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HALF-FRINGE PHASE-STEPPING WITH SEPARATION OF THE PRINCIPAL STRESS DIRECTIONS

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Abstract. It is shown that if optical retardation is less than half the wavelength, the true values of optical retardation and of the parameter of the isoclinic, including the direction of the first principal stress, can be determined uniquely using phase-stepping. The result is especially important for integrated photoelasticity where a priori information about the stress field is limited. The influence of the measurement errors on the results is investigated and application of the method described.

Key words: photoelasticity, phase-stepping.

1. INTRODUCTION

Voloshin and Burger [1] showed that optical retardation can be comparatively easily determined automatically if it is less than half the wavelength. However, they did not consider determination of the isoclinics.

Nowadays phase-stepping is most widely used in automatic photoelastic measurements. The idea of the method was introduced by Hecker and Morche [2] and later considerably developed by Patterson and Wang [3–5], Kihara [6], Asundi [7], Umezaki et al. [8,9], Quiroga and Gonzáles-Cano [10,11], Ramesh and Mangal [12], and many others. A review of different phase-stepping methods is given in [13].

A number of problems are associated with the phase-stepping technique. Both the optical retardation, Δ, and the isoclinic angle, φ, are found using an arctangent operator. The data obtained lie in the ranges

$$-\frac{\pi}{4} \leq \varphi \leq \frac{\pi}{4}, \tag{1}$$

$$-\frac{\pi}{2} \leq \Delta \leq \frac{\pi}{2}. \tag{2}$$

The following questions arise. First, which of the principal stress axes, σ_1 or σ_2, is determined by φ? Second, how to find the true value of Δ? Besides, optical retardation is determined by the formula

$$\Delta = C\frac{2\pi}{\lambda}(\sigma_1 - \sigma_2)t, \tag{3}$$

where C is the photoelastic constant, λ is the wavelength, and t is the thickness of the model. Thus, Δ is essentially a positive parameter, although phase-stepping algorithms may give it also a negative value.

In interpreting the measurement data, the operator has to identify optical retardation at least at one point [14]. Besides, the sign and the first derivative of optical retardation are required to generate its continuous distribution [5]. For the separation of the σ_1 and σ_2 directions, no algorithm has been described in the literature on phase-stepping. The method for determining the fast axis of a wave plate, described in [15], needs interferometric measurements and cannot be applied with standard phase-stepping polariscopes.

Thus, in applying phase-stepping in two-dimensional photoelasticity, some a priori information about the stress field is needed for interpretation of the measurement data. Measurement data are interpreted for the whole field, using conditions of continuity, boundary values, etc.

Integrated photoelasticity [16] is nowadays mostly used for residual stress measurement in axisymmetric glass articles [17,18]. Since stress distribution is determined after solving an inverse problem for a system of Fredholm integral equations, a priori information about the stress field is very poor.

Generally, in integrated photoelasticity the characteristic parameters are to be measured [16,17]. A phase-stepping method for this purpose has been elaborated by Tomlinson and Patterson [19]. However, if birefringence is weak, optical measurements are similar to those applied in two-dimensional photoelasticity [17,20], only interpretation of the measurement data is different. In measuring stresses in axisymmetric glass articles which are not tempered (e.g., bottles, neck tubes of CRT bulbs, electric lamps, optical fibre preforms, etc.), optical retardation is usually small (less than half the wavelength). In this case the algorithms of integrated photoelasticity demand measurement of the isoclinic angle and of optical retardation. Thus, ordinary phase-stepping can be used.

Figure 1 shows experimental set-up in integrated photoelasticity. In two-dimensional photoelasticity measurement data on the boundaries can be interpreted directly, but in integrated photoelasticity the light ray, which is tangent to the

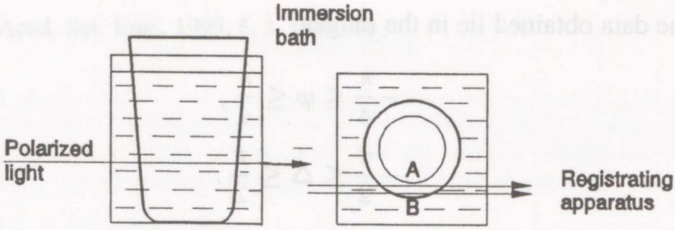

Fig. 1. Experimental set-up in integrated photoelasticity.

boundary at the point B, does not give any information. Stresses can be determined only after measurements in the entire section have been carried out. Besides, algorithms of integrated photoelasticity demand that the direction of σ_1 should be unambiguously known.

Therefore, using phase-stepping in integrated photoelasticity, it is of paramount importance to reveal from the measured values of φ and Δ their true values by identification of the direction of the first principal stress without any a priori information about the stress (and birefringence) field. Our goal will be to develop such a phase-stepping algorithm.

2. ALGORITHM FOR CALCULATING TRUE VALUES OF φ AND Δ

2.1. Circularly polarized incident light

We shall use a polariscope with two polaroids and two quarterwave plates supplied with a CCD camera for recording the distribution of light intensity. Thus, incident light is circularly polarized. However, to be able to establish unique values for φ and Δ, we have to know whether the circularly polarized incident light is right-handed or left-handed [21]. Therefore, we have to distinguish between fast and slow axes of the quarterwave plates as well as of the model. Which principal stress in the model corresponds to the slow axis can be established beforehand. In the following, azimuths of the quarterwave plates and of the model are always given relative to the slow axis which is considered to be the axis of σ_1 in the model.

Optical arrangement of the polariscope is shown in Fig. 2. The intensity of light transmitted for arbitrary values of the angles φ, ψ, and β is obtained as [13]

$$I = I_b + \frac{1}{2} I_0 [1 - \sin 2(\beta - \psi) \cos \Delta + \sin 2(\varphi - \psi) \cos 2(\beta - \psi) \sin \Delta] \,, \quad (4)$$

where I_0 accounts for the amplitude of light and I_b for the background light.

Usually six recordings of I with different values of ψ and β are used for evaluating φ and Δ. They can be chosen as follows [3,4,12]:

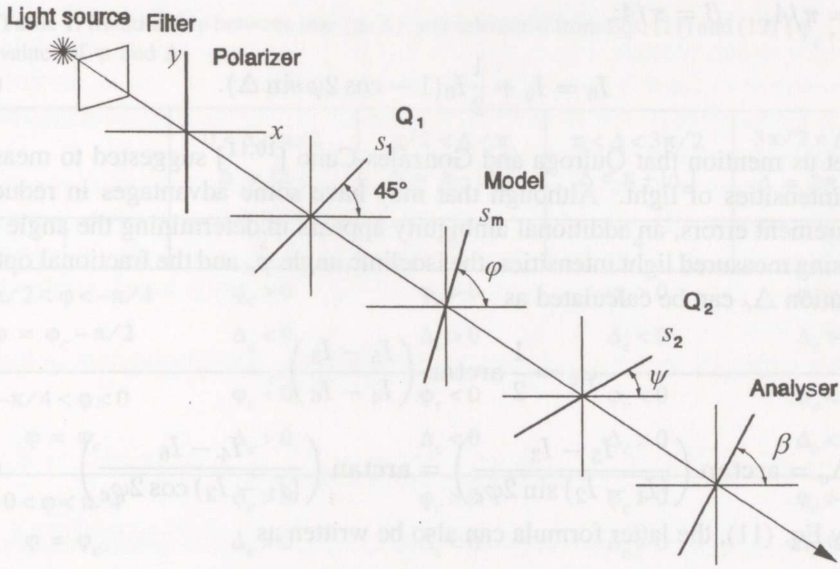

Fig. 2. Optical arrangement of the polariscope with circularly polarized incident light; Q_1, Q_2, quarterwave plates; s_1, s_2, s_m, slow axes of optical elements.

1) $\psi = 0, \quad \beta = -\pi/4$:

$$I_1 = I_b + \frac{1}{2} I_0 (1 + \cos\Delta), \tag{5}$$

2) $\psi = 0, \quad \beta = \pi/4$:

$$I_2 = I_b + \frac{1}{2} I_0 (1 - \cos\Delta), \tag{6}$$

3) $\psi = \pi/2, \quad \beta = \pi/2$:

$$I_3 = I_b + \frac{1}{2} I_0 (1 - \sin 2\varphi \sin\Delta), \tag{7}$$

4) $\psi = -\pi/4, \quad \beta = -\pi/4$:

$$I_4 = I_b + \frac{1}{2} I_0 (1 + \cos 2\varphi \sin\Delta), \tag{8}$$

5) $\psi = 0, \quad \beta = 0$:

$$I_5 = I_b + \frac{1}{2} I_0 (1 + \sin 2\varphi \sin\Delta), \tag{9}$$

6) $\psi = \pi/4, \quad \beta = \pi/4$:

$$I_6 = I_b + \frac{1}{2} I_0 (1 - \cos 2\varphi \sin \Delta). \tag{10}$$

Let us mention that Quiroga and Gonzáles-Cano [10,11] suggested to measure eight intensities of light. Although that may have some advantages in reducing measurement errors, an additional ambiguity appears in determining the angle φ.

Using measured light intensities, the isoclinic angle φ_c and the fractional optical retardation Δ_c can be calculated as

$$\varphi_c = \frac{1}{2} \arctan \left(\frac{I_5 - I_3}{I_4 - I_6} \right), \tag{11}$$

$$\Delta_c = \arctan \left(\frac{I_5 - I_3}{(I_1 - I_2) \sin 2\varphi_c} \right) = \arctan \left(\frac{I_4 - I_6}{(I_1 - I_2) \cos 2\varphi_c} \right). \tag{12}$$

By Eq. (11), the latter formula can also be written as

$$\Delta_c = \arctan \frac{\sqrt{(I_5 - I_3)^2 + (I_4 - I_6)^2}}{I_1 - I_2}. \tag{13}$$

With subscript "c" we underline that the calculated values φ_c and Δ_c may not be equal to the true values of φ and Δ. The angle φ_c gives the direction of the principal stress axis which is closest to the x-axis. The true angle φ, which determines the direction of the slow axis, may be φ or $\varphi \pm \pi/2$.

The true value of optical retardation Δ may be $|\Delta_c|$, $\pi + |\Delta_c|$, $\pi - |\Delta_c|$, or $2\pi - |\Delta_c|$. Besides, Δ has always a positive value, but Eq. (12) reveals Δ_c which may be positive or negative.

Thus, the problem is how to extract from the calculated values φ_c and Δ_c the true values of φ and Δ. Considering the values of φ and Δ in the ranges

$$-\frac{\pi}{2} < \varphi < \frac{\pi}{2}, \tag{14}$$

$$0 < \Delta < 2\pi, \tag{15}$$

the corresponding values of light intensities can be calculated from Eqs. (5)–(10). After that φ_c and Δ_c can be calculated from Eqs. (11) and (12). The results are qualitatively shown in Table 1.

In case $\Delta < \pi$, only columns 2 and 3 are important. In both columns all possible four combinations of the signs of φ_c and Δ_c are present. Thus, the first problem is to determine whether Δ is smaller or bigger than $\pi/2$. Numerical calculations have shown that for a certain value of Δ_c two values of Δ are possible:

$$\Delta_1 = |\Delta_c|, \tag{16}$$

$$\Delta_2 = \pi - |\Delta_c|. \tag{17}$$

Table 1. Relationship between true (φ, Δ) and calculated from Eqs. (11) and (12) (φ_c , Δ_c) values of φ and Δ

	$0<\Delta<\pi/2$ $\Delta = \lvert\Delta_c\rvert$	$\pi/2<\Delta<\pi$ $\Delta = \pi-\lvert\Delta_c\rvert$	$\pi<\Delta<3\pi/2$ $\Delta = \pi+\lvert\Delta_c\rvert$	$3\pi/2<\Delta<2\pi$ $\Delta = 2\pi-\lvert\Delta_c\rvert$
1	2	3	4	5
$-\pi/2<\varphi<-\pi/4$ $\varphi = \varphi_c-\pi/2$	$\varphi_c>0$ $\Delta_c<0$	$\varphi_c>0$ $\Delta_c>0$	$\varphi_c>0$ $\Delta_c<0$	$\varphi_c>0$ $\Delta_c>0$
$-\pi/4<\varphi<0$ $\varphi = \varphi_c$	$\varphi_c<0$ $\Delta_c>0$	$\varphi_c<0$ $\Delta_c<0$	$\varphi_c<0$ $\Delta_c>0$	$\varphi_c<0$ $\Delta_c<0$
$0<\varphi<\pi/4$ $\varphi = \varphi_c$	$\varphi_c>0$ $\Delta_c>0$	$\varphi_c>0$ $\Delta_c<0$	$\varphi_c>0$ $\Delta_c>0$	$\varphi_c>0$ $\Delta_c<0$
$\pi/4<\varphi<\pi/2$ $\varphi = \varphi_c+\pi/2$	$\varphi_c<0$ $\Delta_c<0$	$\varphi_c<0$ $\Delta_c>0$	$\varphi_c<0$ $\Delta_c<0$	$\varphi_c<0$ $\Delta_c>0$

The true value of Δ can be found by calculating the values of I_1 for Δ_1 and Δ_2; let us denote them I_{11} and I_{12}. These values of light intensity should be compared with the experimentally measured value of I_1 which we denote I_1^e. Dependence of I_1 on optical retardation is shown in Fig. 3.

The true value of Δ is found by comparing the following expressions:

$$V_1 = \lvert I_{11} - I_1^e \rvert , \qquad (18)$$

$$V_2 = \lvert I_{12} - I_1^e \rvert . \qquad (19)$$

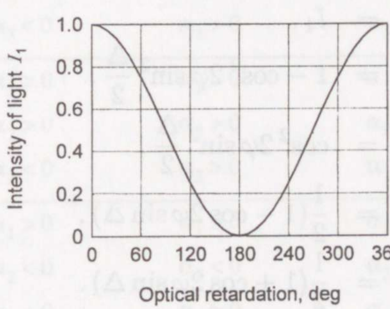

Fig. 3. Dependence of I_1 on optical retardation.

If $V_1 < V_2$, then $\Delta = \Delta_1$; in the opposite case $\Delta = \Delta_2$. In such a way we have determined which column, 2 or 3, in Table 1 is related to the measurement data.

We can see in Table 1 that inside one column the signs of φ_c and Δ_c determine uniquely the range of φ. It is shown how the true value of φ must be calculated from φ_c, depending on the signs of φ_c and Δ_c.

The described algorithm for single-valued determination of φ and Δ is easy to realize in software. The algorithm does not need any interference of the operator. Since the angle φ gives the azimuth of the slow axis of the model, the directions of σ_1 and σ_2 are determined unambiguously. The same algorithm is valid when four light intensities are recorded [22].

Table 1 may be useful for interpreting measurement data also in case Δ has an arbitrary value.

2.2. Linearly polarized incident light

In the phase-stepping method proposed by Kihara [6], twelve light intensities are recorded in a polariscope with linearly polarized incident light and a quarterwave plate before the analyser:

$$I_1 = 1 - \sin^2 2\varphi \sin^2 \frac{\Delta}{2}\,, \tag{20}$$

$$I_2 = \sin^2 2\varphi \sin^2 \frac{\Delta}{2}\,, \tag{21}$$

$$I_3 = \frac{1}{2} + \sin 2\varphi \cos 2\varphi \sin^2 \frac{\Delta}{2}\,, \tag{22}$$

$$I_4 = \frac{1}{2} - \sin 2\varphi \cos 2\varphi \sin^2 \frac{\Delta}{2}\,, \tag{23}$$

$$I_5 = \frac{1}{2}(1 + \sin 2\varphi \sin \Delta)\,, \tag{24}$$

$$I_6 = \frac{1}{2}(1 - \sin 2\varphi \sin \Delta)\,, \tag{25}$$

$$I_7 = I_3\,, \tag{26}$$

$$I_8 = I_4\,, \tag{27}$$

$$I_9 = 1 - \cos^2 2\varphi \sin^2 \frac{\Delta}{2}\,, \tag{28}$$

$$I_{10} = \cos^2 2\varphi \sin^2 \frac{\Delta}{2}\,, \tag{29}$$

$$I_{11} = \frac{1}{2}(1 - \cos 2\varphi \sin \Delta)\,, \tag{30}$$

$$I_{12} = \frac{1}{2}(1 + \cos 2\varphi \sin \Delta)\,. \tag{31}$$

Here we have ignored the background intensity of light. Denoting

$$I_{i,j} = I_i - I_j\,, \tag{32}$$

we obtain expressions for calculating φ_c and Δ_c in the form

$$\varphi_c = \frac{1}{2} \arctan \frac{I_{5,6}}{I_{12,11}} , \tag{33}$$

$$\Delta_c = 2 \arccos A = 2 \arctan \frac{\sqrt{1 - A^2}}{A} , \tag{34}$$

where

$$A = \sqrt{\frac{1}{2}(I_{1,2} + I_{9,10})} . \tag{35}$$

If we take in Eq. (35) always $A \geq 0$, also $\Delta_c \geq 0$. If $\Delta \leq \pi$, Eq. (34) gives for Δ_c always the true value, i.e., $\Delta = \Delta_c$.

Dependence of φ_c on φ and Δ is the same as shown in Table 1.

Table 2. Dependence of α_1, α_2, and α_3 on Δ and φ

	$0 < \Delta < \pi/2$ $\Delta = \Delta_c$	$\pi/2 < \Delta < \pi$ $\Delta = \Delta_c$	$\pi < \Delta < 3\pi/2$ $\Delta = 2\pi - \Delta_c$	$3\pi/2 < \Delta < 2\pi$ $\Delta = 2\pi - \Delta_c$
1	2	3	4	5
$-\pi/2 < \varphi < -\pi/4$ $\varphi = \varphi_c - \pi/2$	$\alpha_1 < 0$ $\alpha_2 < 0$ $\alpha_3 < 0$	$\alpha_1 < 0$ $\alpha_2 < 0$ $\alpha_3 > 0$	$\alpha_1 > 0$ $\alpha_2 > 0$ $\alpha_3 > 0$	$\alpha_1 > 0$ $\alpha_2 > 0$ $\alpha_3 < 0$
$-\pi/4 < \varphi < 0$ $\varphi = \varphi_c$	$\alpha_1 < 0$ $\alpha_2 > 0$ $\alpha_3 < 0$	$\alpha_1 < 0$ $\alpha_2 > 0$ $\alpha_3 > 0$	$\alpha_1 > 0$ $\alpha_2 < 0$ $\alpha_3 > 0$	$\alpha_1 > 0$ $\alpha_2 < 0$ $\alpha_3 < 0$
$0 < \varphi < \pi/4$ $\varphi = \varphi_c$	$\alpha_1 > 0$ $\alpha_2 > 0$ $\alpha_3 < 0$	$\alpha_1 > 0$ $\alpha_2 > 0$ $\alpha_3 > 0$	$\alpha_1 < 0$ $\alpha_2 < 0$ $\alpha_3 > 0$	$\alpha_1 < 0$ $\alpha_2 < 0$ $\alpha_3 < 0$
$\pi/4 < \varphi < \pi/2$ $\varphi = \varphi_c + \pi/2$	$\alpha_1 > 0$ $\alpha_2 < 0$ $\alpha_3 < 0$	$\alpha_1 > 0$ $\alpha_2 < 0$ $\alpha_3 > 0$	$\alpha_1 < 0$ $\alpha_2 > 0$ $\alpha_3 > 0$	$\alpha_1 < 0$ $\alpha_2 > 0$ $\alpha_3 < 0$

Let us denote

$$\alpha_1 = 2I_5 - 1 = 1 - 2I_6\,, \tag{36}$$

$$\alpha_2 = 2I_{12} - 1 = 1 - 2I_{11}\,, \tag{37}$$

$$\alpha_3 = I_2 - \frac{1}{2}\,. \tag{38}$$

Table 2 shows how the signs of α_i depend on the values of Δ and φ. For $0 < \Delta < \pi$ the value of φ can be unambiguously determined from this table.

3. ANALYSIS OF MEASUREMENT ERRORS

Intensities of light are measured with certain errors. It is of practical interest to establish the influence of these errors on the precision of Δ_c and φ_c. We shall find in the classical way maximum errors of φ_c and Δ_c as total differentials of the corresponding functions. We consider the case when the incident light is circularly polarized.

From Eq. (11) the absolute value of the maximum possible error of φ_c, $D\varphi_c$, can be expressed as

$$D\varphi_c \leq \left|\frac{\partial\varphi_c}{\partial I_{5,3}}\right| DI_{5,3} + \left|\frac{\partial\varphi_c}{\partial I_{4,6}}\right| DI_{4,6}\,, \tag{39}$$

and $DI_{i,j}$ is the absolute maximum error of $I_{i,j}$.

From Eqs. (11) and (39) we obtain

$$D\varphi_c \leq \frac{1}{2}\left|\frac{\cos 2\varphi_c}{\sin\Delta_c}\right| \frac{DI_{5,3}}{I_0} + \frac{1}{2}\left|\frac{\sin 2\varphi_c}{\sin\Delta_c}\right| \frac{DI_{4,6}}{I_0}\,. \tag{40}$$

The maximum possible error of Δ_c, $D\Delta_c$, is obtained from Eq. (13):

$$D\Delta_c \leq \left|\frac{\partial\Delta_c}{\partial I_{1,2}}\right| DI_{1,2} + \left|\frac{\partial\Delta_c}{\partial I_{5,3}}\right| DI_{5,3} + \left|\frac{\partial\Delta_c}{\partial I_{4,6}}\right| DI_{4,6}\,. \tag{41}$$

Equations (40) and (41) can be written as

$$D\varphi_c \leq \frac{|\cos 2\varphi_c| + |\sin 2\varphi_c|}{|\sin\Delta_c|}\varepsilon\,, \tag{42}$$

$$D\Delta_c \leq 2[|\sin\Delta_c| + |\cos\Delta_c|\,(|\sin 2\varphi_c| + |\cos 2\varphi_c|)]\varepsilon\,, \tag{43}$$

where we have assumed that all the light intensities are measured with the same error ε:

$$\varepsilon = \frac{DI_i}{I_0}\,, \quad \frac{DI_{i,j}}{I_0} = 2\varepsilon\,. \tag{44}$$

From Eq. (42) it follows that the error $D\varphi_c$ is large when Δ_c is small or close to π. The function

$$F_1 = | \cos 2\varphi_c | + | \sin 2\varphi_c | \tag{45}$$

obtains a maximum at $\varphi_c = \pi/8$ when $F_1 = \sqrt{2}$. Thus,

$$\max D\varphi_c = \frac{\sqrt{2}}{| \sin \Delta_c |} \varepsilon \, . \tag{46}$$

The function

$$F_2 = | \sin \Delta_c | + \sqrt{2} \, | \cos \Delta_c | \tag{47}$$

obtains its maximum value at $\Delta_c \cong 35°$ when $F_2 \cong 1.73$. From Eq. (43) we get

$$\max D\Delta_c \cong 3.46\varepsilon \, . \tag{48}$$

Formulae (46) and (48) give the maximum possible values of the absolute errors of φ_c and Δ_c. Similar analysis may be carried out for the case when incident light is linearly polarized.

4. EXPERIMENT

The above theory was implemented in a computer controlled polariscope. The polarizer, analyser, and both quarterwave plates were supplied with stepper motors for their rotation. Light intensities in the field of view ($D = 40$ mm) were recorded using a CCD camera and frame grabber. The algorithm described in Section 2.1 was realized in the software.

The test object was a pharmaceutical bottle. First, in a section of the specimen the distribution of Δ and of the azimuth of σ_1 was measured with a manual polariscope using a Berek-type compensator. The latter enables unambiguous determination of the azimuth of σ_1.

The same section was measured also by applying the phase-stepping technique described above, using circularly polarized incident light. The comparison of the results is shown in Figs. 4 and 5. Agreement of the results is fairly good. Figure 6 shows that distributions of the axial stress, obtained by the two methods, are very close. In Fig. 7 the distribution of the axial stress in an area of the bottle wall is shown.

Let us mention that the algorithm described in this paper gives reliable results when, before calculating the values of φ and Δ, the background light intensity is deducted from the measured intensities and the intensities are normalized, e.g., dividing them by $I_1 + I_2 = I_0$.

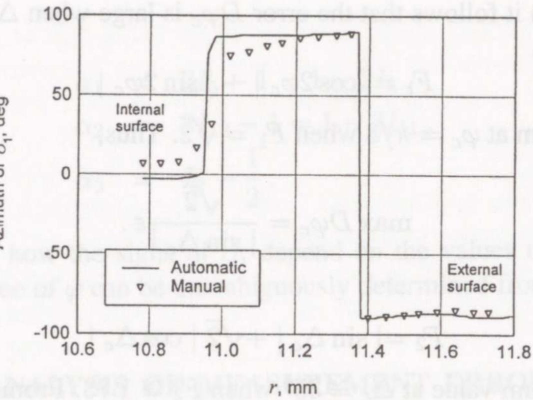

Fig. 4. Distribution of the azimuth of σ_1 in a section of a pharmaceutical bottle.

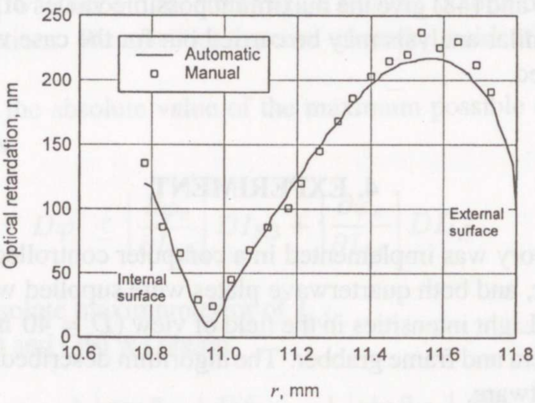

Fig. 5. Distribution of optical retardation in a section of a pharmaceutical bottle.

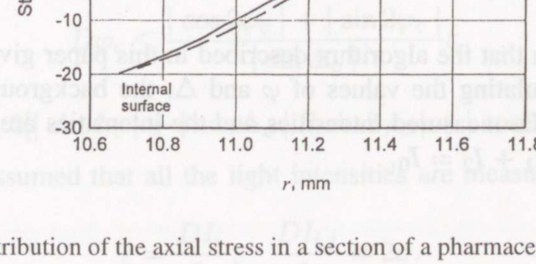

Fig. 6. Distribution of the axial stress in a section of a pharmaceutical bottle.

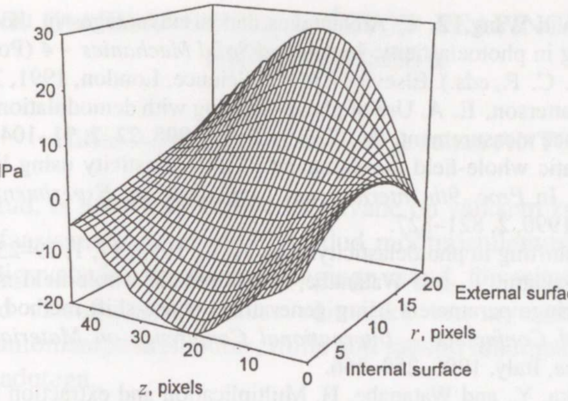

Fig. 7. Distribution of the axial stress in an area of the wall of a pharmaceutical bottle: z, axial coordinate; 10 pixels equal approximately 0.5 mm.

5. SUMMARY

An algorithm for single-valued interpretation of the phase-stepping measurement data for the case of optical retardation being less than half the wavelength was elaborated. The algorithm gives also the direction of the first principal stress. The influence of the measurement errors was analysed. The method was implemented into a computer controlled polariscope. Comparison of the new method with manual measurements showed that it would give reliable results.

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POOLLAINE FAASISAMMUDE MEETOD PEAPINGETE SUUNDADE ERALDAMISEGA

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On näidatud, et juhul kui optiline käiguvahe on väiksem kui pool lainepikkust, on võimalik faasisammude meetodil saadud mõõtmistulemustest üheselt määrata nii optiline käiguvahe kui ka esimese peapinge suund. On esitatud meetodi algoritm ning tuletatud valemid maksimaalse võimaliku vea arvutamiseks. Meetod on realiseeritud automaatpolariskoobis, mille abil saadud tulemusi on võrreldud käsitsi mõõtmisel saadutega.