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GUARDBANDS IN RANDOM TESTING

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Abstract. The fault coverage of a random test can be estimated by fault simulation. If the simulation is performed by a random sequence differing from those used under test or a fault sample, a random difference between the simulation result and the fault coverage has to be considered. The simulation result must exceed the fault coverage that has to be guaranteed. The difference is called a guardband. In this paper, the distribution of the fault coverage and that of the difference were derived by the mathematical model of independently detectable faults. Then it was verified using experimental data. The comparison between theory and experiment has unveiled features of the random test which were neglected in the past. However, the correlations in the fault detection process cannot be ignored in determining guardbands. As the final result, relations for guardband calculation are given.

Key words: test of digital circuits, random test, fault simulation.

1. INTRODUCTION

The most important parameter of a digital test is the fault coverage. It is the fraction of detectable faults from a set of assumed faults. Using random patterns as stimuli, the fault coverage depends mainly on the number of unique test patterns and less on the test patterns themselves. However, it is a random variable.

The aim of the paper is to study what kind of fault coverage can be guaranteed if the fault simulation has been performed with randomly selected input patterns, differing from those used under test. This question is interesting in practice. Random patterns are often used in self-test functions, but also in low cost test systems. Defining the test only by the number of test patterns has many advantages over the alternative, computing, storing and processing a large quantity of deterministic patterns [1, 2]. For the test of a circuit under operation, the input patterns are not known in advance. The fault simulation with an appropriate sample of patterns is the only way to estimate the fault coverage. In many applications, it is not enough to know the average fault coverage. The value that can be guaranteed is required. An akin situation arises, if the fault simulation has been done with a sample of faults. Simulation result and fault coverage differ by a random amount, and a lower bound has to be guaranteed for the fault coverage.

The term guardband has been taken from analogue testing. Testing a parameter, e.g., voltage, the measured value must be better than the value that should be guaranteed by the test [3]. The difference, the so-called guardband, is necessary to reduce the probability that noise and other disruptions during measuring will cause bad devices to be classified as good ones. The problem with the fault coverage is akin. The fault coverage should not be lower than a given bound. Otherwise, the number of bad devices classified as good ones will be too large.

A guardband calculation needs the distribution of the parameter under investigation. Section 2 develops a mathematical model to calculate the distribution, the mean value and the variance of the fault coverage out of detection probabilities. Basic features are discussed. Section 3 deals with the guardband size.

2. DISTRIBUTION OF THE FAULT COVERAGE

The detection probability $p_i(1)$ of a fault i is the probability that the fault will be detected by a single randomly selected input pattern. More detailed and more general explanations can be found in [1,2,4,5]. To calculate the detection probabilities $p_i(n)$ for a sequence of n input patterns, generally, the binomial model is used. A fault will be detected by n input patterns if at least one of the input patterns detects the fault

$$p_i(n) = 1 - (1 - p_i(1))^n . \quad (1)$$

Equation (1) can be simplified

$$p_i(n) = 1 - \mathrm{e}^{-n \cdot q_i} \quad \text{with} \quad q_i = -\ln(1 - p_i(1)) \geq p_i(1) . \quad (2)$$

For small detection probabilities it is

$$p_i(n) = 1 - \mathrm{e}^{-n \cdot p_i(1)} . \quad (3)$$

In the context of guardband calculation, the fault coverage is a random variable. By chance, it can take values between zero and one. To distinguish the random variable fault coverage from an exactly known fault coverage, the Greek letter ξ will be used.

Here is our new idea. We introduce auxiliary random variables, one for each fault i that should be one if the fault is detected and zero if it is not detected. The idea behind this is that the fault coverage is the mean value of these auxiliary variables

$$\xi(n) = \frac{1}{M} \sum_{i=1}^{M} \zeta_i(n), \tag{4}$$

where M is the number of assumed faults. The distribution of each of the auxiliary variables $\zeta_i(n)$ is

$$\begin{aligned} P(\zeta_i(n)=0) &= 1-p_i(n) \quad \text{fault undetectable,} \\ P(\zeta_i(n)=1) &= p_i(n) \quad \text{fault detectable.} \end{aligned} \tag{5}$$

Their mean values are equal to the detection probabilities

$$E(\zeta_i(n)) = p_i(n). \tag{6}$$

The variance is

$$D^2(\zeta_i(n)) = (1-p_i(n)) \cdot p_i(n). \tag{7}$$

The following presupposes that the faults in the circuit are detected independently of each other. Properly speaking, it is not true. Many faults share control and observation conditions. Resulting from that, the same logical values, at least at a part of the inputs, are eligible for fault detection. On the other hand, it would not be possible to calculate the distribution of the fault coverage without this assumption. Additional probabilities would be needed: a probability that fault i is detectable if fault j is (un)detectable. Those data are not available. Therefore, first, the model of independently detectable faults is used. Second, the resulting equations are verified by experiments.

A fast algorithm to calculate the distribution is shown in Fig. 1. Taking the distribution of $M=i$ faults and the detection probability of fault $i+1$, the distribution of $M=i+1$ faults is calculated. The starting distribution is that of the auxiliary random variable for the first fault $\zeta_1(n)$ with the realizations zero and one. From this, the distribution of the first and the second fault

$$P\left(\frac{\zeta_1(n)+\zeta_2(n)}{2} = \frac{m}{2}\right) \text{ with the realizations } \frac{m}{2} \in \{0, 1/2, 1\}$$

is calculated, ... The calculation time of this algorithm grows with the square of the number of faults.

$$P(\xi_1(n)=0)=1-p_1(n)$$

$$P(\xi_1(n)=1)=p_1(n)$$

DO for $i=2$ to M

$$P(\xi_i(n)=0)=P(\xi_{i-1}(n)=0)\cdot(1-p_i(n))$$

$$P(\xi_i(n)=1)=P(\xi_{i-1}(n)=1)\cdot p_i(n)$$

DO for $m=1$ to $i-1$

$$P\left(\xi_i(n)=\frac{m}{i}\right)=$$

$$P\left(\xi_{i-1}(n)=\frac{m}{i-1}\right)\cdot(1-p_i(n))$$

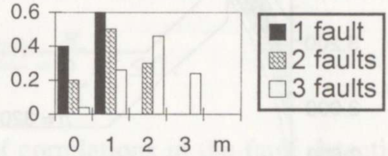
$$+\ P\left(\xi_{i-1}(n)=\frac{m-1}{i-1}\right)\cdot p_i(n)$$

OD

OD

Distribution for 1 to 3 faults

$p_1(n)=0.6$ | $p_2(n)=0{,}5$ | $p_3(n)=0.6$



Distribution for 6, 14 and 26 faults

$p_i(n)$: 2× 0.5; 0.6; 0.7; 2×0.8 | 3×0.4; 3×0.7; 0.8; 0.9 | 0.4; 3×0.5; 6×0.8; 2×0.9

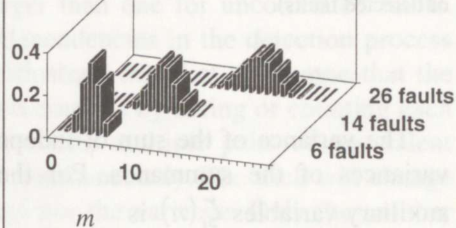


Fig. 1. The algorithm to calculate the distribution of the fault coverage with examples $\xi_i(n)$ – distribution of the fault coverage for i faults; $p_i(n)$ – detection probability of fault i; m – number of detectable faults; M – number of assumed faults; n – number of test patterns.

The example in Fig. 1 shows that the fault coverage converges to a normal distribution with a growing number of faults. A proof presupposing independently detectable faults can be found in [6]. Figure 2 shows the fault coverage of a larger combinational circuit with stuck-at faults. It is also nearly normally distributed.

The mean value of a sum of independent random variables is the sum of the mean values of the summands. The fault coverage is the mean value of the auxiliary variables $\zeta_i(n)$. Thus, the sum has still to be divided by the number of assumed faults

$$E(\xi(n))=\frac{1}{M}\cdot\sum_{i=1}^{M}p_i(n)\approx 1-\frac{1}{M}\sum_{i=1}^{M}\mathrm{e}^{-n\cdot p_i(1)}. \qquad (8)$$

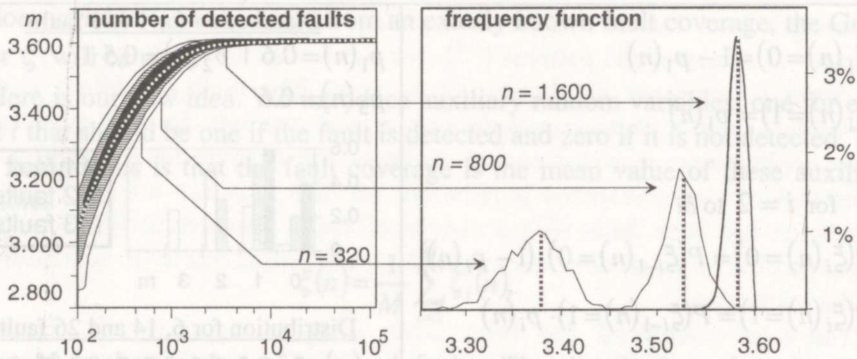


Fig. 2. Fault coverage of the benchmark circuit c3540 [7] (result of a fault simulation with 1,000 different random sequences; number of faults $M = 3{,}605$; n – number of test patterns; m – number of detected faults).

The variance of the sum of independent random variables is the sum of the variances of the summands. For the fault coverage, the mean value of the auxiliary variables $\zeta_i(n)$ is

$$D^2(\xi(n)) = \frac{1}{M^2} \cdot \sum_{i=1}^{M} p_i(n) \cdot (1 - p_i(n)). \tag{9}$$

Without knowing the single detection probabilities, an upper bound of the variance can be given. If all detection probabilities are equal, the variance is maximal:

$$D^2(\xi(n)) \leq \frac{1}{M} \cdot E(\xi(n)) \cdot (1 - E(\xi(n))). \tag{10}$$

Equation (10) has been derived as follows.

Let us substitute all detection probabilities by the sum of the mean value and the difference:

$$p_i(n) = E(\xi(n)) + \delta_i \quad \text{with} \quad E(\xi(n)) = \frac{1}{M} \sum_{i=1}^{M} p_i(n) \quad \text{and} \quad \sum_{i=1}^{M} \delta_i = 0.$$

Inserting this in (10), we obtain

$$D^2(\xi(n)) = \frac{\sum_{i=1}^{M} (E(\xi(n)) + \delta_i)(1 - E(\xi(n)) - \delta_i)}{M^2}$$

$$\leq \frac{E(\xi(n))(1 - E(\xi(n)))}{M} - \sum_{i=1}^{M} \delta_i^2 \leq 0 .$$

Equation (10) allows to measure the effect of correlations in the fault detection process. Let us introduce a parameter ε:

$$\varepsilon = \frac{M \cdot D^2(\xi(n))}{E(\xi(n)) \cdot (1 - E(\xi(n)))} . \quad (11)$$

According to Eq. (10), ε cannot be larger than one for uncorrelated faults. However, it can exceed the bound if interdependencies in the detection process exist. The following train of thought will illustrate this. Let us assume that the number of faults for a given circuit has been doubled by listing or counting each fault twice. This corresponds to a fault set with multiple pairs of equivalent faults or faults that will be detected always simultaneously. The trick will change neither the mean value of the fault coverage nor the variance. Only the number of assumed faults M doubles. Thus ε becomes larger than a set of independent faults. Obviously, by a further increase of the number of equivalent faults, ε may become larger than one.

Column 4 in the table shows the values of ε for the experiment in Fig. 2. Although of each class of equivalent faults only one has been used, the numbers look as if multiple faults would have been detected in each random sequence with the same pattern.

Mean value, standard deviation and ε for a complete stuck-at fault set and for two fault samples (circuit c3540 [7], simulations with 1,000 different random sequences)

	Fault simulation with all 3,606 stuck-at faults			Fault simulation with a sample of 1,000 faults			Fault simulation with a sample of 300 faults		
n	$E(\xi(n))$, %	$\sqrt{D^2(\xi(n))}$, %	ε	$E(\xi(n))$, %	$\sqrt{D^2(\xi(n))}$, %	ε	$E(\xi(n))$, %	$\sqrt{D^2(\xi(n))}$, %	ε
160	88.5	1.28	5.8	88.2	1.41	1.9	89.6	1.87	1.1
320	93.5	0.88	4.6	93.2	1.04	1.7	94.6	1.42	1.2
800	97.6	0.48	3.5	97.5	0.63	1.6	98.4	0.76	1.1
1,600	99.2	0.20	1.8	99.2	0.28	1.0	99.7	0.36	1.2
3,200	99.7	0.08	0.8	99.7	0.11	0.4	99.9	0.11	1.0
6,400	99.8	0.05	0.5	99.8	0.08	0.4	100	0	–

With a growing test length and a growing fault coverage, the interdependencies decrease. All faults with high detection probabilities are detected almost by each random sequence of the corresponding test length. They do not contribute to the variance. Between the faults harder to detect are probably also some interdependencies left. But the effect of the safely detectable faults outweighs them.

3. GUARDBANDS

The fault coverage of a test set is a quality parameter, a manufacturer's guarantee. It must be at least as large as a given lower bound $FC_{\min}$. However, the fault coverage is a random variable which can take a value between zero and one by chance. This leads to a one-side interval estimation

$$P(\xi < FC_{\min}) < \alpha, \tag{12}$$

where α is the error probability.

3.1. Simulation with another random sequence

The fault coverage will be determined by a fault simulation with a random sequence different from those used under test. The number of random patterns should be the same. In this case, the simulation result and the fault coverage are two independent random variables with the same distribution. The variance of the difference of two independent random variables is the sum of the variances, and it doubles. The standard deviation as the square root of the variance increases by the factor $\sqrt{2}$.

As shown in the last section, the fault coverage is often nearly normally distributed. Thus, also the difference is normally distributed. The guardband for a normal random variable must be about two to four times larger than the standard deviation. Multiplied with $\sqrt{2}$, the guardband between the simulation result and the fault coverage that can be guaranteed must be about three to six times larger than the standard deviation. Using the relation between the variance and the mean value in Eq. (11), the guardband has to be

$$G \ge k \cdot \sqrt{\frac{2\varepsilon}{M} \cdot E(\xi(n)) \cdot (1 - E(\xi(n)))} \quad \text{with} \quad \alpha = \Phi(-k), \tag{13}$$

where ε is the parameter to describe the interdependencies; $\Phi(-k)$ is the value of the standardized normal distribution.

3.2. Simulation with a fault sample

The simulation with a fault sample has reduced simulation time. In return, the variance of the simulation result is higher. The distribution becomes broader and flatter (Fig. 3).

The guardband size depends upon whether the simulation is performed with the same or with a different random sequence than the test. Using different random sequences, the simulation result and the fault coverage are two independent random variables. The variances add. Using the upper bounds for the variances, the guardband has to be

$$G_{s,diff} \geq k \cdot \sqrt{\left(\frac{\varepsilon}{M} + \frac{\varepsilon_s}{M_s}\right) \cdot E(\xi(n)) \cdot (1 - E(\xi(n)))}, \quad (14)$$

where M_s is the size of the fault sample; ε_s is ε-value of the fault coverage for the fault sample. If the size of the fault sample M_s is equal to the number of all assumed faults M, Eq. (14) is equal to Eq. (13).

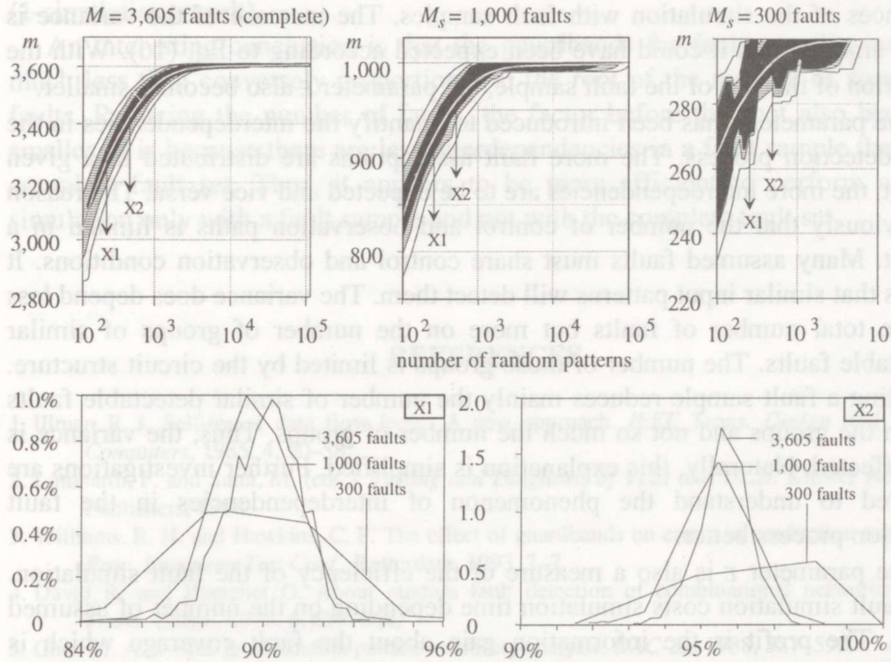


Fig. 3. Distribution of the fault coverage of different samples of stuck-at faults (circuit c3540 [7], simulation with 1,000 random test sequences per fault set).

If the fault simulation is performed by the same random patterns as the test, the simulation result is already the exact coverage for M_s of M faults. This fraction does not contribute to the variance of the difference. In comparison to Eq. (14), the variance of the difference is smaller by the root of the factor $(M - M_s)/M$

$$G_{s,eq} \geq k \cdot \sqrt{\frac{M - M_s}{M} \cdot \left(\frac{\varepsilon}{M} + \frac{\varepsilon}{M_s} \right) \cdot E(\xi(n)) \cdot (1 - E(\xi(n)))}. \tag{15}$$

For a small fault sample $M_s << M$, the variance of the fault coverage can be neglected. The summand ε/M is much smaller than the summand ε_s/M_s, and the factor $(M - M_s)/M$ in Eq. (15) is close to one. The required guardband for small fault samples is

$$G_s \approx k \cdot \sqrt{\frac{\varepsilon_s}{M_s} \cdot E(\xi(n)) \cdot (1 - E(\xi(n)))}. \tag{16}$$

Equation (16) gives the impression that the guardband must be increased conversely proportional to the root of the number of simulated faults. The real proportions are much better. The table shows also the mean values and the variances of the simulation with fault samples. The increase of the variance is much smaller than it could have been expected according to Eq. (16). With the reduction of the size of the fault sample, the parameter ε also becomes smaller.

The parameter ε has been introduced to quantify the interdependencies in the fault detection process. The more fault assumptions are distributed in a given circuit, the more interdependencies are to be expected and vice versa. The reason is obviously that the number of control and observation paths is limited in a circuit. Many assumed faults must share control and observation conditions. It means that similar input patterns will detect them. The variance does depend less on the total number of faults but more on the number of groups of similar detectable faults. The number of those groups is limited by the circuit structure. Selecting a fault sample reduces mainly the number of similar detectable faults within the groups and not so much the number of groups. Thus, the variance is less effected. Naturally, this explanation is simplified. Further investigations are required to understand the phenomenon of interdependencies in the fault detection process better.

The parameter ε is also a measure of the efficiency of the fault simulation. The fault simulation costs simulation time depending on the number of assumed faults. The profit is the information gain about the fault coverage which is conversely proportional to the standard deviation. A conclusion is that a complete fault simulation does not repay. The reduction of the standard deviation is not related to the increase of the number of simulated faults. A fault simulation with a fault sample is more economical.

SUMMARY

The fault coverage of a random test is a normally distributed random variable. An upper bound of the variance, which depends on the mean value and the number of assumed faults, has been found. This bound holds if all assumed faults are independently detectable. Interdependencies in the detection process increase the variance far beyond this bound. Thus, interdependencies can be quantified and measured by simulation experiments.

The guardband is the maximum difference between the result of the fault simulation and the fault coverage of the test. Both the simulation result and the test result are random variables. The variances add. The final result is that the guardband must be approximately

$$G = 4 \ldots 15 \cdot \sqrt{\frac{\text{simulation\_result} \cdot (1 - \text{simulation\_result})}{\text{number\_of\_simulated\_faults}}}\,.$$

In particular, if a fault coverage close to 100% has to be guaranteed, the required guardband is large in comparison to the allowed difference of 100%. It is because the guardband reduces only proportional to the root of the term (1-simulation_result).

An interesting conclusion is that the guardbands for fault samples increase much less than conversely proportional to the root of the number of simulated faults. Reducing the number of faults, the factor before the root also becomes smaller. It is because there are less interdependencies in a fault sample than in a complete fault set. Thus, it appears to be more efficient to perform a fault simulation only with a fault sample and not with the complete fault set.

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STOHHASTILISE TESTIMISE VARUTEGUR

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Stohhastilisel testimisel saavutatavat veakatet on võimalik hinnata vigade simuleerimisega. Kui simuleerimine on tehtud mõne testimisel mittekasutatud stohhastilise jadaga või kasutades vigade etteantud jaotust, tuleb arvesse võtta stohhastilist erinevust simuleerimise tulemuse ja veakatte vahel. Simuleerimise veakatte protsent peab olema suurem nõutavast protsendist. Seda erinevust nimetatakse varuteguriks. Sõltumatult tuvastatavate vigade matemaatilise mudeli põhjal on tuletatud veakatte ja varuteguri jaotused. Hiljem on seda lähenemist korrigeeritud katsetulemuste põhjal. Teoreetiliste tulemuste ja katsetulemuste võrdlus avab stohhastiliste testide omadusi, millele varem pole tähelepanu pööratud. Varuteguri kindlakstegemisel ei saa jätta tähelepanuta korrelatsioone vea avastamise protsessis. On esitatud avaldised varuteguri arvutamiseks.