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MECHANICS AT THE INSTITUTE OF CYBERNETICS

Mati KUTSER

Institute of Cybernetics, Tallinn Technical University, Akadeemia tee 21, 12618 Tallinn, Estonia; Mati.Kutser@cs.ioc.ee

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Abstract. A review of the research in the field of mechanics at the Institute of Cybernetics from 1960 to 2000 is presented. A selection of most important publications is given in the references.

Key words: theory of plates and shells, acoustic echo-signals, wave scattering by elastic objects, nonlinear wave motion, nonlinear dynamics, mathematical modelling of physiological processes, integrated photoelasticity.

1. INTRODUCTION

Research in the field of mechanics has long traditions in Estonia. Many well-known scientists have been working in Tartu University and Tallinn Technical University and their contribution to the progress of mechanical sciences has been remarkable. In this paper the review of research in mechanics is limited with the activities of the Department of Mechanics and Applied Mathematics of the Institute of Cybernetics which was founded 40 years ago by the Estonian Academy of Sciences. Additional information about research in the field of mechanics in Estonia can be found in [1–3].

At the end of the fifties, academician Nikolai Alumäe, who was at that time the head of the Department of Mechanics and Applied Mathematics at the Institute of Energetics of the Estonian Academy of Sciences, started to investigate the problems of the dynamics of elastic shells. The need for complicated calculations in the shell theory was one of the motives, besides the problems of automatics, telemechanics, and operation research, which inspired N. Alumäe to organize a new research centre. The kernel of the Department of Mechanics and Applied Mechanics was formed by Hillar Aben, Leo Ainola, and Uno Nigul.

From the very beginning the studies expanded into two main directions: dynamics of solids and integrated photoelasticity.

2. DYNAMICS OF SOLIDS

2.1. Dynamic problems in the theory of plates and shells

In several areas of technology and structural mechanics detailed knowledge about processes in constructions under dynamic and impact loading is needed. At the Institute of Cybernetics the studies of these processes started with the analysis of the modes and the spectra of free vibrations of thin elastic shells. The possibilities of separated construction of different components of the stress state were elucidated and proved by N. Alumäe. Several new approaches to this problem resulted in elegant solutions explaining the physical essence of the dynamic behaviour of shells [4,5].

In the sixties the studies were devoted to the analysis of linear transient deformation wave processes in plates and cylindrical shells under given moving or impact load along the boundaries and to the construction of improved mathematical models for description of the transient deformation waves in elastic plates and shells.

U. Nigul handled the problem of impact loading using the Fourier method [6]. Later, several problems of the behaviour of the plates and shells under impact loading were treated from the viewpoint of propagation of the transient deformation waves on the basis of the 3D theory of elasticity or improved shell theories. Combined application of analytical and numerical methods was used for the integration of the equations of the theory of elasticity by U. Nigul in his doctoral thesis (1968) and in [7,8]. The axisymmetric transient process of deformation, caused by the fast loading of the boundary, was investigated in shells of revolution within the Timoshenko type shell theory. The jump conditions on the wave fronts were determined and the initial stage of the shell motion was examined by the finite difference method by U. Nigul and Naum Veksler [9].

L. Ainola proposed variational principles for several dynamic problems of mechanics and applied them in the theory of elastic shells and plates [10–13]. The corresponding functionals were obtained with the aid of convolution integrals. Using these methods, the nonlinear and linear equations of the Timoshenko type theory were derived in general coordinates and the high frequency part of the vibration spectrum was analysed. These results were generalized by L. Ainola in his doctoral thesis in 1967.

Based on the results of [14], U. Nigul proposed an asymptotic theory for cylindrical circular shells and used it for the analysis of the accuracy of the Kirchhoff–Love type theories [15,16]. He generalized also the Lur'e symbolic method in the dynamic theory of elastic plates [17] and applied it for the analysis of the accuracy of various approximate plate theories. This method was used also for studying the dynamic bending of the plates [18,19].

The Tallinn seminars on mechanics, organized by N. Alumäe, played an important role in the progress of mechanics in Estonia and persuaded young researchers to take up the problems of dynamics. One of the main topics of these seminars was the propagation of deformation waves.

The waves caused in elastic shells and plates by fast loading on the boundary were considered by N. Alumäe, U. Nigul, and N. Veksler [20–22]. Analysis of transient deformation waves in elastic shells under pressure wave type loading was the following step. Aleksei Tymanok studied wave propagation in cylindrical shells caused by pressure moving along the shell axis with constant speed [23,24]. N. Alumäe proposed a procedure for the analysis of the intensity of elastic waves caused by pressure waves in an elastic spherical shell [25]. Later Mati Kutser in his candidate thesis (1970) and U. Nigul used this procedure for the analysis of the wave propagation in elastic membranes and shells of revolution in the case of a pressure wave moving with decaying speed along the shell [26]. An exact solution of the wave equation, with a right-hand side of the type of an arbitrary convex pressure wave, was derived by U. Nigul [27]. Flutter of plates was considered by Jaan Metsaveer [28].

In the late sixties, engineering practice had set up an interesting problem: do the acoustic echo-signals from elastic underwater objects contain information about the elastic properties of the latter. In the following years a great part of the studies was devoted to the theory of acoustic echo-signals and wave scattering by elastic bodies. At the same time, the studies of the nonlinear wave motion continued with growing intensity.

2.2. Theory of acoustic echo-signals and wave scattering by elastic bodies

The problem under consideration was formulated as a direct linear problem of mathematical physics under following conditions. An elastic object is embedded in an infinite acoustic medium. The object is bounded by a smooth convex surface and may have a cavity. This cavity may contain an elastic or liquid filler. A pulse, generated by an independent source in the medium, meets the elastic object and is scattered. The scattered pressure pulse is calculated in order to find its dependence on the geometrical and physical parameters of the object.

The aim of these studies was theoretical foundation of the algorithms for distant determination of the parameters of different objects through the analysis of the echo-signals. The main attention was paid to the following:

– development of new methods for solving the wave scattering problem for elastic bodies of different shape and internal structure;

– analysis of the physical mechanism of formation of the sound pressure field scattered by several elastic bodies;

– how the information about an elastic body is reflected in the scattered field in space and time.

Several analytical and numerical methods were suggested and applied for the analysis of particular cases of this problem. Traditional methods were applied for solving the two-dimensional problems of scattering of the pulses on spherical and cylindrical bodies using the Laplace or Fourier transform in time and separation of the variables. The series solution for a spherical shell was presented by

J. Metsaveer as a first step [29]. The Sommerfeld–Watson transform was applied to series solution to obtain the high-frequency (J. Metsaveer) and near-front asymptotic solutions (N. Veksler, M. Kutser) [30]. The obtained results were generalized for the analysis of the scattering of pulses on smooth convex bodies of cylindrical shape. New asymptotic methods were proposed for the estimation of the high- and low-frequency sound fields, scattered by smooth convex elastic bodies (J. Metsaveer [31]). Diffraction problems of the acoustic plane waves on the elastic spheres were studied by N. Veksler [32,33]. The methods based on the principles of the ray theory were used for the description of the high-frequency field, and the Bubnov–Galerkin method for the low-frequency field. It was shown that the Sommerfeld–Watson transform technique may be used in case of relatively low frequencies (J. Metsaveer [34,35]). The boundary integral equation method based on the integral Kirchoff formula was generalized for the analysis of the transient waves in fluids by Anatoli Stulov [36–38].

Using the integral transform, methods for the determination of the echo-signals from spherical and cylindrical shells (the empty ones and filled with a liquid) were elaborated for the processes in a large interval of frequencies (J. Metsaveer, J. Pikk [39–41], N. Veksler, M. Kutser [42]).

Analysis of the direct problems by J. Metsaveer and N. Veksler elucidated the influence of several parameters of the elastic body on the scattered field. The regions of frequencies, where the two-dimensional shell theories are valid, were established. A new kind of the radiated sound pulse, caused by waves propagating on the boundary of the thin shell and its liquid filler, was discovered. The resonant nature of the sound pulse scattering was established. Solving a number of direct problems helped to explain the physical mechanism of the sound pulse scattering on smooth convex bodies.

Computer analysis showed several interesting phenomena. The echo-signals consist of a series of pulses retarded in time. Algorithms were elaborated for the determination of the parameters of the object (e.g., thickness, radius, and physical constants of the shell) on the basis of the arrival time and form of these pulses (J. Metsaveer [43–45]). The dependence of the echo-signal on a certain parameter of the object is sufficiently strong for practical usage if the incident pulse has an appropriate form. Therefore an essential element of these algorithms is an iterative procedure of determination of the suitable form of the incident pulse.

Even the short overview of the tackled problems, given above, indicates that the scattering of acoustic pulses from elastic objects and the problems of applying acoustic echo-signals constitute a specific research area. Results in this field formed a basis of two DSc theses (J. Metsaveer – 1979 and N. Veksler – 1982) and of the monograph [46]. In 1982 J. Metsaveer was elected a professor of TTU and his later results are not reflected in this review. N. Veksler's attention was directed mainly to the modal resonance problems and to separating the resonance components. His results from this period on the resonance scattering theory are presented in the monograph [47] and several papers, e.g., [48–51].

2.3. Nonlinear wave motion

Some problems of the geometrically nonlinear shell theory were considered by L. Ainola [52]. Physically and geometrically nonlinear Timoshenko type shell theories were derived and the influence of the nonlinear effects on axisymmetric deformation waves in shells of revolution was analysed by Andres Lahe [53].

In the beginning of the seventies, stress waves in elastic and thermoelastic solids attracted Jüri Engelbrecht's attention. It became evident that in elastic media both nonlinearities, physical and geometrical, should be accounted for simultaneously. Immediately the questions about thermal and viscous effects arose. The governing wave equations are hyperbolic, but the famous Fourier law of thermal effects leads to a parabolic equation with infinite wave speed. Nonlinear theories of thermo-viscoelasticity and thermoelasticity, that take into account the finite velocity of the heat flux, were derived. Following these theories, the evolution of one-dimensional pulses was analysed and the criteria of the shock wave formation and propagation were found [54,55].

The "near field" asymptotic solutions of the one-dimensional quasi-linear wave equation were established making use of the method of successive integration of non-homogeneous linear equations (U. Nigul [56]). The "far field" solutions of quasi-linear systems of equations were found by the modified ray method that leads to a successive nonlinear evolution equation for each wave (J. Engelbrecht [57]). This method took into account the coupling and nonlinear interaction effects and made possible to analyse also one- and two-dimensional shock waves.

In the end of the seventies and beginning of the eighties, the basic attention was paid to the construction of mathematical models for the description of the transient deformation waves in the viscoelastic (hereditary) and active media. The corresponding inverse problems of acoustic evaluation of the properties of such media were studied with the aid of the pulse technique.

It was shown that introducing the modified kernel function, it is possible to construct mathematical models according to which the one-dimensional deformation waves in homogeneous viscoelastic media are described by a first order linear or quasi-linear integro-differential equation instead of the classical second order equations (U. Nigul [58,59]). A method for estimating the kernel characteristics directly from experimental data was elaborated by J. Metsaveer [60]. An asymptotic solution was derived for layered linear media by Arvi Ravasoo and U. Nigul [61]. It was shown that taking into account the nonlinear effects, it is possible to predict weak echo-signals ("noise") from the interface between two media which from the viewpoint of the linear theory are acoustically equivalent.

The traditional model of standard viscoelastic solids does not agree with experimental data about the memory, dispersion, and dissipation processes. Therefore U. Nigul proposed a model of the so-called Ei-memory, using three parameters for describing the memory of the medium [62].

A. Ravasoo investigated the propagation of a one-dimensional longitudinal wave with finite amplitude in inhomogeneous hereditary media with faded memory [63]. Derivation of the constitutive equations of the continuous nonlinear viscoelastic medium [64] paved the way to the idea to extend the possibilities of nondestructive testing by using, besides the wave velocity measurement data, also data about the wave profile evolution [65]. The dependences of the wave characteristics on the parameters of predeformation were determined. These dependences enabled to propose an algorithm for nondestructive evaluation of the inhomogeneous predeformed state of the medium [66,67]. A theoretical model of nonlinear viscoelasticity was checked against experimental data and a modified constitutive equation for nylon threads was proposed [68]. The possibility to distinguish physically homogeneous elastic solids from the homogeneous ones with homogeneous predeformation was demonstrated [69]. A method and an algorithm for acoustodiagnostics of inhomogeneous and inhomogeneously prestressed materials were derived using nonlinear effects by wave interactions [70].

The modified ray method was applied for construction of the two-dimensional evolution equation of wave beams in nonlinear solids, and the corresponding dispersion relations were established by J. Engelbrecht [71]. An efficient numerical algorithm, based on the FFT, for solving one- and two-dimensional evolution equations, was suggested by Tõnu Peipman and J. Engelbrecht [72].

On the basis of the evolution theory (J. Engelbrecht [73]), two-dimensional evolution equations were derived for nonlinear longitudinal (T. Peipman [74]) and transverse (Urmas Valdek [75,76]) waves in solids. U. Valdek derived a novel evolution equation that describes the skew deformation of a harmonic localized shear excitation due to the influence of the second harmonic. The structure of higher harmonics and nonlinear interaction between the waves was explicitly analysed, resulting in a new insight into the contemporary nondestructive testing. In physical experiments, the special pattern of the near field of an ultrasonic transducer must be taken into account and for this purpose the formulation of a model nonlinear problem proved helpful [77]. A more sophisticated mathematical model was elaborated by J. Engelbrecht and Robert Chivers [78,79] for the diagnostics of soft tissues.

In the early eighties, the attention of J. Engelbrecht was focused on the nerve pulse dynamics. The solution of the classical nerve pulse equation has convergence problems. After understanding how the initial telegraph equations were simplified by neglecting certain terms in the parabolic equation, it was clear that in earlier studies something was overlooked. Putting neglected terms back into the initial telegraph equations, the evolution equation was easily derived. It was rather simple but nevertheless there were no problems with convergence and the solution reflected the existence of a threshold, refractory length, etc., all typical properties of a nerve pulse. That gave also an explanation to the existence of the non-oscillating solution for the real nerve pulse [80]. The stationary pulse in a nerve fibre was described by a nonlinear Liénard type equation for which the absence of limit cycles was proved by Teet Tobias [81].

The fact that in seismology some waves are not always attenuated as they should, draw attention to a model explaining the fracture of materials. This model was based on the notion of dilatons that are short-lived microdynamic density fluctuations able to absorb energy from the surrounding medium (J. Engelbrecht, Y. Khamidullin [[82,83]]). When the accumulated energy in a dilaton has reached a certain critical value, the dilaton breaks up, realizing the stored energy. This approach was generalized from microdilatons, characterizing fracture, to macrodilatons that may exist in solids with internal large-scale structure like the block structure of the Earth crust. The dilaton concept was taken into account phenomenologically by means of body forces depending upon the deformation. This leads to a modified Korteweg–de Vries (KdV) equation with right-hand side depending upon the deformation and nonlinearity of longitudinal and shear waves.

The concept of internal variables proved useful by studying complicated dynamic processes. It is known how the damage processes, dynamics of liquid crystals, etc., are described using this concept. The point is that the internal variables have no inertia, therefore their governing equations are parabolic. On the other hand, the wave equation is given in terms of observable variables (stress, deformation) and therefore their governing equations are hyperbolic. That leads to mathematical difficulties when the two processes are coupled and the rates of changes (relaxation times) must be carefully accounted for. This is exactly the process in a nerve fibre where the recovery variables (phenomenological, auxiliary) are internal variables in terms of the continua.

One of the physical effects of microstructures is the dispersion of waves. A general model for a wave in a dispersive medium is the famous KdV equation. The problems of solitary waves were studied from the end of the eighties in collaboration with the University Paris 6. Just before that, some ideas of splitting the solitons were described bearing in mind the control of solitary waves. This was an attempt to rule the process of soliton formation analytically by changing the dispersion parameter so that its influence would split up a soliton into two, etc. [[84]].

All these results were summed up in a theory of asymmetric solitary waves for energetically open systems [[85,86]].

Perturbation method was used to solve the general problem of soliton formation governed by different types of the right-hand sides [[87]]. However, it was clear that a reliable numerical code was to be implemented in order to solve more complicated (nonintegrable) evolution equations. For that Andrus Salupere chose a pseudo-spectral method due to its good accuracy and additional information about the spectral densities at every time step [[88,89]]. The test problem of the KdV equation gave a new result. High accuracy in the analysis showed the existence of short-lived solitons appearing in the process of soliton formation from the harmonic excitation due to the fluctuation reference level. These solitons were called virtual and their existence was shown also by the inverse scattering method, using asymptotics and truncation ideas. Emerging of solitons from the harmonic excitation showed spectral ordering: the maxima of spectral densities over time are ordered along Farey tree distribution with changes due to phase shifts during interaction. An evolution equation for martensitic-austenitic alloys has terms with quartic nonlinearity (c.f. quadratic for the KdV equation) and cubic plus quintic dispersion (c.f. only cubic for the KdV equation). The numerical analysis carried out by A. Salupere [90] showed that several types of positive and negative solitons and multiple solitons can exist depending on the dispersion parameters, and spatio-temporal chaotic behaviour of waves is possible. Typical regions were found for a strong perturbation field in the subspace of parameters of the given KdV system (Pearu Peterson, A. Salupere [91]).

An interesting problem is how to find the amplitudes of the surface waves from an observation of the wave pattern that results from the nonlinear interaction of these waves. It was shown by P. Peterson and E. van Groesen that the problem can be solved. They constructed an explicit unique solution for the case of two interacting waves that were modelled by the Kadomtsev–Petviashvili equation [92], which may be generalized for an arbitrary number of solitons.

2.4. General problems

The experience obtained by analysing nonlinear waves has shown that nonlinear waves and nonlinear oscillations are neighbouring facets of dynamics and have common roots. The coherent structures of spatio-temporal processes were studied in detail, but their chaotic counterparts are not so explicitly analysed yet. J. Engelbrecht in his monograph [93] presented a general philosophical treatment of complexity and simplicity of nonlinear dynamics. The thread of his monograph is the following: simple basic arguments result in a complicated theory that, in turn, needs certain simplifications in order to grasp the physical phenomena involved. Special attention was paid to the description of the sources of nonlinearities.

General theory of nonlinear waves in nonlocal media was summed up and an overview of nonlocal theories in solid mechanics was given by J. Engelbrecht and Manfred Braun [94].

The problems of the thermodynamic theory of complex systems were in the centre of Arkadi Berezovski's interests. The structuring in complex systems was studied and the thermodynamical condition of structuring was derived [95]. An algorithm based on the ideas of cellular automata was developed for the simulation of one-dimensional heat conduction and thermoelasticity problems [96,97]. This algorithm was used for solving several problems of thermomechanics of the continuum. The method of cellular automata was developed also for two-dimensional thermomechanics [98]. A thermodynamically consistent method was derived for analysing the two-dimensional waves in materials with discretely varying parameters [99]. The novelty consists in the representation of integral balance laws of thermoelasticity in terms of contact quantities that describe the non-equilibrium state of discrete elements which represent a continuous medium.

Several interesting results were obtained by Jaan Kalda in diffusion theory [100–103]. The early evolution magnetic field in high energy plasma devices is governed by the Hall effect. It was established earlier that in 2D geometry the magnetic field penetrates into plasma in the form of a shock wave. J. Kalda showed that this phenomenon persists also in the 3D geometry [104]. It was shown that, in the collisionless case, the magnetic field penetrates plasma in the form of an electron vortex and the magnetic field can be nonlinearly enhanced during the penetration [105]. The propagation of a passive tracer in a 2D solenoidal turbulent flow was studied, assuming existence of a certain spectral law. Depending on the parameters, both ordinary and super diffusion is possible. The approach, based on statistical topography of the stream function, was used [106] and the multifractal structure of passively converted scalar fields was studied [107].

The cooperation with Tallinn Piano Factory initiated an investigation of piano hammers. As a result, a new hysteretic model of piano hammers was proposed which showed a good agreement with experimental data (A. Stulov [108,109]). On the basis of this model a device for measuring the piano hammer parameters was constructed. This device gives a possibility to investigate the dynamic force–compression characteristics of the hammer and, using the hereditary (hysteretic) hammer model, to find the hammer parameters by numerical simulation of the dynamic experiments. The analysis of the hammer–string interaction shows that the nonlinear hysteretic model of the piano hammer represents the vibration spectra of the struck strings for real pianos which are closer to measured data than spectra of the nonhysteretic model.

2.5. Mathematical modelling of physiological processes

The investigations of nerve fibre dynamics initiated a wide spectrum of research of the physiological processes. It was shown that mathematical modelling may be successfully used in cardiac research. The studies of the cardiac phenomena were focused on three aspects: 1) regulation of the heart by means of electrical activation of the cardiac conducting system; 2) energy transformation from different chemical forms to mechanical ones by means of oxidative phosphorylation, intracellular energy transport, and mechanical contraction of the myofibrils; and 3) mechanical contraction of the heart wall leading to the efflux of blood into the coronary system.

Modelling of the cardiac conducting system. On the basis of the evolution equation for the nerve pulse, a mathematical model for heart dynamics was derived [110,111]. The main result was successful detection of the non-reentrant bistability on the mathematical model of cardiac Purkinje cell. The bistability of the Purkinje cell seems to be mostly affected by the driving conditions and the level of its supernormality. As the cardiac Purkinje cells mediate the signal delivery from the pace-making nodes to the ventricles, the bistability on the cellular level can in principle induce the bistability on the tissue level leading to the bistability of the cardiac conducting system during tachyarrhythmias. Therefore, according to the results of Olav Kongas, J. Engelbrecht, and Raimo von Hertzen [112–115], the bistability phenomenon can complicate the interpretation of ECG recordings and it should be considered as a factor in modelling on tissue level. Using the Floquet theory, an analysis of the stability and bifurcation of the model was performed. Analytical approximations of the largest Lyapunov exponents, which characterize the stability of a given solution, have been derived. The results were summed up in O. Kongas's PhD thesis in 1998.

Mathematical modelling of intracellular energy fluxes. The energy metabolism is the basis of the cell life. The energy fluxes in cardiac cells were studied on the basis of available data on intracellular diffusion and compartmentation of enzymes and substrates in the cells. The purpose of this research was to develop mathematical models of compartmentalized energy fluxes of living cells, using the results of modern experimental research. The mathematical model of a reaction – diffusion type two-dimensional system was developed by O. Kongas and Marko Vendelin in close cooperation with researchers from other scientific centres [116,117]. This model is based on the synthesis of the Aliev–Saks model of intracellular energy transduction and the Korzeniewsky model of the oxidative phosphorylation. The first results obtained in this project clearly indicate importance of the non-equilibrium state of the creatine kinase system and intracellular diffusion resistance of the outer mitochondrial membrane. It was shown on the basis of the study *in silico* of compartmentalized energy transfer by the phosphocreatine/creatine system that there exist multiple parallel regulatory factors controlling the rate of oxygen consumption in dependence of the workload [118]. A model that describes quantitatively the published experimental data on dependence of the rate of oxygen consumption and metabolic state of a working isolated perfused rat heart on workload over its physiological range was developed [119].

Mechanical contraction of the cardiac muscle. The simulation of the heart wall contraction and energy consumption has to be based on a good mathematical description of the properties of the heart muscle tissue, active stress development, and energy consumption by the heart muscle. Since the mechanical deformation is a macroscopic phenomenon, a macroscopic mathematical model of the mechanical contraction and intracellular energy turnover is required to describe the experimental results. The basic assumption used in the model is that the stress developed in the heart muscle tissue may be divided into two components: active and passive. The passive stress component is determined by elastic response of the tissue to the deformation. The active stress is generated by muscle fibres. The description of the active stress is based on the Huxley-type equations and on the general mechanochemical formalism of the cross-bridge model of the muscle contraction (M. Vendelin, J. Engelbrecht [120]). The governing parameters were obtained by comparing the theoretical solution with experimental data, measured in several scientific centres (isometric stress, quick-release shortening velocity, and muscle shortening during isotonic contraction).

Besides the cardiac phenomena, the tree-like fractal biological networks were studied. A fractal model of the human blood-vessel system was proposed as a generalized Scheiddeger's model of rivers, and its fractal properties were determined by J. Kalda [121–123]. Transport processes in biological fractal structures have been analysed and the governing scaling laws established. Fractal dimension of blood-vessel systems was calculated and the propagation of infection in the lung analysed. Similarity dimensions were calculated for biological tree-like structures. Optimal way for Monte-Carlo calculation of fractal dimensions was studied [124].

A method for analysis of heart rate variability was proposed, based on the theory of Holter-monitoring, using the theory of fractals. A new Zipf-law-based multiscaling behaviour of the heart rate variability was discovered [125].

3. INTEGRATED PHOTOELASTICITY

The aim of the investigations in the field of photoelasticity has been elaboration of methods which allow the 3D state of stress to be determined on the basis of integral optical measurements, similarly to tomography. In his candidate thesis H. Aben used the method of photoelasticity for investigating the stress state in buckled plates [126]. Trying to interpret the photoelastic measurement data in this particular case, H. Aben and Endel Saks had to deal with a complicated phenomenon. Namely, in the buckled plates two stress fields are present: membrane stresses and bending stresses. Since in the general case the principal stress directions of these fields do not coincide, a rotation of the principal stress directions through the plate thickness takes place. Accordingly, the principal directions of the birefringence, caused by stresses, also rotate through the plate thickness [127, 128]. Rotation occurs also in shells and in three-dimensional stress states.

Since at this time three-dimensional problems were experimentally mostly investigated using the frozen stress method, the rotation of the principal stress directions was usually ignored and theoretical aspects related to this phenomenon had been only superficially investigated. In integrated photoelasticity, rotation of the principal stress and birefringence directions cannot be ignored and this phenomenon causes a lot of complications by interpreting the measurement data. Actually, three-dimensional photoelastic models belong to the so-called twisted birefringent media [129].

By investigating optical phenomena in twisted birefringent media, H. Aben and E. Saks showed that there always exist two perpendicular directions of the polarizer by which the light emerging from the model is linearly polarized [126,128]. Experimental determination of these so-called characteristic directions gives information about the state of stress of a three-dimensional model. The theory of the method of characteristic directions was elaborated and several applications were considered (investigation of stress in shells and plates [130], determination of the initial birefringence in twisted fibres [131], etc.). The main results of this theory were presented in H. Aben's doctoral thesis (1966).

In the sixties Aivo Saar and Maie Uffert participated in developing methods for investigating shells of revolution. Namely, A. Saar elaborated a method for residual stress measurement in shells of revolution, including sandwich shells [132] which is important in glass industry by manufacturing sandwich lamp shades.

In the end of sixties it was shown that if the photoelastic models are investigated in a magnet field, due to the Faraday effect, qualitatively new possibilities arise to determine states of stress which are not constant on the wave normal (so-called magnetophotoelasticity) [133–138]. For investigation of the possibilities of magnetophotoelasticity, a magnetooptical polariscope was designed and built [135].

In the seventies Siim Idnurm took actively part in elaboration of the theory of magnetophotoelasticity and applying it for investigation of stress concentration in plates under cylindrical bending. Application of other physical fields (the Kerr effect and the Cotton–Mouton effect) in integrated photoelasticity was also considered [139].

Edvard Brosman developed a method for the determination of the stress distribution in cubic single crystals of cylindrical and prismatic form, using for that method of integrated photoelasticity. This method found application by evaluating the quality of KCl and NaCl single crystals used as scintillators in space research [140]. Scattered light method was elaborated for stress measurement in cubic single crystals by Jüri Josepson [141].

The results of the investigations in the field of integrated photoelasticity have been summarized in H. Aben's monograph [142].

In integrated photoelasticity, the test object is a three-dimensional inhomogeneous and birefringent continuum which transforms in a certain manner the polarization of light. Polarization transformations are most complicated when the principal stress directions rotate along the light beam. In polarization optics, optical systems which consists of birefringent plates and rotators (rotator is an optical element which rotates the plane of polarization), are often used. Several theoretical results obtained in integrated photoelasticity can be applied also for discrete optical systems. The results involve a theory of the artificial quarter-wave plates [143,144], theory of a defective polariscope [145], general theory of the pile of birefringent plates [146], etc.

Since the specimen in integrated photoelasticity is inhomogeneous, certain bending of the light rays takes place. Usually this phenomenon has been considered as a source of errors. At the same time, measurement of the bending of the light rays gives also additional information about the stress field. This is named gradient photoelasticity [147]. Integrated gradient photoelasticity was used for complete determination of stresses in planes of symmetry of axisymmetric bodies by H. Aben and Kalle-Jüri Kell [148,149]. Bending of the light rays in tempered drinking glasses was also investigated [150].

In the general case interpretation of the measurement data in integrated photoelasticity is complicated. It was shown that in the case of weak birefringence it is possible to determine on each light ray the average direction of the principal stresses and the integral optical retardation (H. Aben, J. Josepson, K.-J. Kell [151–153]). Using these data, for each light ray it is possible to calculate two integrals: of the normal stress difference and of the shear stress in the plane perpendicular to the light ray. These integrals permit to obtain information about some components of the axisymmetric (H. Aben, S. Idnurm, Alfred Puro [154]) and, also of the general 3D stress distribution. The methods developed have been used for the determination of stresses in various glass products like optical fibre preforms [155], bottles, drinking glasses, etc. [156].

Since in the case of weak birefringence stress measurement in integrated photoelasticity is based on two integrals of the stress components, analogy between integrated photoelasticity and tomography is evident. However, stress field tomography has many specific features. Classical tomography is scalar field tomography, i.e., every point of the field is characterized by a scalar. Since stress is a tensor, integrated photoelasticity is actually optical tensor field tomography [151,152]. Application of the magnetic field in the tensor field tomography has been considered by A. Puro [157]. Usually it is assumed that residual stress distribution in axisymmetric glass products is axisymmetric. The measurements have shown that in axisymmetric glass products residual stress distribution deviates from the axisymmetric one, often considerably. A method for taking this phenomenon into account was elaborated by Johan Anton [158].

Optical equations of integrated photoelasticity are in the general case nonlinear. Therefore the behaviour of the characteristic directions and characteristic optical retardation are somehow unpredictable and by recording integrated fringe patterns a curious optical phenomenon was observed by H. Aben and J. Josepson. Namely, besides interference fringes sometimes also interference blots appear [159,160]. Interference blots are areas where the interference fringes lose their contrast or disappear at all. It has been shown that the reason of appearance of the interference blots is rotation of the principal stress axes. It is curious that the number of fringes, which enter into an interference blot, is different from the number of fringes that emerge from the latter. This phenomenon is named fringe dislocation and it is related to optical vortices. Theory of the interference blots and fringe dislocations was elaborated by L. Ainola and H. Aben [161].

Algorithms of integrated photomechanics for the interpretation of experimental data in the case of axisymmetric problems of viscous flow and plastic deformation have been elaborated by A. Puro, H. Aben, L. Ainola and Karl-Hans Laermann [162–165].

The laboratory of photoelasticity has wide experience in using the methods of integrated photoelasticity for determining stresses in glass objects. This experience has been generalized in a monograph by H. Aben and Claude Guillemet [156]. Some specific problems are considered in [166].

An automatic computer controlled polariscope for stress measurement in axisymmetric glass articles has been elaborated by J. Anton [167]. For correct interpretation of the measurement data obtained with the automatic polariscope, an original version of the phase-stepping method was elaborated [168]. This method gives unambiguously the value of the optical retardation and the direction of the first principal stress if optical retardation is less than half the wave-length. Polariscopes, manual or automatic, have been installed in several glass factories and university laboratories in Italy, France, Holland, and Japan.

For complete determination of complicated stress fields it is often reasonable to use hybrid mechanics, i.e. a combination of experimental and analytical-numerical methods. Idea of hybrid mechanics lies in measuring distribution of some stress or displacement components on the test object, and in calculating other components by using the measurement data and the relationships of the theory of elasticity. Integrated photoelasticity allows to determine nondestructively in axisymmetric objects the distributions of the axial and the shear stress. A hybrid mechanics method for complete residual stress measurement in axisymmetric glass articles was elaborated. The method is based on the generalized sum rule that relates axial, radial and circumferential stress, and contains also an integral of the gradient of the shear stress (H. Aben, L. Ainola [169]). The possibilities of this method have been demonstrated in several papers [170,171]. A. Puro and K.-J. Kell have elaborated a hybrid technique for complete residual stress determination in optical fibre preforms of arbitrary cross-section [155].

H. Aben and L. Ainola have shown that there exists a duality relationship between well-known equations that describe transformation of the polarization of light in twisted birefringent media. This duality relationship is induced from the duality between the two different parametric representations of the unimodular matrix that describes the transformation of the state of polarization in a twisted birefringent medium [172]. This work helps to understand better the nonlinear phenomena that appear in integrated photoelasticity.

4. SUMMARY

The results of studies in various fields of mechanics have been presented in numerous papers and generalized in 20 monographs and books. The following monographs should be picked out [46,47,59,73,85,86,93,142,156]. The monographs by J. Engelbrecht [86] and H. Aben [156] were awarded the Estonian Science Prize.

This overview shows how deep analysis, started four decades ago in one field of science, has initiated a successive generation of new ideas. “Problems should be difficult, then investigators are happy” – this famous saying by N. Alumäe has been carried along for all these years. Nowadays the studies in mechanics have direct links with many other fields of science – biophysics and cardiology, fractality and signal analysis, solitonics and microstructrured materials – proving again the importance of interdisciplinarity. The traditional fields – photoelasticity, impact analysis, continuum theory – involve new problems. In 1999, at the Institute of Cybernetics a Centre for Nonlinear Studies (CENS) was founded, uniting the research within the Institute with studies of other colleagues in Tartu

University, the Centre of Biomedical Engineering of Tallinn TU, and Estonian Marine Institute (see http://cens.ioc.ee). CENS has an International Advisory Board, many international contacts, and what is actually the most important facet, many young people.

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REFERENCES

1. *Ehitusinsenerid Tallinna Polütehnilisest Instituudist.* Valgus, Tallinn, 1986.
2. *Mehaanika meil ja mujal* (Engelbrecht, J., ed.). Valgus, Tallinn, 1992.
3. Lepik, Ü. *Tartu Ülikool minu elus.* Vanemuise seltsi kirjastus, Tartu, 1998.
4. Alumäe, N. On the decomposition of the stress state in axisymmetrical problems of motion for circular cylindrical shell. *Eesti NSV Tead. Akad. Toim. Füüs.-matem. tehn.*, 1961, **10**, 3, 171–181 (in Russian).
5. Alumäe, N. On the fundamental system of integrals of the equation of small steady axisymmetrical vibrations of elastic conical shell. *Eesti NSV Tead. Akad. Toim. Tehn. füüs.-matem.*, 1960, **9**, 1, 3–15 (in Russian).
6. Nigul, U. K. Vibrations of the circular cylindrical elastic shell excited by a concentrated pulse. *Tallinna Polütehn. Inst. Toim.*, 1960, A 171, 37–57 (in Russian).
7. Nigul, U. On methods and results of investigation on transient bending waves in an elastic plate. *Eesti NSV Tead. Akad. Toim. Füüs.-matem. tehn.*, 1965, **14**, 3, 345–384 (in Russian).
8. Nigul, U. Regions of effective application of the methods of three-dimensional and two-dimensional analysis of transient stress waves in shells and plates. *Int. J. Solids Struct.*, 1969, **5**, 607–627.
9. Veksler, N. D, Männil, A. J., and Nigul, U. K. Application of the networks method and Timoshenko type theory for investigation of transient deformation wave processes in finite plates. *Prikl. Mekh.*, 1965, 1, 38–49 (in Russian).
10. Ainola, L. On Castigliano's variational principle in the theory of dynamics of the finite elastic deformation. *Eesti NSV Tead. Akad. Toim. Füüs.-matem. tehn.*, 1961, **10**, 1, 22–27 (in Russian).
11. Ainola, L. The nonlinear Timoshenko type theory of elastic shells. *Eesti NSV Tead. Akad. Toim. Füüs.-matem. tehn.*, 1965, **14**, 3, 337–344 (in Russian).
12. Ainola, L. J. On geometrically nonlinear theories of elastic plates for dynamical problems. *Prikl. Mekh.*, 1965, 1, 7–16 (in Russian).
13. Ainola, L. J. Integral variational principle for mechanics. *Prikl. Mat. Mekh.*, 1966, **30**, 5, 946–949 (in Russian).
14. Nigul, U. K. Linear equations free from hypothesis for dynamics of a circular cylindrical shell. *Tallinna Polütehn. Inst. Toim.*, 1960, A 176, 1–68 (in Russian).
15. Nigul, U. K. Asymptotic theory of statics and dynamics of elastic circular cylindrical shells. *Prikl. Mat. Mekh.*, 1962, **26**, 5, 923–930 (in Russian).
16. Nigul, U. K. Asymptotic theory of statics and dynamics of elastic circular cylindrical shells and analysis of the accuracy of several versions of the Kirchhoff–Love theory. In *Proc. 4th All-Union Conference on Theory of Shells and Plates.* Erevan, 1964, 738–742 (in Russian).

17. Nigul, U. K. On the application of the A. I. Lur'e symbolic method to analysis of stress states and two-dimensional theories of elastic plates. *Prikl. Mat. Mekh.*, 1963, **27**, 3, 583–588 (in Russian).
18. Nigul, U. On the application of A. I. Lur'e symbolic method in the three-dimensional theory of motion of elastic plates. *Eesti NSV Tead. Akad. Toim. Füüs.-matem. tehn.*, 1963, **12**, 2, 146–155(in Russian).
19. Kutser, M. and Nigul, U. On the application of the A. I. Lur'e symbolic method in the theory of motion of plates for deformation symmetric about the middle plane. *Eesti NSV Tead. Akad. Toim. Füüs.-matem. tehn.*, 1965, **14**, 3, 385–392 (in Russian).
20. Alumäe, N. and Poverus, L. Transient stresses in a semi-infinite elastic cylindrical shell under nonaxial loading. *Eesti NSV Tead. Akad. Toim. Füüs.-matem. tehn.*, 1963, **12**, 1, 13–23 (in Russian).
21. Nigul, U. K. Application of three-dimensional theory of elasticity to the analysis of the wave process of bending of semi-infinite plate under instant boundary loading. *Prikl. Mat. Mekh.*, 1963, **27**, 6, 1044–1056 (in Russian).
22. Veksler, N. D. and Nigul, U. K. On the theory of wave processes of axisymmetric deformation of a spherical shell. *Mekh. Tverd. Tela*, 1966, 1, 74–78 (in Russian).
23. Tymanok, A. On transient axisymmetric motion of membrane cylindrical shell under moving load. *Mekh. Mashinostr.*, 1964, 4, 47–53 (in Russian).
24. Tymanok, A. Transient axisymmetric oscillation of cylindrical shells excited by a moving load. *Eesti NSV Tead. Akad. Toim. Füüs.-matem. tehn.*, 1965, **14**, 3, 414–422 (in Russian).
25. Alumäe, N. A. Discontinuities in accelerations of a spherical shell caused by the plane pressure wave. In *Proc. 6th All-Union Conference on Theory of Shells and Plates*. Baku, 1966, 44–47 (in Russian).
26. Kutser, M. E. and Nigul, U. K. A method for the analysis of discontinuities on wavefronts, caused by pressure waves in membranes and shells. *Prikl. Mat. Mekh.*,1969, **33**, 4, 609–621 (in Russian).
27. Nigul, U. Plane stress waves in membranes caused by an arbitrary pressure wave. *J. Acoust. Soc. Am.*, 1972, **51**, 1, part 2, 241–248.
28. Metsaveer, J. On the flutter of fixed plates. *Mekh. Tverd. Tela*, 1969, 4, 179–180 (in Russian).
29. Metsaveer, J. On the scattering of waves from spherical shells in acoustic medium. *Eesti NSV Tead. Akad.Toim. Füüs. Matem.*, 1970, **19**, 4, 415–422 (in Russian).
30. Nigul, U. K., Metsaveer, J. A., Veksler, N. D., and Kutser, M. E. *Echo-Signals from Elastic Objects, II.* Valgus, Tallinn, 1974 (in Russian).
31. Metsaveer, J. A. Calculation of the echo-signal from noncircular cylindrical shell. *Akust. Zh.*, 1976, **22**, 6, 906–913 (in Russian).
32. Veksler, N. D. Diffraction of a plane acoustic wave on the hollow elastic sphere. *Akust. Zh.*, 1975, **21**, 5, 694–700 (in Russian).
33. Veksler, N. D. Axisymmetric diffraction of a plane acoustic wave on the closed thin elastic sphere. *Mekh. Tverd. Tela*, 1975, 1, 69–78 (in Russian).
34. Metsaveer, J. Echo-signal of a finite spherical pulse from elastic cylindrical shell. *Prikl. Mat. Mekh.*, 1973, **37**, 2, 274–284 (in Russian).
35. Metsaveer, J. Echo-signals from a cylindrical shell with an arbitrary cross-section. *Eesti NSV Tead. Akad. Toim. Füüs. Matem.*, 1977, **26**, 2, 173–177 (in Russian).
36. Stulov, A. Diffraction of an acoustical pulse by a rigid elliptical cylinder. *Eesti NSV Tead. Akad. Toim. Füüs. Matem.*, 1977, **26**, 2, 222–225 (in Russian).
37. Stulov, A. Diffraction of an acoustical pulse by an elastic elliptical cylinder. *Eesti NSV Tead. Akad. Toim. Füüs. Matem.*, 1977, **26**, 3, 313–316 (in Russian).
38. Stulov, A. S. Calculation of the echo-signal from a noncircular cylinder. *Mekh. Tverd. Tela*, 1977, 4, 197–198 (in Russian).
39. Metsaveer, J. Calculation of the echo-signal of a directed pulse from the spherical shell. *Akust. Zh.*, 1976, **22**, 6, 939–941 (in Russian).

40. Metsaveer, J. and Pikk, J. Echo-signal from a cylindrical shell filled with liquid, caused by a pulse of finite length. *Eesti NSV Tead. Akad. Toim. Füüs. Matem.*, 1976, **25**, 3, 260–268 (in Russian).
41. Metsaveer, J. A. and Pikk, J. P. Echo-signal of a finite spherical pulse from a cylindrical shell filled with liquid. *Prikl. Mat. Mekh.*,1976, **40**, 6, 648–654 (in Russian).
42. Veksler, N. D. and Kutser, M. E. Asymptotics of the pressure field in the definite jump for diffraction of a plane acoustic wave on the elastic cylindrical shell. *Prikl. Mat. Mekh.*, 1976, **40**, 3, 509–519 (in Russian).
43. Metsaveer, J. A. and Nigul, U. K. Determination of the thickness and radius of a spherical shell by echo-signals. *Mekh. Tverd. Tela*, 1972, 6, 60–66 (in Russian).
44. Metsaveer, J. A. Theoretical basis of algorithms for determination of parameters of spherical and cylindrical shells by echo-signals. *Akust. Zh.*, 1975, **21**, 1, 141–143 (in Russian).
45. Metsaveer, J. A. On the determination of parameters of liquid filled shells by echo-signals. *Akust. Zh.*, 1977, **23**, 1, 170–171 (in Russian).
46. Metsaveer, J. A., Veksler, N. D., and Stulov, A. S. *Diffraction of Acoustic Pulses on Elastic Bodies*. Nauka, Moscow, 1979 (in Russian).
47. Veksler, N. D. *Resonance Acoustic Spectroscopy*. Springer-Verlag, Berlin, 1993.
48. Veksler, N. Modal resonances in problems of acoustic waves scattering by elastic shells. *Proc. Estonian Acad. Sci. Phys. Math.*, 1993, **42**, 1, 14–21.
49. Veksler, N. D. Intermediate procedures in resonance scattering theory. *Acustica*, 1994, **80**, 1, 35–41.
50. Veksler, N. D. and Izbicki, J.-L. Modal resonances of the Franz waves. *Acustica*, 1995, **81**, 6, 612–622.
51. Veksler, N. D. and Izbicki, J.-L. Modal resonances of peripheral waves. *ACUSTICA. Acta Acust.*, 1996, **82**, 401–410.
52. Ainola, L. J. Variational principles of dynamics of the shell theory. *Dokl. AN SSSR*, 1967, **172**, 6, 1296–1298 (in Russian).
53. Lahe, A. J. Physically and geometrically nonlinear theory of elastic shells of revolution under axisymmetric deformation. In *Proc. Symposium "Nonlinear and Thermal Effects in Transient Wave Processes"*. Gorkij–Tallinn, 1973, 3–10 (in Russian).
54. Engelbrecht, J. The formation of shock waves in thermoelastic media. In *Proc. 2nd Bulgarian National Congress on Theoretical and Applied Mechanics (Varna, 1973)*. Sofia, 1975, 154–161.
55. Nigul, U. and Engelbrecht, J. Evolution of the finite pulses propagating in viscoelastic media due to nonlinear, viscous and thermal effects. In *Proc. IUTAM Symposium on the Mechanics of Viscoelastic Media and Bodies (Gothenburg, 1974)*. Berlin et al., 1975, 119–130.
56. Nigul, U. K. Deviation of the solution of the quasilinear wave equation from the solution of the linear equation in the region of continuous first derivatives. *Prikl. Mat. Mekh.*, 1973, **37**, 3, 434–447 (in Russian).
57. Engelbrecht, J. One-dimensional deformation waves in nonlinear viscoelastic media. *Wave Motion*, 1979, **1**, 1, 65–74.
58. Nigul, U. K. Asymptotic description of the process of weak distortion of the shape of the longitudinal wave in the nonlinear hereditary elastic medium. *Dokl. AN SSSR*, 1979, **247**, 1, 60–63 (in Russian).
59. Nigul, U. K. *Non-linear Acoustic Diagnostics. One-dimensional Problems.* Korablestroenie, Leningrad, 1981 (in Russian).
60. Metsaveer, J. On dynamic models of viscoelastic media. *Dokl. AN SSSR*, 1981, **260**, 3, 564–566 (in Russian).
61. Nigul, U. K. and Ravasoo, A. A. Application of the second asymptotic approximation of the pulse distortion in acoustodiagnostics of a layer on the half-space. *Prikl. Mat. Mekh.*, 1979, **43**, 5, 933–939 (in Russian).
62. Nigul, U. K. and Ravasoo, A. Transient longitudinal waves in an inhomogeneous medium with Ei-memory. *Mekh. Tverd. Tela*, 1991, 2, 66–74 (in Russian).

63. Ravasoo, A. Propagation of non-linear waves in inhomogeneous hereditary media. *Int. J. Non-Linear Mech.*, 1989, **24**, 1, 57–64.
64. Ravasoo, A. Some remarks on the quasi-linear theory of viscoelasticity. *Proc. Estonian Acad. Sci. Phys. Math.*, 1991, **40**, 2, 121–129.
65. Ravasoo, A. Transient longitudinal waves in inhomogeneously predeformed viscoelastic medium. *Mekh. Tverd. Tela*, 1993, 6, 91–99.
66. Ravasoo, A. Ultrasonic nondestructive evaluation of inhomogeneous plane strain in elastic medium. *Res. Nondestruct. Eval.*, 1995, **7**, 1, 55–68.
67. Ravasoo, A. Nonlinear longitudinal waves in inhomogeneously predeformed elastic media. *J. Acoust. Soc. Am.*, 1999, **106**, 6, 3143–3149.
68. Blanc, R. H. and Ravasoo, A. On the nonlinear viscoelastic behaviour of nylon fiber. *Mech. Mater.*, 1996, **22**, 301–310.
69. Ravasoo, A. Nonlinear waves in characterization of inhomogeneous elastic material. *Mech. Mater.*, 1998, **31**, 3, 205–213.
70. Ravasoo, A. and Braunbrück, A. Nonlinear interaction of longitudinal waves in elastic material. *Proc. Estonian Acad. Sci. Phys. Math.*, 1999, **48**, 3/4, 252–264.
71. Engelbrecht, J. Two-dimensional nonlinear evolution equations: The derivation and the transient wave solution. *Int. J. Non-Linear Mech.*, 1981, **16**, 2, 199–212.
72. Peipman, T. and Engelbrecht, J. Two-dimensional evolution equations of transient waves – an algorithm for analysis by means of Fourier transform. *Acta Comment. Univ. Tartuensis*, 1982, **627**, 107–112 (in Russian).
73. Engelbrecht, J. *Nonlinear Wave Processes of Deformation in Solids.* Pitman, London, 1983.
74. Peipman, T. Propagation of two-dimensional deformation waves in inhomogeneous medium. *Acta Comment. Univ. Tartuensis*, 1987, **772**, 150–157 (in Russian).
75. Valdek, U. A. and Engelbrecht, J. K. On asymptotic description of nonlinear longitudinal and transverse deformation waves in half-space. *Mekh. Tverd. Tela*, 1986, 4, 101–105 (in Russian).
76. Peipman, T., Valdek, U., and Engelbrecht, J. Nonlinear two-dimensional longitudinal and shear waves in solids. *Acustica*, 1992, **76**, 1, 84–94.
77. Feldman, M. J. and Engelbrecht, J. K. On calculation of the near field of an ultrasonic transducer. *Akust. Zh.*, 1987, **33**, 2, 367–369 (in Russian).
78. Engelbrecht, J. K. and Chivers, R. C. An evolution equation approach for modelling nonlinear ultrasonic processes in tissue. *Acoustic Lett.*, 1988, **11**, 7, 125–130.
79. Engelbrecht, J. and Chivers, R. Evolution equations and ultrasonic wave propagation in biological tissues. *Phys. Med. Biol.*, 1989, **34**, 11, 1571–1592.
80. Engelbrecht, J. On the theory of pulse transmission in a nerve fibre. *Proc. R. Soc. Lond. A, Math. Phys. Sci.*, 1981, **A375**, 195–209.
81. Engelbrecht, J. and Tobias, T. On a model stationary nonlinear wave in an active medium. *Proc. R. Soc. Lond. A, Math. Phys. Sci.*, 1987, A411, 139–154.
82. Engelbrecht, J. K. and Khamidullin, Y. N. Dilaton mechanism for the amplification of seismic waves. *Dokl. AN SSSR*, 1989, **308**, 1, 71–74 (in Russian).
83. Engelbrecht, J. and Khamidullin, Y. On the possible amplification of nonlinear seismic waves. *Phys. Earth Planet. Inter.*, 1988, **50**, 1, 39–45.
84. Engelbrecht, J. On the KdV soliton splitting. *Wave Motion*, 1987, **9**, 2, 127–132.
85. Engelbrecht, J., Fridman, V., and Pelinovski, E. *Nonlinear Evolution Equations.* Longman, London, 1988.
86. Engelbrecht, J. *An Introduction to Asymmetric Solitary Waves.* Longman, Harlow, 1991.
87. Engelbrecht, J. A note on hierarchical equations of the KdV soliton. *C. R. Acad. Sci.*, 1992, II, **314**, 1273–1278.
88. Salupere, A., Maugin, G. A., and Engelbrecht, J. Korteweg–de Vries soliton detection from a harmonic input. *Phys. Lett. A.*, 1994, **192**, 5–8.
89. Salupere, A. On the application of the pseudospectral method for solving the Korteweg–de Vries equation. *Proc. Estonian Acad. Sci. Phys. Math.*, 1995, **44**, 1, 73–87.

90. Salupere, A., Maugin, G. A., Engelbrecht, J., and Kalda, J. On the KdV soliton formation and discrete spectral analysis. *Wave Motion*, 1996, **23**, 1, 49–66.
91. Peterson, P. and Salupere, A. Solitons in a perturbed Korteweg–de Vries system. *Proc. Estonian Acad. Sci. Phys. Math.*, 1997, **46**, 1/2, 102–110.
92. Peterson, P. and van Groesen, E. A direct and inverse problem for wave crests modelled by interactions of two solitons. *Physica D*, 2000, **141**, 3–4, 316–332.
93. Engelbrecht, J. *Nonlinear Wave Dynamics: Complexity and Simplicity*. Kluwer, Dordrecht et al., 1997.
94. Engelbrecht, J. and Braun, M. Nonlinear waves in nonlocal media. *Appl. Mech. Rev.*, 1998, **51**, 8, 475–788.
95. Berezovski, A. and Rozenblum, V. Basic concepts of thermodynamics of complex systems. *Proc. Estonian Acad. Sci. Phys. Math.*, 1993, **42**, 2, 178–194.
96. Berezovski, A. Simulation of nonlinear heat conduction by means of thermodynamics based algorithm. *Int. J. Numer. Methods Heat Fluid Flow*, 1997, **7**, 7, 711–721.
97. Berezovski, A. Continuous cellular automata for simulation of thermoelasticity. *Proc. Estonian Acad. Sci. Phys. Math.*, 1997, **46**, 1/2, 5–43.
98. Berezovski, A. Modelling of two-dimensional elastic wave propagation with continuous cellular automata. *Proc. Estonian Acad. Sci. Eng.*, 1999, **5**, 2, 112–121.
99. Berezovski, A. and Maugin, G. A. Material formulation of finite-strain thermoelasticity and applications. *J. Therm. Stresses*, 1999, **22**, 4/5, 421–449.
100. Gruzinov, A. V., Isichenko, M. B., and Kalda, J. L. Two-dimensional turbulent diffusion. *Zh. Eksp. Teor. Fiz.*, 1990, **70**, 2, 263–269 (in Russian).
101. Isichenko, M. B. and Kalda, J. L. Anomalous resistance of randomly inhomogeneous Hall media. *Zh. Eksp. Teor. Fiz.*, 1991, **72**, 1, 126–132.
102. Isichenko, M. B. and Kalda, J. Statistical topography. Part I. Fractal dimension of coastal lines and number-area rule for islands. *J. Nonlin. Sci.*, 1991, **1**, 3, 255–277.
103. Isichenko, M. B. and Kalda, J. Statistical topography. Part II. 2D transport of passive scalar. *J. Nonlin. Sci.*, 1991, **1**, 4, 375–397.
104. Kalda, J. On the fast penetration of magnetic field into plasma. *Phys. Fluids B*, 1993, **5**, 12, 1–6.
105. Kalda, J. On transport of passive scalar in 2D turbulent flow. In *Transport, Chaos and Plasma Physics* (Benkadda, S., Doveil, F., and Elskens, Y., eds.). World Scientific, Singapore, 1994, 2720–2725.
106. Kalda, J. Fast penetration of magnetic field into a collisionless plasma. *Phys. Rev. E.*, 1996, **54**, 2, 1824–1828.
107. Kalda, J. Simple model of intermittent passive scalar turbulence. *Phys. Rev. Lett.*, 2000, **84**, 3, 471–474.
108. Stulov, A. A simple grand piano hammer felt model. *Proc. Estonian Acad. Sci. Eng.*, 1995, **1**, 2, 172–182.
109. Stulov, A. Hysteretic model of the grand piano hammer felt. *J. Acoust. Soc. Am.*, 1995, **97**, 4, 2577–2585.
110. Engelbrecht, J. and Kongas, O. Mathematical modelling of the heartbeat. *Proc. Estonian Acad. Sci. Phys. Math.*, 1993, **42**, 1, 124–127.
111. Kongas, O. and Engelbrecht, J. Bifurcation diagram of the driven asymmetric van der Pol equation. *Proc. Estonian Acad. Sci. Phys. Math.*, 1994, **43**, 2, 123–126.
112. Engelbrecht, J. and Kongas, O. Driven oscillators in modelling of heart dynamics. *Appl. Anal.*, 1995, **57**, 1/2, 119–144.
113. Kongas, O. Stability and torsion in period doubling cascade. *Phys. Lett.*, 1998, **241A**, 3, 163–167.
114. Kongas, O. A global map of local bifurcations. In *Proc. IUTAM Symposium "Synthesis of Nonlinear Dynamical Systems"* (Lavendelis, E., ed.). Dordrecht, 1999, 179–188.
115. Kongas, O., Hertzen, R. von, and Engelbrecht, J. Bifurcation structure of a periodically driven nerve pulse equation modelling cardiac contraction. *Chaos Solitons Fractals*, 1999, **10**, 1, 119–136.

116. Saks, V., Aliev, M., Dos Santos, P., Vendelin, M., and Kongas, O. Mathematical model of energy transfer in hearts with inhibited or ablated creatine kinase system. *MAGMA*, 1998, **6**, 2/3, 124–125.
117. Kongas, O., Vendelin, M., and Saks, V. Modeling of intracellular compartmentalized energy and metabolic fluxes in the heart. *Med. Biol. Eng. Comput.*, 1999, **37**, Suppl. 1, 27–32.
118. Saks, V. A., Kongas, O., Vendelin., M., and Kay, L. Role of the creatine/phosphocreatine system on the regulation of mitochondrial respiration. *Acta Physiol. Scand.*, 2000, **168**, 4, 635–641.
119. Vendelin, M., Kongas, O., and Saks, V. Regulation of mitochondrial respiration in heart cells analyzed by reaction-diffusion model of energy transport. *Am. J. Physiol. Cell Physiol.*, 2000, **278**, 4, C747–C764.
120. Vendelin, M., Bovendeerd, P., Arts, T., Engelbrecht, J., and van Campen, D. H. Linear dependence of myocardium oxygen consumption on stress-strain area predicted by cross-bridge model. *Med. Biol. Eng. Comput.*, 1999, **37**, Suppl. 1, 63–66.
121. Kalda, J. Fractal model of blood vessel system. *Fractals*, 1993, **1**, 2, 191–197.
122. Kalda, J. Fractality of blood-vessel system and lung. *Med. Biol. Eng. Comput.*, 1996, **34**, Suppl. 1, 375–376.
123. Kalda, J. Transport processes in fractal biological networks. *Proc. Estonian Acad. Sci. Eng.*, 1999, **5**, 4, 270–280.
124. Kalda, J. On the optimization of Monte-Carlo simulations. *Physica A*, 1997, **246**, 646–658.
125. Kalda, J., Säkki, M., Vainu, M., and Laan, M. The methods of nonlinear dynamics in the analysis of heart rate variability for children. *Med. Biol. Eng. Comput.*, 1999, **37**, Suppl. 1, 69–72.
126. Aben, H. K. On application of the method of photoelasticity for investigation of buckled plates. *Eesti NSV Tead. Akad. Toim. Tehn. Füüs.-matem.*, 1957, **6**, 1, 28–40 (in Russian).
127. Aben, H. Optical phenomena in photoelastic models by the rotation of principal axes. *Exp. Mech.*, 1966, **3**, 9, 1414–1421.
128. Aben, H. K. and Saks, E. G. Optical phenomena by passing of light through shells. In *Proc. Conference "Polarization-optical Method for Investigation of Stresses" (Leningrad, 1958).* Leningrad, LGU, 1960, 208–221 (in Russian).
129. Aben, H. K. Characteristic directions in optics of twisted birefringent media. *J. Opt. Soc. Am.*, 1986, **3**, 9, 1414–1421.
130. Aben, H. K. Investigation of plates and shells with method of photoelasticity. In *Proc. All-Union Conference on Theory of Shells and Plates.* Leningrad, 1975, 348–353 (in Russian).
131. Aben, H. The optical investigation of twisted fibers. *J. Textile Inst.*, 1968, **59**, 11, 523–527.
132. Aben, H. K. and Saar, A. J. Determination of temperature stresses in two-layered shells of revolution by method of photoelasticity. In *Proc. Conference "Polarization-optical Method for Investigation of Stresses"*. Leningrad, 1966, 305–308 (in Russian).
133. Aben, H. K. Using of natural and magnetic rotation of the plane of polarization in photoelasticity. *Dokl. AN SSSR,* 1970, **192**, 3, 541–543 (in Russian).
134. Aben, H. K., Veigel, M., Idnurm, S. J., Saar, A. J., and Uffert, M. M. Determination of stresses in bended plates by method of photoelasticity. In *Proc. VII All-Union Conference* on *Polarization-optical Method for Investigation of Stresses*. Tallinn, 1971, 111–120.
135. Aben, H. K., Veigel, M., and Idnurm, S. J. Photoelectrical magnitopolarimeter for investigation of inhomogeneous optically anisotropic media. *Opt.-Mekh. Prom.*, 1977, 9, 21–23 (in Russian).
136. Aben, H. K., Idnurm, S. J., and Tatarinov, A. C. Method of photoelasticity in strong magnetic field. In *Proc. IV National Congress on Theoretical and Applied Mechanics.* Varna, 1981, **2**, 58–63 (in Russian).
137. Aben, H. Magnetophotoelasticity – photoelasticity in a magnetic field. *Exp. Mech.*, 1970, **10**, 3, 97–105.
138. Aben, H. Principles of magneto-photoelasticiy. In *Experimental Stress Analysis and Its Influence on Design*. Institution of Mech. Eng., London, 1970, 175–183.

139. Aben, H., Idnurm, S., Klabunovski, E., and Uffert, M. Photoelasticity with additional physical fields and optically active models. *Exp. Mech.*, 1974, **14**, 9, 361–366.
140. Aben, H. and Brosman, E. Integrated photoelasticity of cubic single crystals. *VDI-Berichte*, 1978, 313, 45–50.
141. Josepson, J. I. Stress distribution in cubic single crystals of cylindrical form by scattered-light photoelasticity. *Eesti NSV Tead. Akad. Toim. Füüs. Matem.*, 1979, **28**, 2, 146–150 (in Russian).
142. Aben, H. *Integrated Photoelasticity*. McGraw-Hill, New York, 1979.
143. Aben, H. K. On theory of composite quarterwave plate. *Opt. Spektrosk.*, 1962, **13**, 5, 746–750 (in Russian).
144. Aben, H. Towards the achromatic quarterwave plate. *Exp. Mech.*, 1974, **14**, 6, 249–250.
145. Aben, H. K. On theory of defective circular polariscope. *Opt. Spektrosk.*, 1962, **13**, 2, 256–258 (in Russian).
146. Aben, H. K. On the optical phenomena by passing of light through a pile of double-refracting plates. *Eesti NSV Tead. Akad. Toim. Füüs.-matem. tehn.*, 1962, **11**, 1, 16–23 (in Russian).
147. Aben, H. K., Krasnovski, B. K., and Pindera, J. T. Nonrectilinear light propagation in integrated photoelasticity of axisymmetric bodies. In *Proc. 8th Canadian Congress of Applied Mechanics*. Moncton, 1981, 519–520.
148. Aben, H. K. Integrated gradient photoelasticity of bodies of revolution. *Dokl. AN SSSR*, 1984, **247**, 2, 289–291 (in Russian).
149. Aben, H. K. and Kell, K.-J. E. Integrated gradient photoelasticity of bodies of revolution. *Prikl. Mekh.*, 1985, **21**, 9, 11–15 (in Russian).
150. Aben, H. and Josepson, J. On the precision of integrated photoelasticity for hollow glassware. *Opt. Lasers Eng.*, 1995, **22**, 2, 121–135.
151. Aben, H. Integrated photoelasticity as tensor field tomography. In *Proc. International Symposium on Photoelasticity (Tokyo, 1986)*. Springer-Verlag, Tokyo, 1986, 243–250.
152. Aben, H. Tomographie optique des champs de contraintes. *Revue Franç. Méc.*, 1989, 1, 121–130.
153. Aben, H. K., Josepson, J. I., and Kell, K.-J. E. The case of weak birefringence in integrated photoelasticity. *Opt. Lasers Eng.*, 1989, **11**, 3, 145–157.
154. Aben, H., Idnurm, S., and Puro, A. Integrated photoelasticity in case of weak birefringence. In *Proc. 9th International Conference on Experimental Mechanics*. Copenhagen, 1990, 867–875.
155. Puro, A. and Kell, K.-J. Complete determination of stresses in fiber preform of arbitrary cross section. *J. Lightwave Technol.*, 1992, **10**, 88, 1010–1014.
156. Aben, H. and Guillemet, C. *Photoelasticity of Glass*. Springer-Verlag, Berlin, Heidelberg, 1993.
157. Puro, A. On the tomographic method in magnetophotoelasticity. *Opt. Spektrosk.*, 1996, **81**, 1, 132–139.
158. Aben, H., Anton, J., and Josepson, J. Nonaxisymmetric residual stress distribution in axisymmetric glass articles. *Glass Sci. Technol.*, 1996, **69**, 3, 75–81.
159. Josepson, J. Curious optical phenomena in integrated photoelasticity. In *Recent Advances in Experimental Mechanics* (Gomes, S., et al., eds.). Balkema, Rotterdam, 1994, **1**, 91–94.
160. Aben, H. and Josepson, J. Strange interference blots in the interferometry of inhomogeneous birefringent objects. *Appl. Opt.*, 1997, **36**, 28, 7172–7179.
161. Aben, H. and Ainola, L. Interference blots and fringe dislocations in optics of twisted birefringent media. *J. Opt. Soc. Am. A*, 1998, **15**, 9, 2404–2411.
162. Aben, H. and Puro, A. Integrated photoelasticity for axisymmetric flow birefringence. *Proc. Estonian Acad. Sci. Phys. Math.*, 1993, **42**, 1, 7–13.
163. Aben, H., Puro, A., and Laermann, K.-H. Axisymmetric nonlinear problems in integrated photomechanics. In *Recent Advances in Experimental Mechanics* (Gomes, S., et al., eds.). Balkema, Rotterdam, 1994, **1**, 3–86.
164. Aben, H. and Puro, A. Photoelastic tomography for three-dimensional flow birefringence studies. *Inverse Probl.*, 1997, **13**, 215–221.

165. Ainola, L. Integrated photoelasticity for viscous flow in closed conduits. *Proc. Estonian Acad. Sci. Phys. Math.*, 1998, **47**, 1, 48–55.
166. Aben, H. Edge and boundary stress measurement in glass. *Glass Technol.*, 1995, **36**, 6, 561–566.
167. Anton, J. Automatic measurement of residual stresses in tempered tumblers. In *Proc. 18th International Congress on Glass*. San Francisco, 1998, **A11**, 9–13.
168. Aben, H., Ainola, L., and Anton, J. Half-fringe phase-stepping with separation of the principal stress directions. *Proc. Estonian Acad. Sci. Eng.*, 1999, **5**, 3, 198–211.
169. Aben, H. and Ainola, L. Hybrid stress analysis with integrated photoelasticity. In *IUTAM Symposium "Advanced Optical Methods and Applications in Solid Mechanics"* (Lagarde, A., ed.). Poitiers, 1998, **1**, L 10, 1–8.
170. Aben, H., Ainola, L., and Anton, J. Complete residual stress measurement in axisymmetric glass articles. In *Proc. 18th International Congress on Glass*. San Francisco, 1998, **B6**, 46–50.
171. Aben, H., Ainola, L., and Anton, J. Sum rules for photoelastic residual stress measurement in axisymmetric glass articles. In *Proc. International Conference on Advanced Technology in Experimental Mechanics (ATEM'99)*. Ube, 1999, **2**, 629–634.
172. Ainola, L. and Aben, H. Duality in optical theory of twisted birefringent media. *J. Opt. Soc. Am. A*, 1999, **16**, 10, 2545–2549.

MEHAANIKAUURINGUD KÜBERNEETIKA INSTITUUDIS

Mati KUTSER

On antud ülevaade mehaanikaalastest uuringutest Küberneetika Instituudis ajavahemikul 1960–2000. Vaadeldud perioodi temaatika haarab laia probleemide ringi integraalsest fotoelastsusest kuni südametegevuse matemaatilise modelleerimiseni, hõlmates tahke keha dünaamikat, akustiliste kajasignaalide arvutamist, mittelineaarsete lainete levikut ning mitmesuguseid mittelineaarse dünaamika keerukaid ülesandeid.

Bibliograafiaosas on esitatud valik olulisemaid teaduspublikatsioone.