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CALIBRATION OF MEASURING INSTRUMENTS

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Abstract. The method of calibration of measuring instruments presented in the paper describes a new approach to calculating the corrections as well as uncertainty of the correction curve of the measuring instruments. Two examples illustrate application of the new method.

Key words: calibration, correction, uncertainty, traceability, measuring instrument, working standard.

1. INTRODUCTION

Calibration is a set of operations which under specified conditions establish relationship between the values of quantities indicated by a measuring instrument or a measuring system and corresponding values of standards. The general terms "standard" and "calibrated measuring instrument" are used to describe the calibration method presented here, irrespective of the subject of calibration (ordinary measuring instrument, working standard, reference standard, etc.) since in any calibrating procedure one measuring instrument is the standard and the other is the one to be calibrated. The model and the method of calibration described in this paper contain segments of the traceability chain and determination of the corrections as well as characteristics of the corrections, and calculation of uncertainties of the results. The uncertainty of a measurement is expressed numerically by a parameter characterizing the dispersion of the values conceivably assigned to the measurands [1,2].

2. CALIBRATION MODEL

The calibration of a measuring instrument (standard) gives a relationship between the value of the measurand and the values of the output of the measuring instrument (indication, nominal value of the measurand realized by a standard) either as a discrete correction or as the characteristics of corrections.

The calibration correction (characteristic of correction) Y is determined by the functional relationship f of N input quantities (such as the values of the measurand realized by a standard, the indication of the measuring instrument, the correction of the standard, the temperature of the environment during calibration) $X_i (i = 1, \ldots, N)$ as [1,2]

$$Y = f(X_1, X_2, \ldots, X_i, \ldots, X_N). \tag{1}$$

The estimated value of the correction Y, denoted by y, is obtained from Eq. (1) with the estimates of the inputs $x_1, x_2, \ldots, x_i, \ldots, x_N$. The estimate of the output y is described by the relationship

$$y = f(x_1, x_2, \ldots, x_i, \ldots, x_N). \tag{2}$$

The result of the measurement of the correction y may be determined if the estimates of the inputs x_i are presented as arithmetic means of the measurands $\bar{x}_i$:

$$y = f(\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_i, \ldots, \bar{x}_N). \tag{3}$$

It is evident from the above that the result of the measurement y is an estimate of the value of the correction Y. Therefore, the result of the measurement is complete only if it is accompanied by information about the uncertainty of the estimate. Below we describe the calibration process and the methods of estimation of the calibration results and their uncertainties.

3. CORRECTION AND UNCERTAINTY

3.1. Evaluation of the correction

Calibration of a measuring instrument with an indication appliance must be performed as follows. The measurand realized by the standard is measured by the measuring instrument, the indication or arithmetic mean value of deviations is fixed and the systematic deviation of the measurement e is calculated from the formula

$$e = x - x_c, \tag{4}$$

where x is the indication of the calibrated measuring instrument by measuring of the measurand, realized by the standard, and x_c is the conventional value of the measurand realized by the standard, obtained as the result of calibration of the standard.

The indication x in (4) depends on the procedure of reading of the indication: as a single indication or as the mean value of several indications.

The value of the correction K is calculated as

$$K = -e = x_c - x = x_{ns} - K_s - x, \tag{5}$$

where x_{ns} is the nominal value of the standard and K_s is the value of the correction at the calibration of the standard (Fig. 1).

The calculated correction K is related to the indication of the calibrated measuring instrument. The estimation of the standard uncertainty of the correction predestines that the potential measuring deviations of the estimates $\overline{x}_i$ of the input quantities (factors) are summarized in the systematic measuring deviation e, on the basis of which the corresponding correction is calculated from Eq. (5). The correction K, obtained at the calibration, is a sum of the corrections caused by the difference of the indication of the calibrated measuring instrument and the value of the standard, measuring deviation of the calibration method used, the qualities of the measuring instrument (subject) to be calibrated, and the values of the parameters of the measuring environment. The correction K comprises the joint uncertainty calculated from the following formula (Fig. 1a)

$$u(K) = \sqrt{u^2(x_c) + u^2(x_{indic}) + u^2(x_{subj}) + u^2(x_{envir})}, \tag{6}$$

where $u(x_c)$ is the standard uncertainty of the standard, $u(x_c) = u(K_s)$, $u(x_{indic})$ is the standard uncertainty of the indication or arithmetic mean of indications obtained with the calibrated measuring instrument, $u(x_{subj})$ is the standard uncertainty of the calibrated subject converted into the unit of the measurand, and $u(x_{envir})$ is the standard uncertainty caused by the values of the parameters of the measuring environment converted into the unit of the measurand.

The estimate $u(x_c)$ of the standard uncertainty of the standard value in Eq. (6) is obtained from the calibration certificate. Uncertainty of the indication $u(x_{indic})$, obtained at the calibration, is estimated by the dispersion of the indications of the measuring instrument. The pooled standard deviation $s_p(x)$, obtained by the preceding repeating calibrations of the measuring instruments of the same type, must be used for the calculation of $u(x_{indic})$. In this case, the standard uncertainty $u(x_{indic})$ is determined according to [1] by the relationship

$$u(x_{indic}) = \frac{s_p(x)}{\sqrt{n}}, \tag{7}$$

where n is the number of measurements. Here, $u(x_{indic})$ depends only on the uncertainty of the reading of the calibrated measuring instrument if only one indication is fixed at the calibration. Hence

$$u(x_{indic}) = \frac{c}{\sqrt{3}}, \tag{8}$$

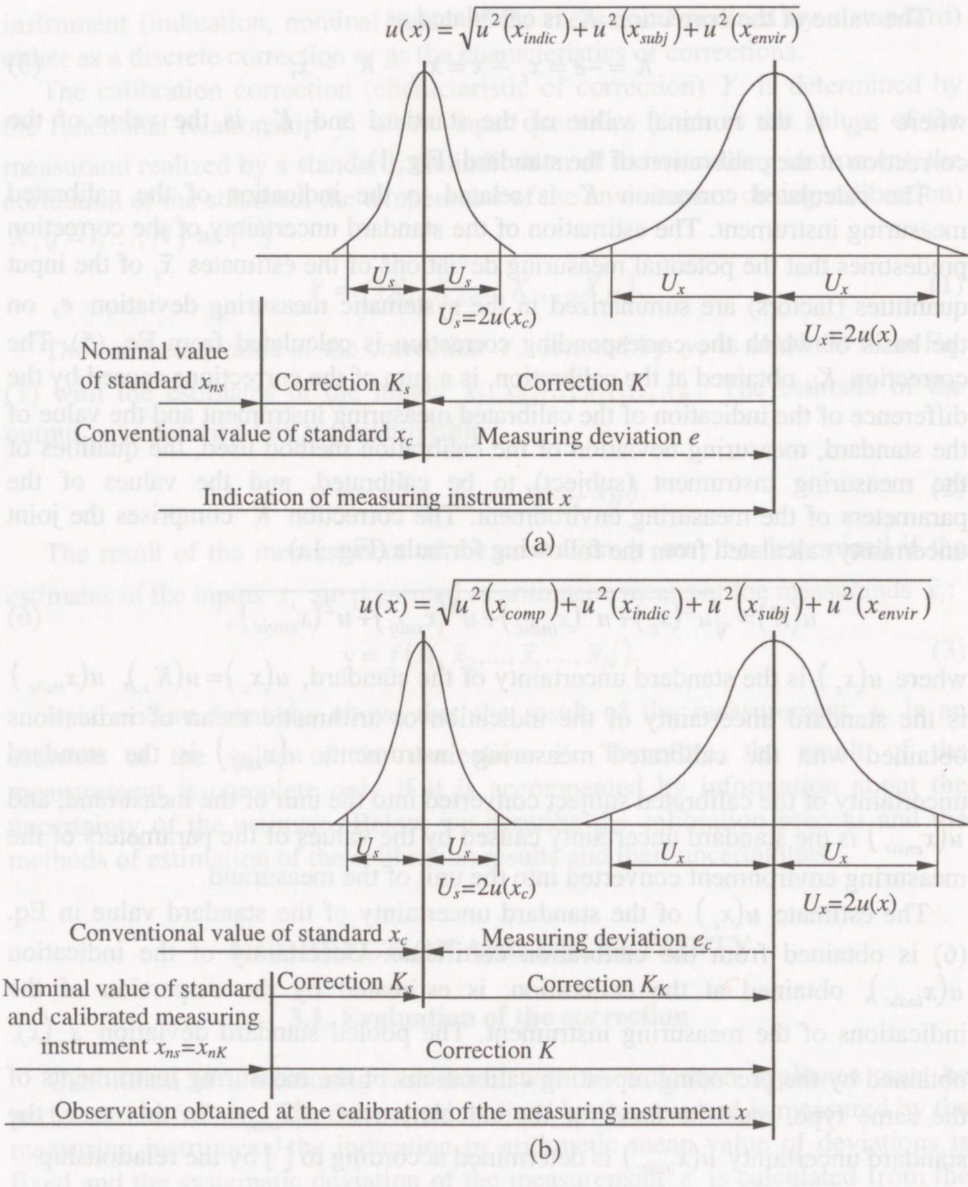

Fig. 1. Scheme of calibration: (a) without comparator; (b) using a comparator.

where c is the scale interval of the calibrated measuring instrument (the digital step in the case of digital indication).

The standard uncertainty $u(x_{subj})$, caused by the quality of the calibrated measuring instrument (subject), can be determined from the influence function of the quality of the subject (e.g., linear expansion coefficient) to the indication. The same statement is valid for the estimate $u(x_{envir})$ of the standard uncertainty of the correction component caused by the measuring environment. The uncertainties of the estimates of the influence quantities must be converted into the uncertainty of the measurand, using the known relationships in the appropriate units to enable one to calculate the estimates of the standard uncertainties $u(x_{subj})$ and $u(x_{envir})$ (because the uncertainties of the estimates of the values of the influence quantity of the subject and environment are given in the inherent units).

The calibration of a digital single or multivalued measuring instrument (such as material measure, working standard, and reference standard), realizing the measurand, must be carried out using simplified calibration scheme. The measurand, realized by the standard, must be compared, using a comparator (comparing measuring instrument), with the measurand realized by the calibrated measuring instrument. The difference of the two measurands must be measured by the comparator giving the difference or the arithmetic mean of the observations, i.e., the deviation of the value of the measurand realized by the calibrated measuring instrument [3]

$$e_c = x_c - x, \tag{9}$$

where e_c is the deviation of the value of the measurand, realized by the calibrated measuring instrument, from the value of the same measurand realized by the standard that is measured by the comparator.

The correction K is related to the nominal value of the measurand x_{nK}:

$$K = -(e_c - K_s) = x - x_{nK}. \tag{10}$$

In this case, the combined uncertainty of the correction K is expressed by the relationship

$$u(K) = \sqrt{u^2(x_c) + u^2(x_{comp}) + u^2(x_{indic}) + u^2(x_{subj}) + u^2(x_{envir})}, \tag{11}$$

where $u(x_{comp})$ is the standard uncertainty of the comparator used for the calibration and $u(x_{indic})$ is the standard uncertainty of indication or arithmetic mean of indications obtained at measuring by the comparator the difference of the measurands, realized by the calibrated measuring instrument and the standard.

The difference of the relationship (11) from (6) involves the standard uncertainty of the comparator $u(x_{comp})$ and the fact that the standard uncertainty of the indication $u(x_{indic})$ is expressed by the indication or dispersion of the indications of the comparator (Fig. 1b). The estimate of the standard uncertainty of the comparator $u(x_{comp})$ can be calculated from the calibration results of the comparator or from the data given in the specification of the comparator.

The estimate of the uncertainty $u(K) = u(y)$ may be presented in more general form as

$$u(y)=\sqrt{\sum_{i=1}^{N} c_i^2 u^2(\overline{x}_i)}, \qquad (12)$$

where $u(\overline{x}_i)$ is the estimate of the mean value of the standard uncertainty associated with the ith input and c_i is the sensitivity coefficient associated with the ith input.

The sensitivity coefficient c_i in Eq. (12) characterizes the dependence of the estimate of the correction y on the change of the input estimates $\overline{x}_1, \overline{x}_2, ..., \overline{x}_i, ..., \overline{x}_N$ in the case of the calibration function f, presented by Eq. (3). It can be calculated as [4]

$$c_i = \frac{\partial f}{\partial \overline{x}_i} = \left.\frac{\partial f}{\partial X_i}\right|_{X_1=\overline{x}_1, ..., X_N=\overline{x}_N}. \qquad (13)$$

Sometimes the sensitivity coefficients must be determined experimentally. Dependence of the correction Y on the input X_i must be measured preserving other input quantities constant.

The influence quantities of the calibrated subject and the environment are often mutually dependent and the expression for the combined uncertainty associated with the correction is

$$u(y)=\sqrt{\sum_{i=1}^{N} c_i^2 u^2(\overline{x}_i)+2\sum_{i=1}^{N-1}\sum_{k=i+1}^{N} c_i c_k u(\overline{x}_i) u(\overline{x}_k) r(\overline{x}_i, \overline{x}_k)}, \qquad (14)$$

where $r(\overline{x}_i, \overline{x}_k)$ is the correlation coefficient of the arithmetic means $\overline{x}_i$ and $\overline{x}_k$.

According to [4], the correlation coefficient characterizes the degree of correlation between $\overline{x}_i$ and $\overline{x}_k$ and is expressed as

$$r(\overline{x}_i, \overline{x}_k)=\frac{u(\overline{x}_i, \overline{x}_k)}{u(\overline{x}_i)\, u(\overline{x}_k)}, \qquad (15)$$

where $r(\overline{x}_i, \overline{x}_k)=r(\overline{x}_k, \overline{x}_i)$, $-1 \le r(\overline{x}_i, \overline{x}_k) \le +1$, and $r(\overline{x}_i, \overline{x}_k)=0$ if the estimates $\overline{x}_i$ and $\overline{x}_k$ are independent.

The expanded uncertainty obtained by multiplying the combined uncertainty with the coverage factor k is recommended in [2] for characterizing the accuracy of the correction or calibration result. Thus, the expanded uncertainty of the correction U_K is described as

$$U_K = k \cdot u(y), \qquad (16)$$

where k is the coverage factor ($k=2$ is recommended in [2]).

The method for evaluating the correction and its uncertainty is recommended for the calibration of the measuring instruments at fixed scale points or for the calibration of a standard for determining the conventional value of the measurand realized by the standard.

3.2. Application

The calibration method described above was used at a test carried out at AS METROSERT in 1998 in the framework of the PRAQ III Intercomparison Program of length measurements. Test subjects were the gauge blocks of the French National Metrology Laboratory (*Laboratoire National D'Essais*) with the nominal values 1 mm (13/95368), 10 mm (8/0092), 10.3 mm (7/23338), 50 mm (23/23658), and 100 mm (36/23534); the identification numbers of the gauge blocks are given in the brackets. The gauge blocks were calibrated by the comparison method, using the comparator E-43 calibrated at the AS METROSERT. The standard gauge blocks with the same nominal values were calibrated at the Rostest–St. Petersburg. The estimates of the corrections K of the gauge blocks and the estimates x of the values of the mean length were calculated on the basis of five measurements of the difference of the mean length of the standard and of the calibrated gauge blocks using simplified relationship (10) in the form

$$y = K = x - x_{nK} = x_c - x_{nK} - e_c - x_c\left(\delta\alpha \cdot \theta + \alpha_s \cdot \delta\theta\right), \qquad (17)$$

where α_s is the linear expansion coefficient of the standard material, $\delta\alpha$ is the difference between the linear expansion coefficients of the standard and the calibrated gauge block, θ is the deviation of the temperature of the calibrated gauge block from the normal temperature 20 °C, and $\delta\theta$ is the difference between the temperatures of the standard and the calibrated gauge block.

The calibration results of the gauge blocks obtained with the calibration model (17) are presented in Table 1.

Table 1. Results of calibration of the gauge block

Nominal value x_{nK}, mm	Conventional value of the standard x_c, mm	Correction K_s, µm	Expanded uncertainty U_s $(k = 2)$, µm	Calibration results x, mm	Correction K, µm	Expanded uncertainty U_K $(k = 2)$, µm
1	1.00000	0	0.03	1.00000	0	0.05
10	9.99994	+0.06	0.04	9.99994	+0.06	0.06
10.3	10.29997	+0.03	0.05	10.29991	+0.09	0.07
50	50.00001	–0.01	0.05	50.00003	–0.03	0.08
100	100.00017	–0.17	0.06	99.99978	+0.22	0.11

The relationship (7) was used to calculate the standard uncertainty of the indications of the comparator $u(x_{indic})$. The expanded uncertainty of the corrections U_K was calculated using Eq. (16), where the combined uncertainty $u(K)$ was calculated from Eq. (11). The value of the correction of the gauge block with the nominal value 100 mm, $K = +0.22$ µm, is problematic since the gauge block was kept in the laboratory of AS METROSERT only for 24 h. The temperature of the massive gauge block probably did not conform to the temperature of the AS METROSERT standard. It was not possible to measure the temperature of the standard and of the gauge block during the calibration. The deviation 0.22 µm could be caused by the difference between the temperatures of the standard and the calibrated gauge block $\delta\theta = 0.2$ K.

4. CORRECTION CURVE

4.1. Theory

To diminish systematic and random deviations of the measuring instrument, the correction curve of the measuring instrument was calculated. The choice of the calibration method, the calculation of the correction values and the analysis of the correction curve, obtained as the result of the calibration, can reduce systematic deviation of indications. Random deviations can be reduced by increasing the number of the measurements at calibration [5]. The result of that is a quantitative estimate of the precision of the measuring instrument.

In the general case, the correction curve of the calibrated measuring instrument [3,5] can be calculated as

$$y(K) = a_0 + a_1 x + \ldots + a_m x^m, \tag{18}$$

where $a_0, a_1, \ldots, a_m$ are coefficients and x is the indication of the measuring instrument or arithmetic mean of the indications.

The calibration involves the determination of the corrections at single indication points of the measuring instrument in the whole measuring range, calculation of the coefficients of the correction curve on the basis of the observations, and the calculation of the measuring uncertainty characterizing the curve.

An overdetermined system of equations is composed on the basis of the observed corrections in the following form:

$$a_0 + a_1 x_{ci} + \ldots + a_m x_{ci}^m = y_i(K), \quad 1 \le i \le L, \quad L > m, \quad D[y_i(K)] = \sigma^2, \tag{19}$$

where x_{ci} is the conventional value of the ith measurand realized by the standard or the known value of the ith measurand, L is the number of the discrete calibration points of the measuring interval and $y_i(K)$ is the value of the correction of the measurements of the ith standard with the calibrated measuring instrument.

The correction curve is obtained after solving the system (19) as

$$y(K) = a_{00} + a_{10}x + ... + a_{m0}x^m, \tag{20}$$

where $a_{00}, a_{10}, ..., a_{m0}$ are the estimated values of the coefficients of the correction characteristic obtained as the result of the calibration and x is the indication of the calibrated measuring instrument.

In this case, the combined standard uncertainty $u[y(K)]$ of the correction curve depends on the indication of the measuring instrument expressed as

$$u[y(K)] = \left[\left(\frac{\partial y(K)}{\partial a_{00}}\right)^2 u^2(a_{00}) + \left(\frac{\partial y(K)}{\partial a_{10}}\right)^2 u^2(a_{10}) + ... \right.$$
$$+ \left(\frac{\partial y(K)}{\partial a_{m0}}\right)^2 u^2(a_{m0}) + 2\left(\frac{\partial y(K)}{\partial a_{00}}\frac{\partial y(K)}{\partial a_{10}}\text{cov}(a_{00}, a_{10})\right.$$
$$\left.\left. + \frac{\partial y(K)}{\partial a_{00}}\frac{\partial y(K)}{\partial a_{20}}\text{cov}(a_{00}, a_{20}) + ... + \frac{\partial y(K)}{\partial a_{m-1,0}}\frac{\partial y(K)}{\partial a_{m0}}\text{cov}(a_{m-1,0}, a_{m0})\right)\right]^{\frac{1}{2}}. \tag{21}$$

4.2. Application

The method of calculation of the correction characteristic described above was used for calibrating the coating thickness gauge MINITEST 600 F3, Type 121-09-00, No. 1925 of the firm ELEKTRO-PHYSIK in the whole measurement range from 0 to 3000 μm, carried out at Tallinn Technical University in 1998. Eighteen observations (the arithmetic mean of three indications is considered as one observation x_i) of the coating thickness gauge, which had different standard uncertainties $u(x_i)$, were compared with the known reference coating thickness x_{ci} and a correction $y_i(K)$ was calculated for every indication x_i. The calibration results are presented in Table 2 where the corrections $y_i(K)$ and the observations x_i of the coating thickness are the inputs of the estimation. The correction curve (20) was used in the form

$$y(K) = a_{00} + a_{10}x + a_{20}x^2 \tag{22}$$

The curve (22) is determined using the least squares fit. The values of the coefficients a_i and the estimates of the dispersions and covariances were calculated assuming that the sum

Table 2. Results of calibration of the coating thickness gauge

i	Value of the standard x_{ci}, μm	Standard uncertainty of the standard value $u(x_{ci})$, μm	Indication of the coating thickness gauge x_i, μm	Correction K_i, μm	Correction by the correction characteristic $y_i(K)$, μm	Standard uncertainty $u[y_i(K)]$, μm
1	0	–	0	0	+1.44	0.78
2	33	0.5	31.5	+1.5	+1.36	0.85
3	87	0.6	85	+2	+1.22	0.96
4	125	0.5	124	+1	+1.11	1.03
5	140	0.5	138	+2	+1.06	1.06
6	152	0.5	152	0	+1.03	1.08
7	165	0.7	160	+5	+0.99	1.10
8	205	0.7	203	+2	+0.87	1.18
9	262	0.8	261	+1	+0.68	1.29
10	306	0.7	305	+1	+0.53	1.37
11	321	0.7	322	–1	+0.48	1.40
12	389	0.8	391	–2	+0.23	1.54
13	515	1.0	513	–2	–0.27	1.81
14	945	3.0	947	–2	–2.34	2.97
15	1080	3.0	1085	–5	–3.12	3.41
16	1800	4.0	1810	–10	–8.22	6.21
17	2100	5.0	2105	–5	–10.82	7.58
18	2780	7.0	2800	–20	–17.79	11.09

$$\sum_{i=1}^{n}\left[y_i(K)-a_{00}-a_{10}x_i-a_{20}x_i^2\right]^2 \tag{23}$$

has a minimum. Dispersion s^2 gives general uncertainty of the fitting and is calculated from the relationship

$$s^2=\frac{1}{n-3}\sum_{i=1}^{n}\left[y_i(K)-a_{00}-a_{10}x_i-a_{20}x_i^2\right]^2, \tag{24}$$

where $n-3$ reflects the fact that three coefficients a_{00}, a_{10} and a_{20} are determined on the basis of n observations, and thus the degree of freedom of s^2 is $n-3$.

The correction curve (Fig. 2) was calculated from the data given in Table 2, using the system of equations (19), where the values of the corrections were calculated as

$$y(K)=1.45-2.52\times10^{-3}\,x-1.58\times10^{-6}\,x^2. \tag{25}$$

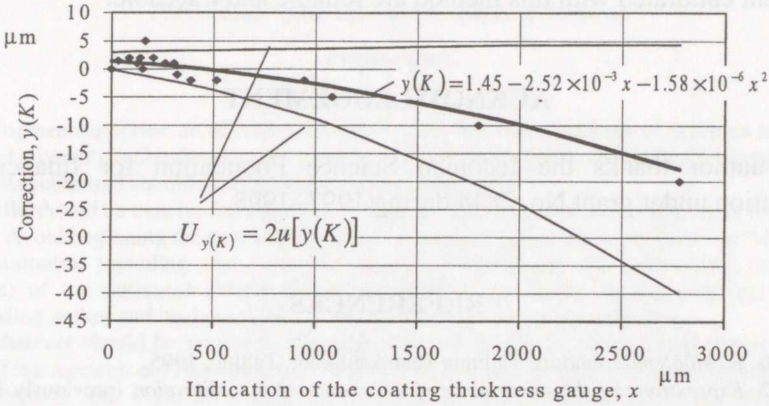

Fig. 2. Dependence of the correction on the indication of the coating thickness gauge.

The following values of the standard uncertainties and covariance estimates of the characteristic were obtained:

$$u(a_{00})=0.79 \qquad \operatorname{cov}(a_{00},a_{10})=1.66\times10^{-3}$$
$$u(a_{10})=6.52\times10^{-6} \qquad \operatorname{cov}(a_{10},a_{20})=2.43\times10^{-9}$$
$$u(a_{20})=9.79\times10^{-13} \qquad \operatorname{cov}(a_{00},a_{20})=5.49\times10^{-7}$$

They are excluded from the calculation of the covariance since their values are relatively small as compared to the estimates of the standard uncertainties, and the relationship (21) is used as

$$u[y(K)]=\left(0.62+3.33\times10^{-3}x+1.10\times10^{-6}x^2+4.87\times10^{-9}x^3+9.59\times10^{-25}x^4\right)^{\frac{1}{2}}. \tag{26}$$

The expanded uncertainty $U_{y(K)}$ of the characteristic $y(K)$ is calculated using Eq. (16) and is shown in Fig. 2.

The correction curve (25) gives the estimated value of the correction $y(K)$ as a function of the indication x of the coating thickness gauge and it must be taken into account by measuring the coating thickness.

5. CONCLUSIONS

The two calibration methods worked out, including determination of the correction or correction characteristic and calculation of their uncertainty, enable one to relate the measuring results with an international standard, the uncertainty of which is established using measuring instruments. Measuring results of an instrument calibrated with this method are reliable and traceable.

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REFERENCES

1. Laaneots, R. *Mõõtemääramatus.* Tallinna Tehnikaülikool, Tallinn, 1995.
2. EA-4/02. *Expressions of the uncertainty of measurement in calibration* (previously EAL-R2). European Co-operation for Accreditation, Utrecht, 1997.
3. Laaneots, R. Methods and instruments for calibration of coating thickness gauges. In *Proc. XIV Imeko World Congress*. Tampere, 1997, Vol. VIII, 284–291.
4. *Guide to the expression of uncertainty in measurement.* ISO, Geneva, 1995.
5. Laaneots, R. *Methods and Instruments for Calibration of Coating Thickness Gauges.* Valgus, Tallinn, 1989 (in Russian).

MÕÕTEVAHENDITE KALIBREERIMINE

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Metroloogias on suur tähtsus niisugusel kalibreerimismetoodikal, mis annab usaldusväärse hinnangu mõõtevahendist tingitud määramatusele. Artiklis on kirjeldatud uut võimalust nii mõõtevahendi parandi kui ka parandi tunnusjoone määramatuse arvutamiseks. Metoodika kasutamist illustreerib kaks näidet.