Proc. Estonian Acad. Sci. Eng., 2004, **10**, 4, 290–299

Torque distribution control unit in automotive propulsion systems

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Received 1 July 2004, in revised form 27 September 2004

Abstract. The paper considers energy flow modelling in automotive propulsion systems to provide basic data for the selection of the parameters of the differential, bearing in mind its impact on the vehicle lateral dynamics. The characteristics and operational conditions of the limited-slip differential gear train are found. A virtual differential (algorithm of energy flow) has been created.

Key words: planetary gear train, limited slip, torque transmission, energy loss, modelling.

1. INTRODUCTION

Development of the control unit for a vehicle (its neural system) in order to improve its safety characteristics needs information about the power flow from the engine to the driving wheels. An essential disadvantage of the conventional differential is that if one wheel of the vehicle slips on a surface with low friction, it is likely to bring the vehicle to a halt. The conventional differential is unable to transmit the necessary torque to the other wheel. The limited-slip differential can in this case transmit more torque, but a decrease in the steering qualities takes place due to the increased understeering.

The aim of this work is to investigate the power flow from the engine to the driving wheels with a limited-slip differential, considering also the lateral dynamics of the vehicle.

2. ANALYSIS OF THE DIFFERENTIAL

2.1. Geometrical interpretation of the gear–tooth losses

By modelling the differential, the influence of gear tooth losses will be considered. The loss of free torque can be expressed as $T = F_t r_\omega$, where F_t denotes the tangential contact force component at the pitch radius r_ω.

Figure 1 shows a teeth pair of the gear mesh in contact at one of the end points of the line of contact.

The distance between P and P_1 is called the force pole offset and is denoted by Δr_ω. In the case of friction, the magnitude of the torque on the driving (subscript "1") and the driven (subscript "2") wheels can be expressed as

$$T_1 = F_t(r_{\omega 1} + \Delta r_\omega), \qquad T_2 = F_t(r_{\omega 2} - \Delta r_\omega). \tag{1}$$

If one of the wheels is internally geared, the corresponding pitch radius should be considered to be negative.

The relationships in Eq. (1) may be transformed into an expression for traditional efficiency η

$$\eta \approx 1 - \Delta r_\omega \left(\frac{1}{r_{\omega 1}} \pm \frac{1}{r_{\omega 2}} \right), \tag{2}$$

where the minus sign applies for the internal gear mesh.

The geometrical handling of the efficiency of epicyclical gear trains is essential by modelling of the power flow.

2.2. Virtual shaft extension

In [1,2] Mägi gave a generalized definition for the common positive sense for velocities and torques (Fig. 2). This is relevant in the description of non-parallel shaft gearings.

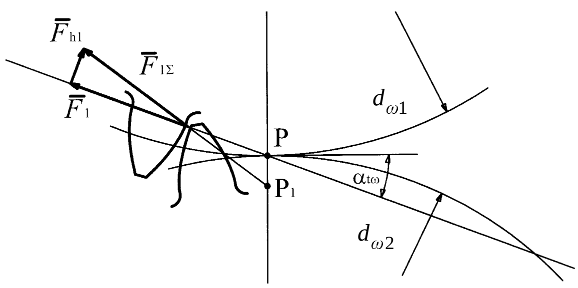

Fig. 1. Diagram of the friction loss by gearing: $\alpha_{t\omega}$ – pressure angle; $d_{\omega 1}$, $d_{\omega 2}$ – diameters of the pitch circles of the driving and the follower gear wheels, respectively; P – the pitch point; P_1 – the force pole, where the tangential force acts; $\overline{F}_1$ – the theoretical loss-free normal contact force; $\overline{F}_{h1}$ – the friction force; $\overline{F}_{1\Sigma}$ – the resultant contact force in the friction-related case.

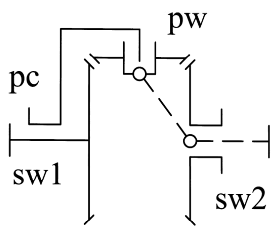

Fig. 2. Virtual shaft extensions (broken lines) for a bevel-wheel planetary system: pc – the planet carrier; pw – the planet wheel; sw1, sw2 – sun wheels.

2.3. Application of the Lagrange multipliers

Using the Lagrange multiplier theorem, [2] the Jacobian of the system constraint matrix is of fundamental importance. It can be easily compiled so that it fully defines the kinematics of the model. This approach implies that originally all shafts of a differential or of a planetary system in general may rotate independently. Existing interconnections, as gear meshes, introduce constraints to the motion.

The constraint approach in the present context implies that the difference between the relative peripheral pitch circle velocities in each gear mesh between two mating gear wheels, ΔV, is zero. For instance, an internal motion constraint of the system contributes to the Jacobian matrix for mesh j between shafts pw and sw1 as follows:

$$\Delta V_j = (\omega_{\text{pw}} - \omega_{\text{pc}})r_{\text{pw}} + (\omega_{\text{sw1}} - \omega_{\text{pc}})r_{\text{sw1}} = 0, \tag{3}$$

where r_{pw} denotes the radius of the planet wheel that mates r_{sw1} and ω is the rotational velocity of the shaft in consideration.

Expressing all differences of relative velocities, ΔV_j, as a vector $\{\Delta V\}$, and velocities of all shafts as another vector $\{\omega\}$, the compatibility with all the constraints can be expressed as

$$\{\Delta V\} = [J_{\text{C}}]\{\omega\} = \{0\}, \tag{4}$$

where $[J_{\text{C}}]$ is the Jacobian of the internal motion constraints in the system, containing various pitch radii as matrix elements. Since Eq. (4) equals to zero, all radii may be pre-multiplied by a constant, allowing the radii to be replaced by the numbers of teeth of the wheels. The contribution of the constraint of tangential forces F at gearing, T_{constr}, to the torque equilibrium for each shaft, is according to the Lagrange multiplier theorem

$$\{T\}_{\text{constr}} = [J_{\text{C}}]^T\{F\}. \tag{5}$$

In dynamic situations, the inertial effects must be included. The change of rotational speeds, i.e., rotational acceleration, is in the sense of d'Alembert equivalent to the action of an external torque $T_{\text{inert}} = -J_{\text{P}}\dot{\omega}$, J_{P} is the polar mass moment of inertia of the particular subsystem, and $\dot{\omega}$ is its angular acceleration. For all rotating elements the inertial torque vector is

$$\{T\}_{\text{inert}} = [J_{\text{P}}]\{\dot{\omega}\}, \tag{6}$$

where $[J_{\text{P}}]$ is the diagonal matrix of all polar mass moments of inertia and $\{\dot{\omega}\}$ is the vector of all angular accelerations.

The total fully dynamic but loss free equilibrium is then given as

$$\{T\} + \{T\}_{\text{constr}} + \{T\}_{\text{inert}} = \{0\} \rightarrow -\{T\}_{\text{inert}} - \{T\}_{\text{constr}} = \{T(t)\}, \tag{7}$$

where $\{T\} = \{T(t)\}$ is the prescribed time-dependent external torque vector. The constraint conditions, Eq. (4), may be differentiated once, yielding

$$\pm [J_{\text{C}}]\{\dot{\omega}\} = \{0\}. \tag{8}$$

All equilibrium and compatibility equations, Eqs. (4) to (8), may now be collected to form a set of equations

$$\begin{bmatrix} [J_{\text{P}}] & [J_{\text{C}}]^T \\ [J_{\text{C}}] & [0] \end{bmatrix} \begin{Bmatrix} \{\dot{\omega}\} \\ \{F\} \end{Bmatrix} = \begin{Bmatrix} \{T(t)\} \\ \{0\} \end{Bmatrix}. \tag{9}$$

The Lagrange multiplier approach eliminates the need for the detailed derivation of torque equilibrium equations.

The above simplifications opened the way to consider the modules of the model in a systematic way. Thus we can also complete the generalized model of the vehicle, which contains different modules. The model describes the distribution of the energy flow.

3. PERFORMANCE CHARACTERISTICS OF A VEHICLE WITH LIMITED-SLIP DIFFERENTIAL

3.1. Characteristics of the tractive effort

At any radii of cornering, the rotational velocity of the driving wheels of a vehicle with a common differential is adjusted to the steering radius. The application of a planetary gear train (locked differential) leads to a resistance at cornering. Actually, the coefficient of efficiency of the common differential is relatively high. Thus in case of the common differential, the torque of the driving wheels is approximately equal. In a loss-free case, ignoring gear ratio at the differential, a certain correlation exists: $T_{\text{in}} = T_{\text{out1}} + T_{\text{out2}}$. Here T_{in} denotes the input moment of the differential gear train and T_{out1} and T_{out2} are the output torques on the corresponding wheels. Besides, $T_{\min} = Gf_{\min} r_{\text{d}}$ (G is the gravitational force on the driving wheel and $f_{\min}$ is the coefficient of friction between the driving wheel with radius r_{d} and the road surface, calculated on the wheel with a lower value of friction on the assumption that an equal gravitation force has an impact on the driving wheels).

The highest torque in case of a common differential can be $T_{\mathrm{in}} = 2T_{\mathrm{min}}$. The locked differential has taken the form of a planetary gear train as a result of locking, thus enabling the transmission of a higher torque on the road:

$$T_{\mathrm{in}} = T_{\mathrm{min}} + T_{\mathrm{max}} = Gr_{\mathrm{d}}(f_{\mathrm{min}} + f_{\mathrm{max}}). \tag{10}$$

Here T_{max} denotes the torque of the wheel with a higher value of friction with a coefficient of friction f_{max} on the road surface. In this case, the driving wheels with equal rotational velocities provide a disadvantage at cornering.

As a rule, in most street and road vehicles, a differential with a relatively high coefficient of efficiency is applied. In extreme situations (e.g., $f_{\mathrm{min}} << f_{\mathrm{max}}$) it is essential to increase the torque on the driving wheels. One has to find a reasonable compromise between the increasing tractive effort and understeering of vehicle. To achieve that, we have tried to limit the relative mutual rotational velocity of the driving wheels.

Theoretically, the torque ratio at the differential can be expressed as

$$k = \frac{T_2}{T_1}, \tag{11}$$

where T_2 and T_1 denote the torque of each output shaft of the differential in the beginning of the switch-off of the brakes. In case of the application of the limited-slip differential, the highest torque value can be expressed as $T_{\mathrm{in}} = T_{\mathrm{min}}(1 + k)$.

In case of jeeps (off-road vehicles) that are driven on rough roads, higher torque values are needed. It is possible to obtain higher torque values by decreasing the cornering abilities. Torm [3] has carried out tests to study the limited-slip differentials in tractors. According to his results, a differential with the torque ratio 2.0 is not likely to increase the cornering radius (in case of tractor DT-20). In case of a trailer, used in a cultivated and a stubble field (with maximum angle of the front tyres), the cornering radius was increased for 10–20% and the total motion resistance grew for 11–17%. However, it is not reasonable to apply a differential with a torque ratio over 2.5–3.0.

Mägi has used the Lagrange multiplier approach [2], which eliminates the need for detailed torque equilibrium equations. This is a considerable simplification of the planetary gear train analysis. The correlation of the velocities and torques of the shafts (rotating elements) of each transmission unit can then be automatically formulated, presented in the form of a matrix and solved by the computer program. This will enable us to calculate the values of the torques and velocities of each shaft in the system.

The torque and the speed losses can be calculated using the Lagrange multipliers technique. The transmission systems with more than one input and one output shaft as well as the epicyclical trains can be calculated. Besides, the possible over-constrained elements of the transmission system can be detected.

3.2. Steering characteristics

The characteristics of the road vehicle by cornering at low velocities have also been examined (with no centrifugal force). At low velocities, a simple relation between the direction of motion and the steering wheel angle has been observed. The prime concern by the design of the steering system is the minimum tyre scrub at cornering. Therefore, at cornering all tires should be in pure rolling without lateral sliding. To satisfy this requirement, the wheels should follow the curved path with different radii originating from a common centre C (Fig. 3). The steer angles δ_o and δ_i should satisfy the relationship:

$$\cot(\delta_o) - \cot(\delta_i) = B/L, \tag{12}$$

where, the subscripts “o” and “i” denote outer and inner wheels at cornering. The steering geometry that satisfies the above equation is usually referred to as the Ackermann steering geometry and is valid as a theoretical reference case, where sideslip of the wheels is disregarded [4].

Modelling of the steering geometry and its influence on the steering characteristics have been considered in [5].

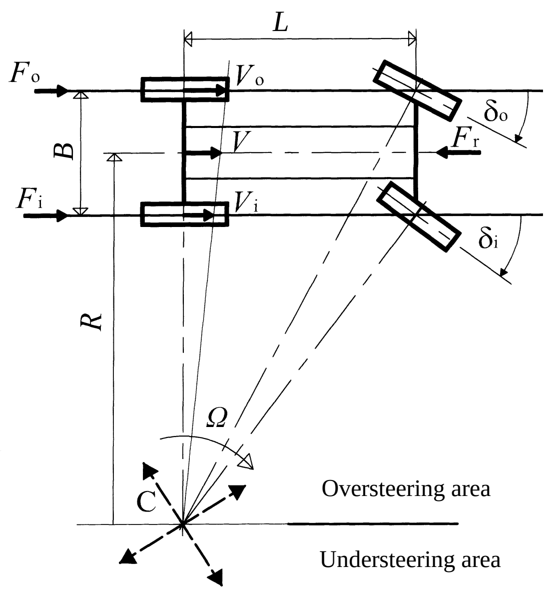

Fig. 3. Steering characteristics of the vehicle: *B* – the track (or tread) of the vehicle; *L* the wheel base; subscript r denotes resistance.

In Fig. 3, C is the instantaneous centre and Ω is the rotational velocity relative to it. The steering angles of the front wheels, δ_o and δ_i, have been used in the calculations with an equal value $\delta = 0.5(\delta_o + \delta_i)$.

According to the value of the understeer coefficient [6] or the relationship between the slip angles of the front and the rear tires, the steady-state handling characteristics may be divided into three categories: neutral steering, under-steering and oversteering.

In the design of the differential it is essential that the differential provides the vehicle with the utmost neutral steering qualities.

3.3. Components of the virtual differential

We have assumed that the vehicle moves with a non-constant radius with a changing angle of the front wheels. The lateral dynamics of the limited-slip differential can be divided into certain stages. As for the characteristics of the differential, the most significant factor is the constraint on the inner slip. This constraint may have a variable or constant impact on the torque ratio. In case it is variable, it can be proportional to the load or velocity. Let us take a limited-slip differential, which is load-proportional. The difference between the velocities of the driving wheels is $\Delta V = V_o - V_i$.

The lateral dynamics of a vehicle equipped with a limited-slip differential undergoes the following processes.

1. The transition from the curvilinear trajectory to the linear one.
2. The transition from the linear trajectory to the curvilinear one with a variable radius.
    - The stage of the relative small torque of the driving wheels – the differential is still locked (Fig. 4, points 1 and 2). The vehicle is moving along a curve and the rotational velocities of its driving wheels are equal $(\omega_i = \omega_o)$. The inner wheel slips back relative to the road surface, whereas the outer wheel slips forward in the direction of motion. However, both driving wheels slip back as a result of the motional resistance.

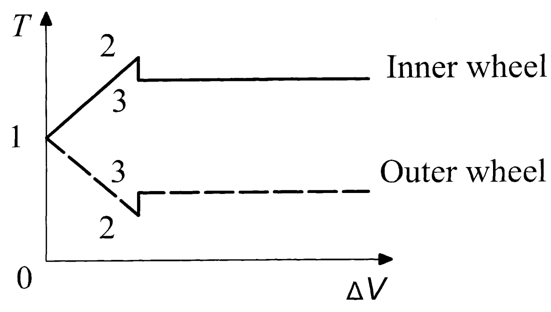

Fig. 4. The torque ratio at the differential during the motion process on a curvilinear trajectory; *T* – the torque of the driving wheel; 1, 2, 3 – characteristic points of the limited-slip differential.

- The intermediate stage – relative slip appears in the friction elements of the differential (Fig. 4, points 2 and 3). It is possible to fix the unlocking phase of the differential, which is characterized by the transition from the static friction to the dynamic one. As a rule, the transition is gradual, depending on the surfaces of friction.
- The stage of the large angle of the front wheel – the differential has been unlocked (Fig. 4, point 3). Further increase in the torque of the driving wheels will not bring about any increase in the relative torque ratio.

These stages can be interpreted as modules of the vehicle-movement models.

For the modelling of the limited-slip differential we need speed and load characteristics of the vehicle as initial parameters. The general equations of the limited-slip differential for the simulation of the lateral dynamics of the vehicle (Fig. 3) can be divided into equilibrium, compatibility and constitutive relations, respectively.

The equilibrium of the force and torque can be expressed as $\Sigma T = 0$, $\Sigma F = 0$. We can balance the motion resistance F_{r} at constant movement by the tangential force of the driving wheels

$$F_{\mathrm{r}} = F_{\mathrm{i}} + F_{\mathrm{o}}. \tag{13}$$

In the general case, the equilibrium condition is

$$m\,{}^{\mathrm{d}V}\!/_{\mathrm{d}t} = F_{\mathrm{i}} + F_{\mathrm{o}} - F_{\mathrm{r}}, \tag{14}$$

where m denotes mass of the vehicle.

Compatibility of the outer and inner driving wheels can be expressed as

$$\omega r(1 - s_{\mathrm{o}}) = V_{\mathrm{o}}, \qquad \omega r(1 - s_{\mathrm{i}}) = V_{\mathrm{i}}, \tag{15}$$

where s denotes slip.

According to Fig. 3, for the curvilinear motion of the vehicle the following equation is valid:

$$\frac{V_{\mathrm{o}}}{V_{\mathrm{i}}} = \left(\frac{R + 0.5B}{R - 0.5B}\right) = \frac{1 - s_{\mathrm{o}}}{1 - s_{\mathrm{i}}}. \tag{16}$$

The constitutive relations of the limited-slip differential for the tangential force relative to the driving wheels can be approximately expressed as

$$F_{\mathrm{i,o}} = \frac{mg\mu}{2\pi}\arctan(20 s_{\mathrm{i,o}}). \tag{17}$$

The motion resistance can be written in the form

$$F_{\mathrm{r}} = \pm k_{\mathrm{r}} mg + k_{\mathrm{a}}\frac{\rho A}{2}V^2, \tag{18}$$

where k_r denotes the rolling resistance, k_a is the coefficient of aerodynamic resistance, A is the vehicle front area, μ is the wheel–road friction coefficient, g is acceleration due to gravity and ρ is air density. The motion process was simulated by MatLab. Results of the model simulation have been described in [6,7]. At the stage of modelling it is important to follow the velocity parameters and to record the load characteristics. Thus the models of the power from the engine to the driving wheels can be improved.

4. CONCLUSIONS

The basic equations for the description of the modules of the virtual differential have been elaborated. Based on these equations, the system of modules of the vehicle model were derived. The model of the differential is a part of the vehicle model. This model enables one to observe and control the distribution of the energy flow in the differential and change the parameters of the vehicle.

ACKNOWLEDGEMENTS

Participation of J. Resev in this research was supported by a grant from the Nordic Council of Ministers, administrated by the Swedish Institute in Stockholm. This grant enabled him to study the problem at Chalmers University of Technology in Göteborg, Sweden. The authors are obliged to the Estonian Scientific Foundation for support (grant No. 5903).

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Pöördemomendi jaotuse juhtmoodul liikuri jõuülekandesüsteemis

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On koostatud diferentsiaali mudel, mis sisaldab piiratud libisemisega diferentsiaali parameetrite määramiseks vajalikku moodulit.