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WAVE INTERACTION FOR CHARACTERIZATION OF NONLINEAR ELASTIC MATERIALS

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Abstract. A method for ultrasonic nondestructive testing of nonlinear elastic materials (structural elements) is elaborated. Simultaneous propagation of two longitudinal waves, their reflection and interaction data are utilized. Convenient choice of the wave frequency makes it possible to analyse the data recorded on the boundaries of the specimen in terms of wave harmonics. A detailed analysis of the harmonics evolution and interaction is presented. It is shown that amplitudes of the harmonics and phase shifts are sensitive to material properties.

Key words: nondestructive testing, elastic material, nonlinearity, longitudinal waves, interaction.

1. INTRODUCTION

The piezoelectric effect discovered by brothers Pierre and Jacques Curie in 1880 [1] made it possible to pose in principle the ultrasonic nondestructive testing (NDT) problem. First attempts to use this possibility were made in 1913 [2] and 1931 [3]. After that, intensive application of ultrasound for material characterization began [1]. The result is that ultrasonics plays nowadays a prominent role in NDT. It permits the development of effective and versatile NDT methods for evaluating mechanical properties of materials, for the determination of their micro- and macrostructure, flaws, inclusions, etc. Most practical applications of ultrasonics are related to solid materials. Ultrasound as a travelling wave is defined by two basic parameters: velocity and attenuation. A majority of NDT methods measure these parameters by the conventional time-of-flight method and treat the recorded data on the level of the linear theory [4]. These methods break down when the sample is very thin, the wave velocity is frequency-dependent, and the material has complicated multiparametric properties. Two first limitations of the time-of-flight method can be eliminated by the use of Fourier transform techniques [5]. The elimination of the last limitation demands extraction of additional information from the wave velocity and attenuation measurement data. One possibility is to record and analyse the nonlinear effects of wave propagation [6,7]. Another one is to make use of the more complicated ultrasonic NDT methods [8,9].

In this paper, a method for NDT of nonlinear elastic materials is proposed. The basic idea is to utilize the nonlinear effects that accompany simultaneous propagation of two waves in the material [10]. The method may be regarded as the first stage of the project to elaborate a relatively simple method for NDT of materials with complicated properties. The data about simultaneous propagation, reflection, and nonlinear interaction of waves in a homogeneous nonlinear elastic material constitute the reference data for the advanced methods. Similar simultaneous wave propagation problems have been studied by several authors [11]. The peculiarity of the approach in this paper is that due to the progress in analytical computation software (Maple V), the wave interaction problem is described by an analytical solution to the nonlinear wave equation. This enables us to follow the whole process of two wave nonlinear interaction in the material analytically and to analyse the evolution of the nonlinear effects in detail.

The proposed NDT method makes use of two harmonic waves excited on the opposite surfaces of the material (structural element) in terms of particles velocity and recorded on the same surfaces in terms of stress. It clears up that the appropriate choice of the wave frequency enables one to analyse the recorded data in terms of wave harmonics amplitudes and phase shifts. The recorded values of these wave characteristics are dependent on material properties. Since the analytical expressions relating wave characteristics with material properties are too cumbersome, the corresponding plots are computed and analysed. As the result, an algorithm for nondestructive material characterization is proposed.

2. PROBLEM FORMULATION

A specimen with two parallel traction free surfaces is considered. The material of the specimen is isotropic, homogeneous, and elastic, characterized by the density ρ, the Lamé coefficients λ and μ, and by the third order elastic coefficients ν_1, ν_2, and ν_3. This characterization corresponds to the five constant physically nonlinear theory of elasticity [12]. The one-dimensional nonlinear wave propagation process in the specimen in the range of small but finite deformations is described by the equation of motion [10]

$$[1 + k_1\, U_{,X}(X,t)]\, U_{,XX}(X,t) - c^{-2}\, U_{,tt}(X,t) = 0, \qquad (1)$$

where $U(X,t)$ denotes displacement vector. The indices after the comma, X and t, indicate differentiation with respect to the Lagrangian rectangular coordinate X

and time t, respectively. Equation (1) is derived using the Kirchhoff pseudostress tensor and it takes the geometrical nonlinearity of the problem into account.

The coefficients of Eq. (1),

$$k_1 = 3[1 + 2k_0(\nu_1 + \nu_2 + \nu_3)], \qquad c^2 = (k_0\rho)^{-1}, \qquad k_0 = (\lambda + 2\mu)^{-1}, \tag{2}$$

are functions of the material properties.

One-dimensional harmonic waves are excited in the specimen of thickness L in correspondence with the initial and boundary conditions

$$\begin{aligned} U(X,0) &= U_{,t}(X,0) = 0, \\ U_{,t}(0,t) &= \varepsilon a_0 H(t) \sin\omega t, \\ U_{,t}(L,t) &= \varepsilon a_L H(t) \sin\omega t. \end{aligned} \tag{3}$$

Here $H(t)$ denotes the Heaviside function, ε, a_0, and a_L are constants, and ω is the frequency. The constant ε is regarded as a small parameter ($\varepsilon \ll 1$). Making use of the perturbation theory, the analytical solution to Eq. (1) is derived in the form of a series [10]

$$U(X,t) = \sum_{n=1}^{\infty} \varepsilon^n U^{(n)}(X,t). \tag{4}$$

Solution (4) describes simultaneous propagation of the two waves in homogeneous isotropic elastic medium. The wave excited on the surface $X = 0$ propagates in positive direction and the wave excited on the surface $X = L$ in negative direction of the X axis (Fig. 1). Solution (4) is valid for the initial stage of the wave profile distortion ($0 \le t < 2L/c$) and it is supposed that in this initial stage the distortion of the wave profile is weak and the shock wave is not generated.

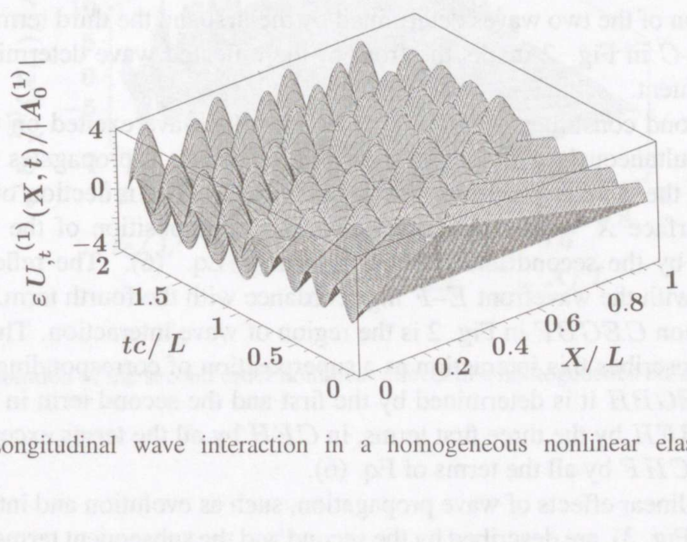

Fig. 1. Longitudinal wave interaction in a homogeneous nonlinear elastic material, $A_0^{(1)} \equiv \varepsilon a_0$.

3. WAVE INTERACTION

The wave process in the material is excited in terms of particles velocity (Eq. (3)). Consequently, it is convenient to analyse the wave propagation, reflection, and interaction on the basis of the solution (4) also in terms of particles velocity:

$$U_{,t}(X,t) = \sum_{n=1}^{\infty} \varepsilon^n U_{,t}^{(n)}(X,t). \tag{5}$$

The three first terms in Eq. (5) will be analysed. Following the perturbation procedure [10], the first term in solution (5)

$$\varepsilon\, U_{,t}^{(1)}(X,t) = A_0^{(1)} \sin\omega\xi + A_L^{(1)} \sin\omega\eta - A_0^{(1)} \sin\omega\theta - A_L^{(1)} \sin\omega\zeta \tag{6}$$

describes the linear wave process in a homogeneous physically linear elastic material. In Eq. (6), $A_0^{(1)} \equiv \varepsilon\, a_0$, $A_L^{(1)} \equiv \varepsilon\, a_L$, and functions ξ, η, θ, and ζ are expressed by formulae

$$\begin{aligned} \xi &= t - X/c, & \eta &= t - L/c + X/c, \\ \theta &= t - 2\,L/c + X/c, & \zeta &= t - L/c - X/c. \end{aligned} \tag{7}$$

The linear propagation, reflection, and interaction of waves are characterized by four constituents of Eq. (6). The first one describes the propagation of the wave excited on the surface $X = 0$. It propagates in the positive direction of the X axis with the wavefront shown by the line A–B in the X/L, tc/L plane plotted in Fig. 2. The reflection of the wave from the surface $X = L$ is described as a superposition of the two waves determined by the first and the third term in Eq. (6). The line B–C in Fig. 2 marks the front of the reflected wave determined by the third constituent.

The second constituent of Eq. (6) describes the wave excited on the surface $X = L$ simultaneously with the wave mentioned above. It propagates in negative direction of the X axis with the front D–E (Fig. 2). The reflection of this wave from the surface $X = 0$ is described also as a superposition of the two waves determined by the second and the fourth term in Eq. (6). The reflected wave propagates with the wavefront E–F in accordance with the fourth term.

The region $CEGBF$ in Fig. 2 is the region of wave interaction. The first term in Eq. (5) describes this interaction as a superposition of corresponding waves. In the region $EGBH$ it is determined by the first and the second term in Eq. (6), in the region BFH by the three first terms, in CEH by all the terms except the third one, and in CHF by all the terms of Eq. (6).

The nonlinear effects of wave propagation, such as evolution and interaction of harmonics (Fig. 3), are described by the second and the subsequent terms in Eq. (5).

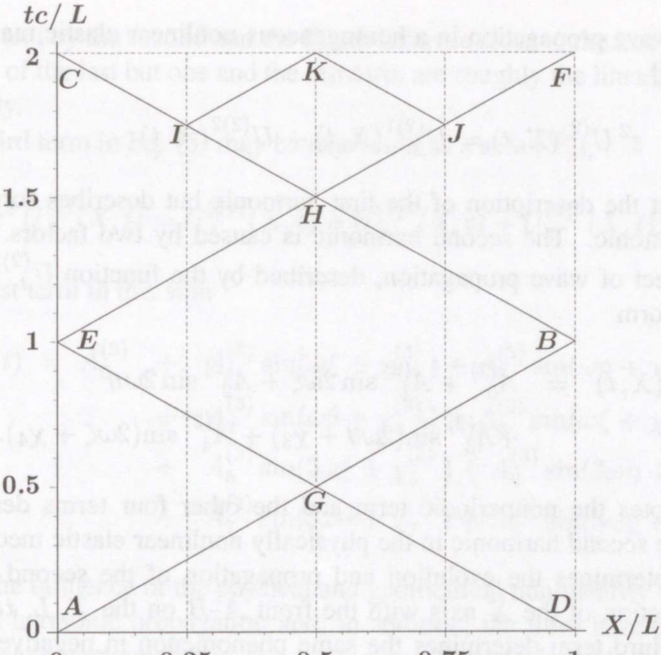

Fig. 2. Front pattern of the wave components; first and second order small terms in Eq. (5).

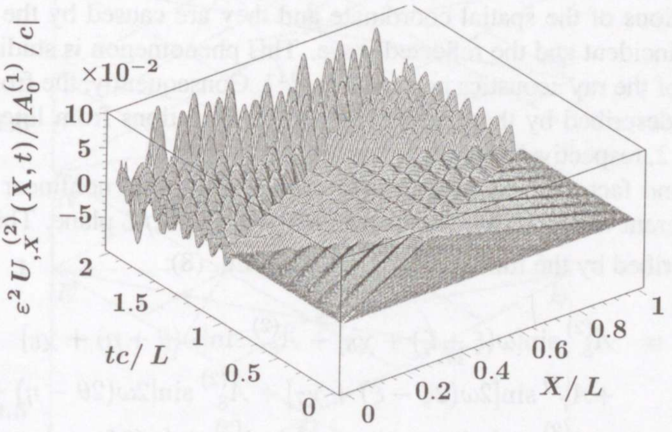

Fig. 3. Evolution of the second order nonlinear effects in a homogeneous elastic material.

In the case of wave propagation in a homogeneous nonlinear elastic material, the second term [10]

$$\varepsilon^2 U_{,t}^{(2)}(X,t) = U_{,t}^{(2)1}(X,t) + U_{,t}^{(2)2}(X,t) \tag{8}$$

does not correct the description of the first harmonic but describes formation of the second harmonic. The second harmonic is caused by two factors. First, as a nonlinear effect of wave propagation, described by the function $U_{,t}^{(2)1}(X,t)$ in Eq. (8), in the form

$$\begin{aligned} U_{1,t}^{(2)1}(X,t) &= A_0^{(2)} + A_1^{(2)} \sin 2\omega\xi + A_2^{(2)} \sin 2\omega\eta \\ &\quad + A_3^{(2)} \sin(2\omega\theta + \chi_3) + A_4^{(2)} \sin(2\omega\zeta + \chi_4). \end{aligned} \tag{9}$$

Here $A_0^{(2)}$ denotes the nonperiodic term and the other four terms describe the evolution of the second harmonic in the physically nonlinear elastic medium. The second term determines the evolution and propagation of the second harmonic in positive direction of the X axis with the front A–B on the X/L, tc/L plane (Fig. 2). The third term determines the same phenomenon in negative direction of the X axis with the front D–E. The absence of phase shifts in arguments of these terms means that the influence of the wave interaction on phase velocity of the second harmonic in the region $EGBH$ (Fig. 2) is a higher order small effect. It is described by the third term in Eq. (5).

Two last terms in Eq. (9) describe the propagation of the considered above part of the second harmonic after reflection. Phase shifts in these terms, χ_3 and χ_4, are functions of the spatial coordinate and they are caused by the interaction between the incident and the reflected wave. This phenomenon is studied in detail on the basis of the ray acoustics approach in [11]. Consequently, the fronts of wave components described by these terms have small deviations from lines B–C and E–F in Fig. 2, respectively [11].

The second factor of the second harmonic formation is nonlinear interaction between different waves in the same region of the X/L, tc/L plane. This nonlinear effect is described by the function $U_{,t}^{(2)2}(X,t)$ in Eq. (8):

$$\begin{aligned} U_{,t}^{(2)2}(X,t) &= A_5^{(2)} \sin[\omega(\xi+\zeta)+\chi_5] + A_6^{(2)} \sin[\omega(\theta+\eta)+\chi_6] \\ &\quad + A_7^{(2)} \sin[2\omega(2\zeta-\xi)+\chi_7] + A_8^{(2)} \sin[2\omega(2\theta-\eta)+\chi_8] \\ &\quad + A_9^{(2)} \sin[\omega(3\zeta-\xi)+\chi_9] + A_{10}^{(2)} \sin[\omega(3\theta-\eta)+\chi_{10}]. \end{aligned} \tag{10}$$

Phase shifts χ_j ($j = 5, 6, ..., 10$) are functions of the coordinate X and they characterize the phase velocity [7]. Consequently, the nonlinear interaction of different waves affects the phase velocity of harmonics formed by this interaction. This result has been described also by other authors [11]. Fronts of wave components described by the first and the third term in Eq. (10), are close to the line E–F, and by the second and the fourth term are close to the line B–C in Fig. 2. The fronts of the last but one and the last term are roughly the lines I–K and J–K, respectively.

The third term in Eq. (5) may be expressed as a sum [10]

$$\varepsilon^3 U_{,t}^{(3)}(X,t) = U_{,t}^{(3)1}(X,t) + U_{,t}^{(3)2}(X,t) + U_{,t}^{(3)3}(X,t). \tag{11}$$

The first term in this sum

$$\begin{aligned} U_{,t}^{(3)1}(X,t) = A_0^{(3)} &+ A_1^{(3)}\sin(\omega\xi + \chi_1^{(3)}) + A_2^{(3)}\sin(\omega\eta + \chi_2^{(3)}) \\ &+ A_3^{(3)}\sin(\omega\theta + \chi_3^{(3)}) + A_4^{(3)}\sin(\omega\zeta + \chi_4^{(3)}) \\ &+ A_5^{(3)}\sin(3\omega\xi + \chi_5^{(3)}) + A_6^{(3)}\sin(3\omega\eta + \chi_6^{(3)}) \\ &+ A_7^{(3)}\sin(3\omega\theta + \chi_7^{(3)}) + A_8^{(3)}\sin(3\omega\zeta + \chi_8^{(3)}) \end{aligned} \tag{12}$$

describes the influence of the physical and geometrical nonlinearity of the problem on the first harmonic propagation and, in addition, the third harmonic evolution. Fronts of wave components in Eq. (12) coincide roughly with the fronts of the terms in Eq. (9) with analogous arguments and they are plotted as lines A–B, B–C, D–E, and E–F in Fig. 4.

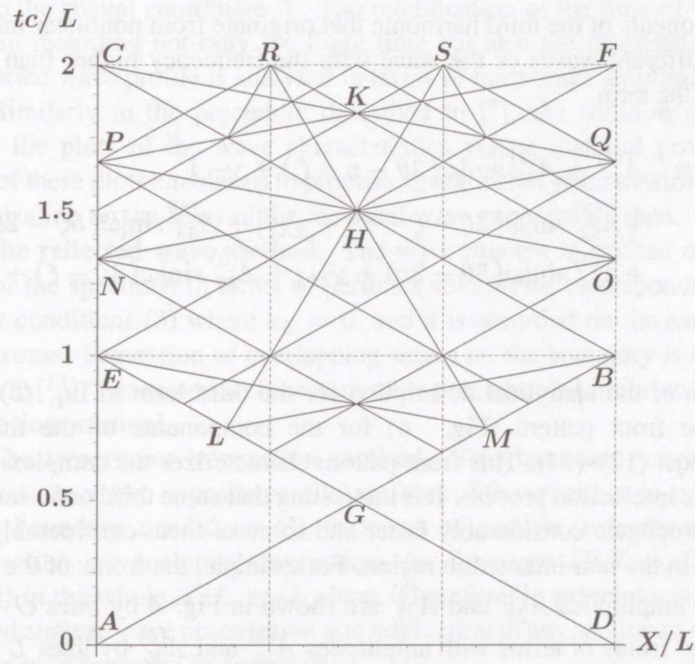

Fig. 4. Front pattern of the wave components. Third order small terms in Eq. (5).

The second term

$$
\begin{aligned}
U_{,t}^{(3)2}(X,t) = A_{9}^{(3)} &+ A_{10}^{(3)}\sin[\omega(2\xi+\zeta)+\chi_{10}^{(3)}] + A_{11}^{(3)}\sin[\omega(2\xi+\theta)+\chi_{11}^{(3)}] \\
&+A_{12}^{(3)}\sin[\omega(2\xi+\eta)+\chi_{12}^{(3)}] + A_{13}^{(3)}\sin[\omega(2\xi-\eta)+\chi_{13}^{(3)}] \\
&+A_{14}^{(3)}\sin[\omega(2\xi-\theta)+\chi_{14}^{(3)}] + A_{15}^{(3)}\sin[\omega(2\eta+\xi)+\chi_{15}^{(3)}] \\
&+A_{16}^{(3)}\sin[\omega(2\eta+\theta)+\chi_{16}^{(3)}] + A_{17}^{(3)}\sin[\omega(2\eta+\zeta)+\chi_{17}^{(3)}] \\
&+A_{18}^{(3)}\sin[\omega(2\eta-\xi)+\chi_{18}^{(3)}] + A_{19}^{(3)}\sin[\omega(2\zeta-\theta)+\chi_{19}^{(3)}] \\
&+A_{20}^{(3)}\sin[\omega(2\eta-\zeta)+\chi_{20}^{(3)}] + A_{21}^{(3)}\sin[\omega(2\theta+\xi)+\chi_{21}^{(3)}] \\
&+A_{22}^{(3)}\sin[\omega(2\theta+\eta)+\chi_{22}^{(3)}] + A_{23}^{(3)}\sin[\omega(2\theta+\zeta)+\chi_{23}^{(3)}] \\
&+A_{24}^{(3)}\sin[\omega(2\theta-\xi)+\chi_{24}^{(3)}] + A_{25}^{(3)}\sin[\omega(2\theta-\zeta)+\chi_{25}^{(3)}] \\
&+A_{26}^{(3)}\sin[\omega(2\zeta+\xi)+\chi_{26}^{(3)}] + A_{27}^{(3)}\sin[\omega(2\zeta+\theta)+\chi_{27}^{(3)}] \\
&+A_{28}^{(3)}\sin[\omega(2\zeta-\eta)+\chi_{28}^{(3)}] + A_{29}^{(3)}\sin[\omega(2\zeta+\eta)+\chi_{29}^{(3)}]
\end{aligned}
\tag{13}
$$

describes propagation of the first and the third harmonic components that originate from nonlinear interaction of various waves in different regions of the X/L, tc/L plane.

The components of the third harmonic that originate from nonlinear interaction of the three different waves or the wave with the frequency higher than $2\,\omega$, are collected into the term

$$
\begin{aligned}
U_{,t}^{(3)3}(X,t) \;=\; & A_{30}^{(3)} + A_{31}^{(3)}\sin[\omega(\,3\theta-\eta+\zeta\,)+\chi_{31}] \\
&+A_{32}^{(3)}\sin[\omega(\,3\zeta-\xi+\theta\,)+\chi_{32}\,] + A_{33}^{(3)}\sin[\omega(\,5\zeta-2\xi)+\chi_{33}\,] \\
&+A_{34}^{(3)}\sin[\omega(\,5\theta-2\eta)+\chi_{34}\,] + A_{35}^{(3)}\sin[\omega(\,4\zeta-\xi)+\chi_{35}\,].
\end{aligned}
\tag{14}
$$

Possession of the analytical description for the third term in Eq. (5) enables us to plot the front pattern (Fig. 4) for the components of the third term specified by Eqs. (12)–(14). This front pattern characterizes the complexity of the nonlinear wave interaction process. It is interesting that some third order interaction components propagate considerably faster and some of them considerably slower than the wave in the non-interaction region. For example, the fronts of the terms in Eq. (13) with amplitudes $A_{23}^{(3)}$ and $A_{27}^{(3)}$ are shown in Fig. 4 by lines $O-P$ and $N-Q$ and the fronts of terms with amplitudes $A_{19}^{(3)}$ and $A_{25}^{(3)}$ by lines $L-S$ and $M-R$, respectively.

4. MATERIAL CHARACTERIZATION

Investigation of wave motion in homogeneous nonlinear elastic materials is of especial importance since the obtained results may be used as reference data for elaboration of methods for characterization of materials with complicated properties. Wave propagation, reflection, and nonlinear interaction in nonlinear elastic materials has been studied by many authors (see references in [11]). Progress in computer technology and in the development of symbolic software (Maple V, Mathematica 3.0, etc.) makes it possible to study this problem on higher level. In this paper the analytical solution derived in [10] is used. The advantage of this approach is that all effects of wave propagation may be analysed on the basis of analytical expressions. The effects that accompany wave propagation are dependent on the physical and geometrical properties of the material. The analytical solution analysed in this paper enables us to derive an analytical description of the wave characteristics as a function of the material properties. This solution may be used as a theoretical basis for the following NDT methods.

1. The modified time-of-flight method. The ultrasound in the form of a harmonic wave is excited on the surface $X = 0$ of the specimen in correspondence with boundary conditions (3) where $a_L = 0$, i.e., the velocity of the material particles on the surface $X = L$ is supposed to be equal to zero. The wave process is recorded on the surface $X = L$ in terms of stress. The stress is characterized by the derivative of the displacement vector $U(X,t)$, determined by Eq. (4), with respect to the spatial coordinate X. The modification of the time-of-flight method consists in recording not only the flight time but also the distorted wave profile. The distorted wave profile is analysed in terms of harmonics amplitudes and phase shifts. Similarly to the procedure described in [6], the solution (4) is used to compose the plots of the wave characteristics versus material properties. The analysis of these plots enables us to propose an algorithm of nondestructive material characterization on the basis of the recorded wave propagation data.

2. The reflected wave method. The wave process is excited on the surface $X = 0$ of the specimen in terms of particles velocity in correspondence with the boundary conditions (3) where $a_L = 0$, and it is recorded on the same surface in terms of stress. Separation of overlapping waves on the boundary is described, for example, in [13]. The material characterization procedure is similar to the procedure of the previous method.

3. The two waves interaction method. Two harmonic waves are excited simultaneously on two opposite parallel surfaces of the specimen in correspondence with the boundary conditions (3) in terms of particle velocity (Fig. 1). In our possession are analytical expressions for functions $U(X,t)$, $U_{,t}(X,t)$, and $U_{,X}(X,t)$ in the whole X/L, tc/L plane. Therefore, in principle, it is possible to record and analyse wave propagation and interaction in any section of the specimen. Two cases may be useful in applications.

First, the NDT of rods. Waves are excited simultaneously at both ends of the rod. The ratio of the rod diameter to the wavelength enables us to describe the wave propagation as one-dimensional. The wave motion is recorded in arbitrary rod sections in terms of $U_{,t}(X,t)$ or $U_{,X}(X,t)$. The plots of the material properties versus wave characteristics for the selected sections of the rod are computed. The analysis of these plots enables us to solve the ultrasonic material characterization problem.

Secondly, the more general case. The two waves interaction method may be applied if there is an access at least to two parallel traction free surfaces of the specimen. Let us discuss this case in more detail.

Two waves are excited simultaneously on parallel surfaces of the specimen in terms of the particle velocity (see boundary conditions (3)) and they are recorded on the same surfaces in terms of stress. The recorded data are analysed in the time interval $0 \leq t\,c\,/\,L < 2$ (Fig. 3) making use of the analytical expression for the function $U_{,X}(X,t)$ determined on the basis of Eq. (4). The nonlinear oscillation on the specimen boundaries may be considered as a sum of the linear constituent (first order effects) described by the first term in Eq. (4), of the second order nonlinear effects (second term in Eq. (4)), of the third order nonlinear effects, etc. It is possible to distinguish two different intervals, $0 \leq t\,c\,/\,L < 1$ and $1 \leq t\,c\,/\,L < 2$, for all these boundary oscillations (Fig. 5). In all cases the maximum amplification of the oscillation amplitude occurs in the wave interaction interval $1 \leq t\,c\,/\,L < 2$. From the point of view of NDT it is essential that the inhomogeneity in material properties may be easily identified by the difference in oscillations on both boundaries. In the considered case of a homogeneous nonlinear elastic material these oscillations are theoretically identical. The profile of the oscillation and its amplitude depend on material properties. The problem is how to determine these dependences.

The analysis of the solution (4) leads to the following method for ultrasonic NDT of homogeneous nonlinear elastic materials. It can be shown that if the initial frequency of the excited waves satisfies the condition

$$\omega = 2\,\pi\,c n\,/\,L, \tag{15}$$

where n in an integer, the approach used in the modified time-of-flight method [6] may be adapted. In this case the analytical solution (4) may be presented on the boundaries of the specimen in the form

$$U_{,X}(s,t) = A_0 + A_1\sin(\omega\tau + \phi_1) + A_2\sin(2\omega\tau + \phi_2) + A_3\sin(3\omega\tau + \phi_3). \tag{16}$$

Here $\tau = t - L/c$, A_0 is the non-periodic term, amplitudes A_j and phase shifts ϕ_j have different constant values in various boundary regions plotted in Figs. 2 and 4. Constant s is equal to zero on the boundary $X = 0$ and to L on the boundary $X = L$.

The physical meaning of the condition (15) is that the frequency of the excited waves must be chosen so that the number of wave periods on the specimen boundary is equal to the integer in the time interval $0 \leq t\,c\,/\,L \leq 1$.

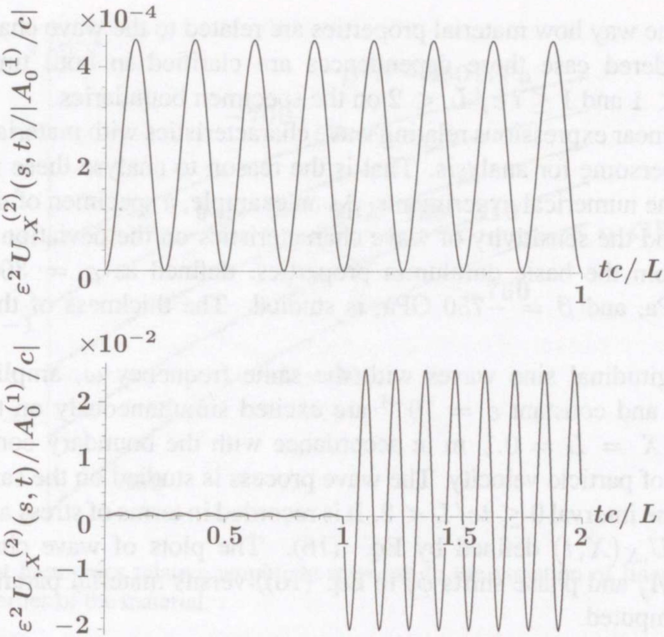

Fig. 5. Second order nonlinear effects on boundaries.

The nondestructive characterization problem of a homogeneous nonlinear elastic material may be solved on the basis of Eq. (16) as follows.

The specimen of the homogeneous nonlinear elastic material is characterized, besides the dimensions, by the density ρ, by the elastic coefficients of the second order, λ and μ, and by the elastic coefficients of the third order, ν_1, ν_2, and ν_3. The peculiarity of the one-dimensional problem is that the elastic coefficients are grouped in the governing equations (1) and (2), and the elastic properties of the material may be characterized by the parameters

$$\alpha = \lambda + 2\mu, \qquad \beta = 2(\nu_1 + \nu_2 + \nu_3). \tag{17}$$

The parameter α characterizes linear elastic properties and the parameter β nonlinear elastic properties of the material, respectively.

The recorded oscillation on the specimen boundaries is described by Eq. (16) and it is characterized by the frequency ω, by the non-periodic term A_0, and by the amplitudes and phase shifts of the harmonics A_1, A_2, A_3, ϕ_1, ϕ_2, and ϕ_3.

If one tries to solve the problem of characterization of the homogeneous nonlinear elastic materials on the basis of one-dimensional longitudinal wave propagation and interaction data, i.e., using Eq. (16), at his disposal are six basic functions A_1, A_2, A_3, ϕ_1, ϕ_2, ϕ_3, and one constant ω. The number of material parameters to be evaluated is three – ρ, α, and β.

The conclusion seems to be that it is possible to determine completely the properties of the material on the basis of wave propagation data. The success depends on the way how material properties are related to the wave characteristics. In the considered case these dependences are clarified in both time intervals $0 \leq tc/L < 1$ and $1 \leq tc/L < 2$ on the specimen boundaries.

The nonlinear expressions relating wave characteristics with material properties are too cumbersome for analysis. That is the reason to analyse these relations on the basis of the numerical experiments. As an example, a specimen of duralumin is considered and the sensitivity of wave characteristics on the deviation of material properties from the basic duralumin properties, defined as $\rho = 3000$ kg/m^3, $\alpha = 100$ GPa, and $\beta = -750$ GPa, is studied. The thickness of the specimen is 0.1 m.

Two longitudinal sine waves with the same frequency ω, amplitude $a_0 = -a_L = -c$, and constant $\varepsilon = 10^{-4}$ are excited simultaneously on the surfaces $X = 0$ and $X = L = 0.1$ m in accordance with the boundary conditions (3), i.e., in terms of particle velocity. The wave process is studied on the same surfaces during the time interval $0 \leq tc/L < 2$. It is recorded in terms of stress and analysed in terms of $U_{,X}(X,t)$ defined by Eq. (16). The plots of wave characteristics (amplitudes A_j and phase shifts ϕ_j in Eq. (16)) versus material parameters ρ, α, and β are computed.

Numerical simulation verifies the fact that the harmonics amplitudes are strongly dependent on the linear (α) and nonlinear (β) elastic properties of the material. These dependences for two first harmonics are plotted in Figs. 6 and 7. They are qualitatively similar in both time intervals $0 \leq tc/L < 1$ and $1 \leq tc/L < 2$. Exception is the dependence of the first harmonics amplitude on the parameter α that has different sign of the curvature in these intervals. The sensitivity of harmonics amplitudes to the variation of the parameters α and β in both time intervals on the boundary is about the same. Only sensitivity of the first harmonics amplitude is about ten times higher in the interval $1 \leq tc/L < 2$ compared with the sensitivity in the interval $0 \leq tc/L < 1$. Essential from the point of view of NDT is that the wave interaction amplifies the first harmonics amplitude about ten times, the second harmonics amplitude about hundred times (Fig. 5) and the third harmonics amplitude about thousand times. Consequently, despite the same sensitivity of the harmonics amplitudes to the variation of elastic properties in both intervals, the absolute value of the amplitude variation is much larger in the wave interaction interval than in the interval $0 \leq tc/L < 1$.

The sensitivity of the first harmonics phase shift to the variation of the parameter β is about three times higher than to the variation of the parameter α in the wave interaction interval. The homogeneity of the elastic material is characterized by the fact that the phase shift of the second harmonic in a homogeneous elastic material is equal to zero. This fact is pointed out also in [6]. In the considered case, the other phase shifts are not sensitive to the variation of the material properties.

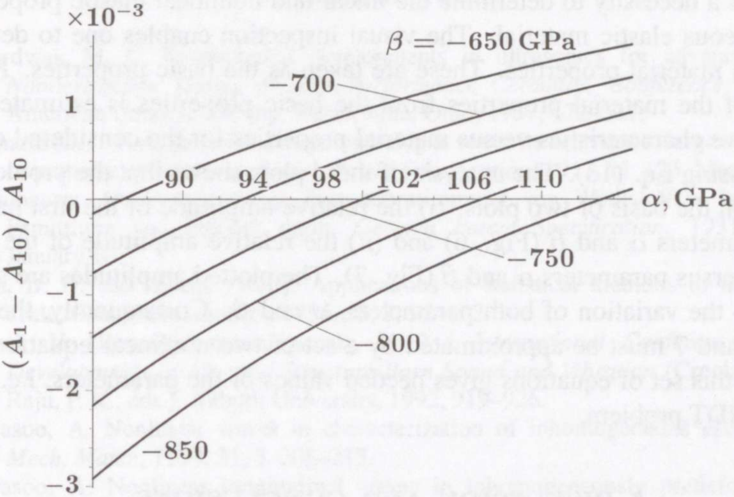

Fig. 6. First harmonics relative amplitude response to the variation of linear and nonlinear elastic properties of the material.

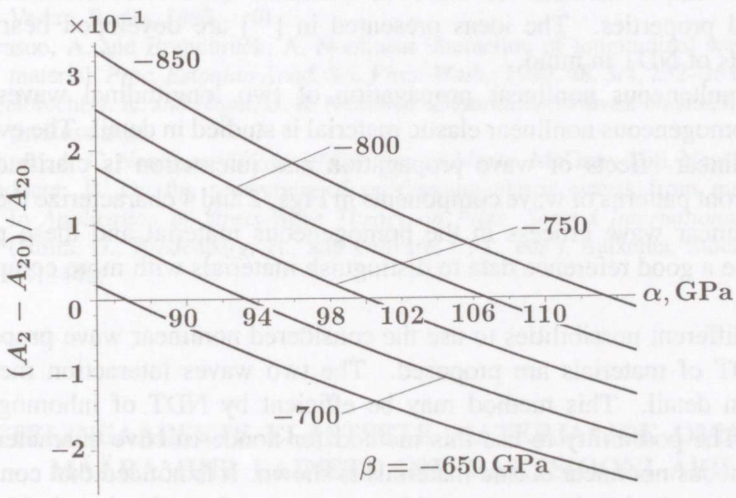

Fig. 7. Second harmonics relative amplitude versus material elasticity parameters.

The analysis of the plots points out that the wave characteristics are not sensitive to the material density variation. The reason is the satisfaction of the condition (15) where it is necessary to compute a new value of the velocity c on the basis of Eq. (2) for every variation of material properties. The result is that by this method of nondestructive material characterization the material density must be determined by a non-acoustic method.

In order to illustrate the possibility to characterize the nonlinear elastic materials on the basis of Eq. (16), the following problem is posed. It is assumed that there is a necessity to determine the linear and nonlinear elastic properties of a homogeneous elastic material. The visual inspection enables one to determine roughly the material properties. These are taken as the basic properties. Possible deviation of the material properties from the basic properties is estimated. The plots of wave characteristics versus material properties for the considered case are computed using Eq. (16). The analysis of these plots shows that the problem may be solved on the basis of two plots: (i) the relative amplitude of the first harmonic versus parameters α and β (Fig. 6) and (ii) the relative amplitude of the second harmonic versus parameters α and β (Fig. 7). The plotted amplitudes are strongly sensitive to the variation of both parameters, α and β. Consequently, the curves in Figs. 6 and 7 must be approximated by a set of two nonlinear equations. The solution of this set of equations gives needed values of the parameters, i.e., solves the posed NDT problem.

5. DISCUSSION AND CONCLUSIONS

The topic of this paper may be considered as a part of the project to elaborate a relatively simple method for NDT of materials (structural elements) with complicated properties. The ideas presented in [10] are developed bearing the requirements of NDT in mind.

The simultaneous nonlinear propagation of two longitudinal waves in an isotropic homogeneous nonlinear elastic material is studied in detail. The evolution of the nonlinear effects of wave propagation and interaction is clarified. The presented front patterns of wave components in Figs. 2 and 4 characterize the nature of the nonlinear wave process in the homogeneous material and these patterns may become a good reference data to distinguish materials with more complicated properties.

Three different possibilities to use the considered nonlinear wave propagation data in NDT of materials are proposed. The two waves interaction method is discussed in detail. This method may be efficient by NDT of inhomogeneous materials. The possibility to use this method for nondestructive characterization of homogeneous nonlinear elastic materials is shown. It is noticed that convenient choice of the excitation frequency enables one to transform the description of the wave process on the boundaries into the form of harmonics. The dependence of harmonics amplitudes and phase shifts on the material properties is analysed on the basis of corresponding plots. An algorithm for nondestructive characterization of homogeneous nonlinear elastic materials is proposed.

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MITTELINEAARSETE ELASTSETE MATERJALIDE OMADUSTE MÄÄRAMINE LAINETE INTERAKTSIOONI ABIL

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On esitatud ultrahelil baseeruv meetod mittelineaarse elastse materjali (konstruktsioonielemendi) mittepurustavaks katsetuseks. Meetod lähtub kahe pikilaine (ultraheli) samaaegse levi, peegelduse ja interaktsiooni mõõtmistulemustest ning võimaldab harmooniliste lainete sageduste sobival valikul analüüsida katsekeha äärtel registreeritud võnkumiste harmoonikuid. Detailselt on uuritud harmoonikute evolutsiooni ja interaktsiooni ning näidatud, et laine harmoonikute amplituudide ja faasinihete väärtused sõltuvad materjali omadustest.