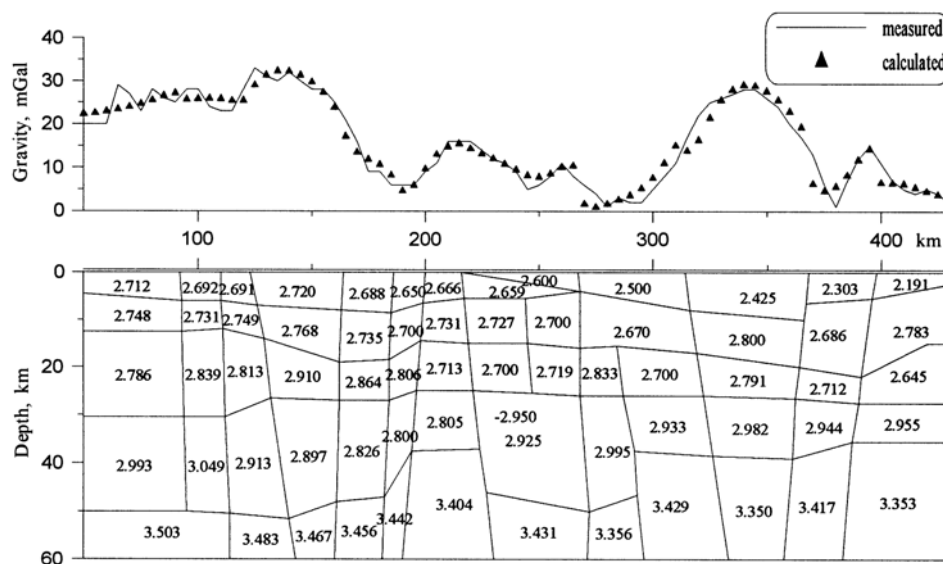
Fig. 1. A density model of the Earth's crust along the seismic profile Putivl–Krivoj Rog.

separate blocks was considered constant, and also the restrictions in possible limits of its variation were imposed in the process of solving the direct and inverse gravity problems. The gravitational field data collected at 77 points with a 5-km step along the structure were used during the calculations. In some versions the calculation was carried out on the field data smoothed by a spline-approximation algorithm. The initial values of density in blocks are given using a velocity section and the known generalized data about velocity–density dependence. The results of the calculations allow us to draw the following conclusions.

1. The developed software essentially accelerates the process of the modelling of a gravitational field along regional structures and enables analysis of various, practically equivalent solutions. The divergence between some parameters of the practically equivalent solutions can be rather large. The number of the practically equivalent solutions can be reduced by narrowing the given restrictions in the limits of the density change in the blocks.

2. With the increase in the dimensions of the fitting problems (a) the restoration accuracy of the fitting parameters decreases, (b) the instability of solutions increases, and (c) receipt of reasonable solutions is impossible without rigid restrictions on the limits of parameter changes. Using the principle of decomposition of a large-dimension problem into some small subproblems can contribute to the solution accuracy.

3. The accuracy of the fitting parameters (limits of dispersion) can be estimated on an accompanying modelling problem that can be built on the basis of any acceptable version of the practical problem solution.

4. The constant component of the background field can be simulated with the help of two formal methods: (a) by the adjustment of the negative density in the block which covers the entire section and (b) by the use of the linear transformation operator in a sliding window for recalculation of initial field components in the transformation δV_{zx}.

The Atlantic Ocean

The magnetic field modelling above considerably massive underwater mountains in the Atlantic Ocean (Fig. 2) is rather interesting because of the methods used. Here the mountain towers like a wall from 4-km depth practically up to the water surface (in some intervals of the vessel's course the depth above the mountain decreases to 25 m). The values of the module of magnetic field intensity above the mountain are highly differentiated. Visual analysis of the diagram of the changes in the magnetic field intensity module above the mountain allows us to assume that a series of rather intensive local positive anomalies (up to 500 nT), caused by local magnetic sources in the mountain itself, is imposed on the negative magnetic anomaly of about the same intensity, caused by the mountain. This idea about the character of a magnetic field change above an underwater mountain served as a basis for the magnetic modelling.

At the first stage of the processing of magnetic data along the tack the transition from the values of the total module intensity to local magnetic anomalies ΔT was carried out by the background component approximation with the first-degree polynomial.

According to the accepted circuit of magnetic modelling the negative magnetic anomaly above the underwater mountain was described as a large magnetic body of trapezoid section covering practically the entire mountain (Fig. 2a). The top edge of the underwater mountain was fixed; the bottom edge coordinates were fitted. The local anomalous objects in the body of the underwater mountain were approximated by separate objects with four-corner sections. At the first step of fitting the rectangular form of local anomalous bodies was fixed, while at the last steps the restriction on preservation of the anomalous body's form was removed. One version of the automated fitting of the parameters of the anomalous source according to the described circuit is presented in Fig. 2a. It shows that the main features of the magnetic field anomaly above the underwater mountain are quite satisfactorily reproduced by a field of the magnetic sources model.

Figure 2b displays another version of anomalous field fitting above the mountain. Unlike in the previous model, here an additional local source is not used for modelling regional negative components of the mountain. For this purpose a horizontal cylindrical body with the section completely identical to that of the underwater mountain along the modelled tack was used. This magnetic source model also reveals quite satisfactorily the basic features of the anomalous field behaviour.
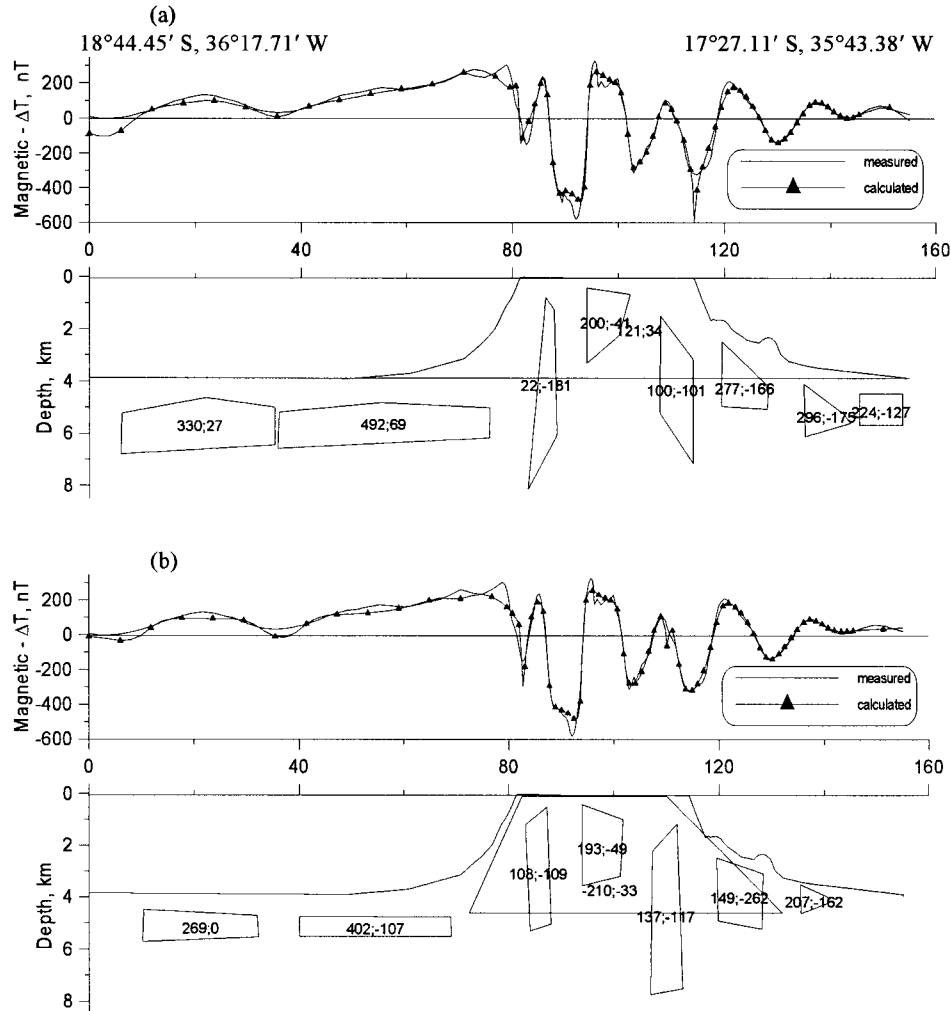
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Fig. 2. The magnetic field along the fragment of tack 1005 in the Atlantic Ocean and the results of the sources fitting by bodies of simple configuration. The numbers on the contours of the anomalous bodies show the values of the vectors of the magnetization intensities of the bodies (in 10^{-5} CGS units).

The Antarctic Peninsula region

Figure 3 shows some versions of the magnetic field modelling along the tack fragment in the Bransfield Strait. At the first stage of modelling the anomalous field was fitted by four prismatic disturbing objects. The geometrical parameters of the objects were selected simultaneously with the value of magnetization intensity of separate bodies and the linear background coefficients. The limitations on geometrical parameters of objects were used. At the second
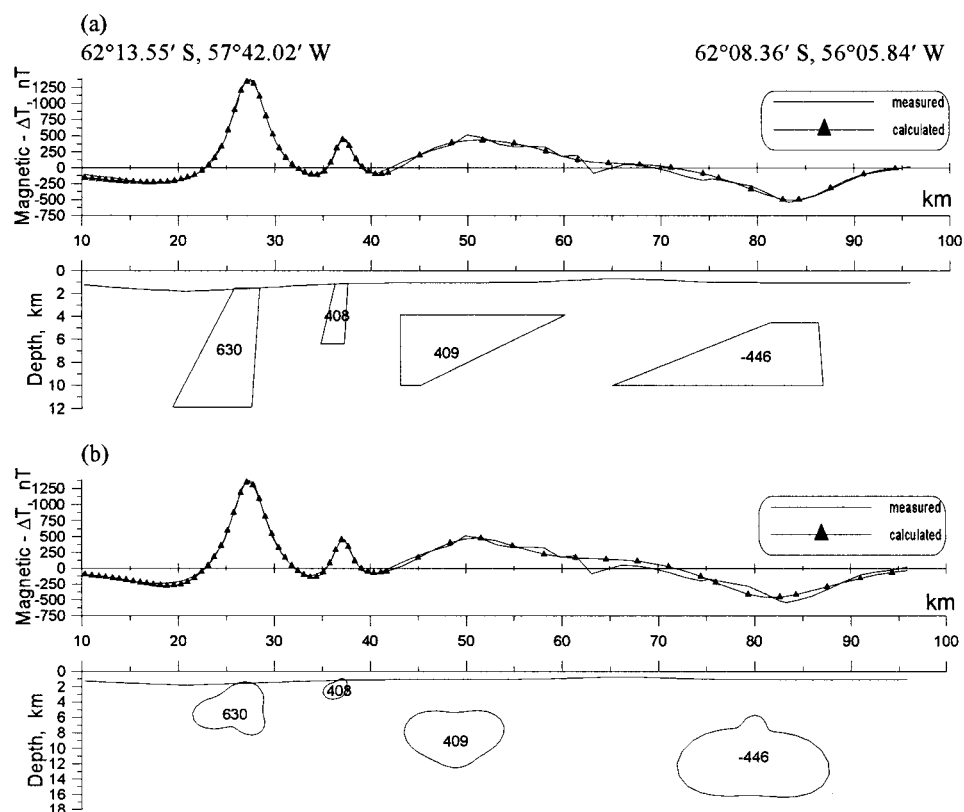
Fig. 3. The magnetic field along the fragment of tack 0704 in the Bransfield Strait and the results of the sources fitting by bodies of trapezoid form (a) and in contours of complex configuration (b). The numbers on the contours of the anomalous bodies show the values of the vectors of the magnetization intensities of the bodies (in 10^{-5} CGS units).

stage the approximation of the objects was performed with the help of contours of trapezoid section (Fig. 3a). Magnetization intensity and linear background coefficients were selected. The approximation of the objects was carried out also by contours of complex configuration. The magnetization intensity of the objects was fixed – the values received at the previous stage of modelling were used. The linear reference coefficients were fitted during solving the problem. The coordinates of the centres of gravity of the anomalous bodies and initial approximation of radius vectors were set in view of the results of the previous solutions. The problem was solved repeatedly, with a different number of members of the truncated trigonometric series, which approximated radius vectors of separate objects. A version of solving this problem is shown in Fig. 3b.

CONCLUSIONS

The $2\frac{1}{2}$D horizontal cylindrical cells with polygonal sections can be used for modelling the 3D geological objects. However, at significant changes in the configuration of such objects the approximation by cylindrical bodies will not be optimum. In such situations it is preferable to use, as approximating cells, body-polyhedrons built on separate vertical sections of the object. The technique of the approximation of the star bodies of complex configuration can be successfully used for automatic construction of polyhedrons on sections.

The modified approximation procedure gives us many opportunities for modelling mineral objects of complex form. On one hand, it is a quite convenient way of parametrizing anomalous sources with complex physical properties. On the other hand, it allows fitting to be carried out by consecutive complication of the model, from the use of simple (singular) approximating cells to more complex ones.

Various techniques, methods, and modifications of fitting can be synthesized in a uniform computer-and-information subsystem of modelling using a local database of the computer model. Basic elements of such a local database can be (a) field catalogues, (b) wells, (c) stratigraphic column, (d) physical properties, and (e) model geometry. The communication of the user with elements of the computer model of the investigated object can be carried out through the dialogue interface, the basic elements of which might be presented by blocks of the following procedures: (a) visualization; (b) optimization; (c) solution of direct problems; (d) automated fitting; (e) formation of fastenings and restrictions; (f) creation, updating, and destruction of separate elements of the model; and (g) helpful and auxiliary information.

The various techniques and strategies of the automated fitting can be performed rather effectively on the basis of the approximation procedure used in the fitting software: by the method of multiparameter functional optimization, by constructing families of the equivalent solutions, by using various modifications of algorithms, and by consecutive complication of the model using the transition from simple singular approximating cells to more complicated ones.

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Keerulise kujuga anomaaliaallikate gravitatsiooniline ja magnetiline modelleerimine

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On käsitletud geoloogiliste objektide modelleerimise põhielemente automaatsobitamise meetodite raames. Vaadeldav metoodika võimaldab tõlgendamise käigus kasutada magnet- ja raskuskiirenduse väljade mõõdetud komponente, nende lineaarteisendusi, geoidi anomaaliaid ja magnetvälja mõõdetud suundgradienti. Tarkvara põhineb mõõdetud ja modelleeritud väljakomponentide hälvete ruutude summa optimeerimisel. Anomaaliaallikate parameetrimise kõikehaarav aproksimeeriv konstruktsioon võimaldab efektiivselt kirjeldada modelleeritava ruumi erinevaid elemente. Optimeerimisprotseduur funktsioonide minimeerimiseks sisaldab gradiendi tüüpi algoritme ja singulaarmaatriksi dekompositsioonimeetodit. On esitatud ka väljatöötatud tarkvara praktilise kasutamise näiteid.

Гравитационное и магнитное моделирование аномальных источников сложной конфигурации

Микола Якимчук, Игнат Корчагин, Татьяна Михеева, Марина Орлова и Юрий Якимчук

Рассматриваются основные элементы технологии моделирования гравитационных и магнитных полей в рамках методов автоматизированного подбора. Технология позволяет использовать в процессе интерпретации наблюдаемые компоненты гравитационного и магнитного полей, линейные трансформанты наблюдаемых компонент, аномалии геоида, данные измерения курсового градиента магнитного поля. Программное обеспечение

базируется на оптимизации суммы квадратных расхождений между измеренной и модельной компонентами полей. Универсальная аппроксимирующая конструкция для параметризации аномальных источников позволяет эффективно описывать различные элементы моделируемого пространства. Оптимизационная процедура для минимизации функций цели включает алгоритмы градиентного типа, а также процедуру, построенную на базе алгоритма сингулярного разложения матриц. Приводятся практические примеры использования разработанной технологии.