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OPTIMAL DIGITAL SIGNAL POST-PROCESSING FOR PRIMARY RADAR

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Abstract. The paper describes synthesis of an algorithm which creates optimal conditions to discover a primary radar signal and object parameters inside the signal – the direction and radial distance estimates. Optimal estimation theory and specifics of digital signal processing have been taken into account. By digital signal processing, the optimal correlation receiver has been considered as a multichannel system where each channel creates an independent output signal as a result of processing the input realization. Estimates for the propagation time and object azimuth can be obtained by processing simultaneously the output signals of all the channels.

Key words: primary radar, digital signal processing, ambiguity function, optimal receiver.

1. INTRODUCTION

Fast development of modern technology creates possibilities for elimination of the human factor in complicated and probabilistic situations. This applies also to modern radar technology.

In case of a primary radar with scanning antenna the direction (azimuth) of an object is given by the azimuth of the maximum value of the antenna diagram at a given initial sensing moment. The delay time is given by the received signal reflected from the object. The maximum of the ambiguity function is optimal estimate of the delay time [1].

In real situations there is a need to process the received signal realizations, i.e., the reflected signals as well as realizations which are caused by the random noise. The noise causes deviations at the maximum of the ambiguity function. Since the ambiguity function changes slowly near its maximum value, relatively small noise causes considerable changes in the estimation of the delay time. Owing to this fact, in practice delay extractors are used, in which the delay time estimation is associated with the time moment when an optimal receiver output signal exceeds the predefined level. The value of this level is dependent on the desired trustworthiness of the estimate. This process is also called front-end detection.

The main problem with the primary radar is that for every position of the antenna tens of scans are performed. This means that the same object (mark) causes many signals at the output of the optimal receiver. In this way we get many delay estimates for the same mark. These estimates have different values caused by the noise, present in the received signals. The extractor gives tens of different pairs of the delay time and azimuth estimates, creating ambiguity in the location of the object. Because of that, large data amounts are required from the primary radar.

Our goal is to develop a coordinate extractor which describes every discovered object with only one pair of delay time and azimuth estimates. This gives the possibility to associate the amount of data directly with the amount of maximum number of objects in the visibility space of the primary radar and to reduce the amount of the data to that used in public digital communication (e.g., 9600 bit/s).

Primary scanning radar consists of two channels. The first is for vertical and the second for horizontal scanning (Fig. 1). Generally, the horizontal scanning period $T_{s\alpha}$ is greater than or equal to the vertical scanning period $T_{s\beta}$, $T_{s\alpha} \geq T_{s\beta}$. In a typical situation, the signal, created by an object, appears at different time moments in the vertical and horizontal channels. Because of that, processing of measurements is done in unified time. In the case of discovering an object in the horizontal channel the parameters $t_\alpha, \hat{\alpha}$, and $\hat{\tau}_\alpha$, and in the case of the vertical channel the parameters $t_\beta, \hat{\beta}$, and $\hat{\tau}_\beta$ are recorded. Here $\hat{\alpha}$ and $\hat{\beta}$ are estimates of the object azimuth and elevation angle, respectively, $\hat{\tau}_\alpha$ and $\hat{\tau}_\beta$ are the object delay period estimates in horizontal and vertical channels, and t_α and t_β are the time moments when the estimates have been taken. All the estimates are from the same object if the following conditions are satisfied:

$$\left| t_\alpha - t_\beta \right| \leq {T_{s\alpha}}/{2}, \quad \left| \hat{\tau}_\alpha - \hat{\tau}_\beta \right| \leq \varepsilon,$$

where ε is radius of the uncertainty region of the ambiguity function of the scanning signal.

Typical object parameters of a primary radar are

$$t_\alpha, \ \Delta t_{\alpha\beta} = t_\alpha - t_\beta; \quad \hat{\tau}_\alpha, \ \Delta\hat{\tau} = \hat{\tau}_\alpha - \hat{\tau}_\beta; \quad \hat{\alpha}, \ \hat{\beta}.$$

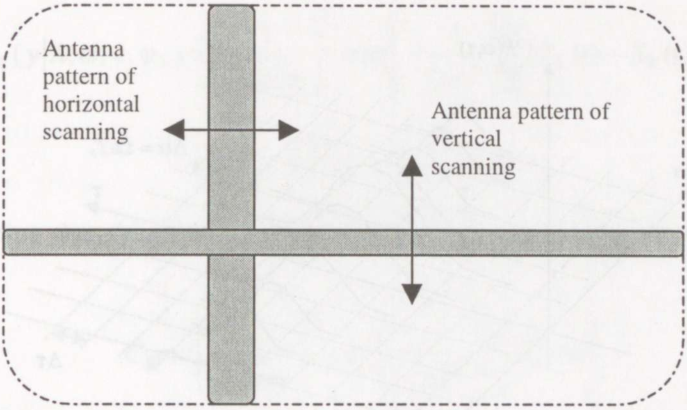

Fig. 1. Antenna patterns of the scanning radar.

In the case of analogue processing we get signals and respective estimates of an object in each scanning period when the object is in the antenna pattern of the scanning radar [2]. It is necessary to find the estimates which are independent of the scanning period. We shall show that optimal digital processing guarantees such estimates.

Since processing in horizontal and vertical channels is identical from the point of view of signal processing, we derive the algorithms only for the horizontal channel. We assume that the optimal correlation receiver is a *M*-channel system where each channel creates independent output signal as a result of processing an input realization.

2. SIGNAL DESCRIPTION AND AMBIGUITY FUNCTION

We describe the output signal of the channel k as

$$Z_k(t) = S_k(t) + N_k(t), \tag{1}$$

where

$$S_k(t) = U_k(A, \alpha, \tau, t)\cos(\omega t + \varphi_k),$$

$N_k(t)$ is white Gaussian noise, ω is circular frequency, t is discrete time, and φ_k is random initial phase of the realization k. Hereby we assume that the processing channels have been tuned to discrete values α and τ with steps

$$\Delta\alpha = \Omega_A T_S, \quad \Delta\tau = \Delta t,$$

where Ω_A is angular velocity of the antenna and T_S is period of the signal (Fig. 2).

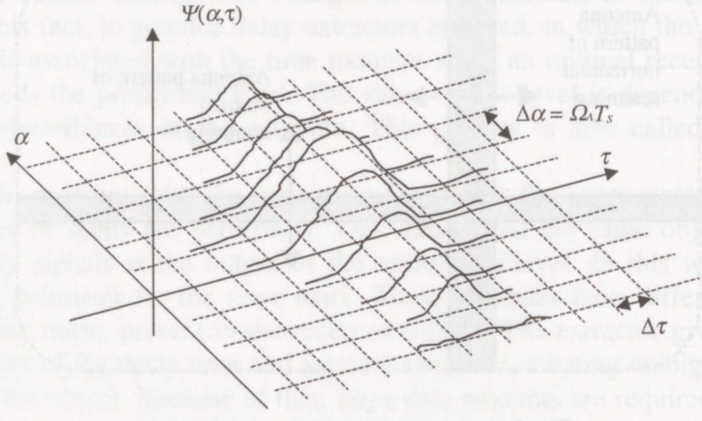

Fig. 2. Ambiguity function of the signal reflected from the object.

The base-band signal is expressed as a multiplication of the normalized ambiguity function $\Psi_k(\alpha, t)$ and amplitude [ ]

$$U_k(A, \alpha, \tau, t) = A(t)\Psi_k(\alpha, t),$$

where

$$A(t) = \begin{cases} A, & 0 \le t < T \\ 0, & 0 > t \ge -T \end{cases},$$

$$\Psi_k(\alpha, \tau) = \Psi(\alpha' - \alpha_k, \tau' - \tau_k),$$

and α', τ' are object parameters (continuous), α_k, τ_k are channel parameters (discrete), and $\Psi_k(0,0) = 1$.

According to the method of maximum probability [1], the measurements are optimal if $\hat{\alpha}$ and $\hat{\tau}$ are calculated from probability equations on condition that a dispersed signal from the object has been discovered. Together with $\hat{\alpha}$ and $\hat{\tau}$ we must also give an estimate of the amplitude $\hat{A}$. The latter is compared with the decision level to find out whether there is an object or not [4].

3. PROBABILITY FUNCTIONS

In order to get probability equations for parameters A, α, and τ, we first derive an appropriate probability functional. Probability density for any realization y in the channel k in the case of fixed parameters A, α, τ, and φ_k is expressed as follows [1]:

$$W_k(y|A,\alpha,\tau,\varphi_k)=\left(\frac{1}{\sqrt{2\pi\sigma}}\right)^T \exp\left(-\frac{1}{2\sigma^2}\sum_{t=0}^{T-1}(Z_k(t)-S_k(t))^2\right),$$

$$\sigma^2=\overline{N_k^2(t)}, \quad \overline{N_k(t)}=0.$$

Assuming that the output realizations of all channels have equal energy and denoting

$$C_W=\left(\frac{1}{\sqrt{2\pi\sigma}}\right)^T \exp\left(-\frac{1}{2\sigma^2}\sum_{t=0}^{T-1}Z_k^2(t)\right),$$

we get

$$W_k(y|A,\alpha,\tau,\varphi_k)$$
$$=C_W \exp\left(-\frac{A^2T}{4\sigma^2}\Psi_k^2(\alpha,\tau)+\frac{A}{\sigma^2}\Psi_k(\alpha,\tau)\sum_{t=0}^{T-1}Z_k(t)\cos(\omega t+\varphi_k)\right). \quad (2)$$

Using notations

$$Z_{kc}=\sum_{t=0}^{T-1}Z_k(t)\cos\omega t, \quad Z_{ks}=\sum_{t=0}^{T-1}Z_k(t)\sin\omega t,$$

$$R_k=(Z_{kc}^2+Z_{ks}^2)^{\frac{1}{2}}, \quad \phi_k=\arctan\frac{Z_{ks}}{Z_{kc}},$$

we obtain

$$W_k(y|A,\alpha,\tau,\varphi_k)=C_W \exp\left(-\frac{A^2T}{4\sigma^2}\Psi_k^2(\alpha,\tau)+\frac{A}{\sigma^2}\Psi_k(\alpha,\tau)R_k\cos(\phi_k+\varphi_k)\right). \quad (3)$$

To develop an algorithm which is independent of the initial phase φ_k, we need to average $W_k(y|A,\alpha,\tau,\varphi_k)$ with weight $W(\varphi_k)$. In order to get adequate results, the synthesis of post-processing algorithms must follow the specifics of functioning of the primary radar.

Definition 1. *In any fixed direction all channels process the same input realization. In the case of a fixed direction,* $\varphi_k=\varphi_p=\text{const}$.

Definition 2. *Two-dimensional ambiguity function* $\Psi_k(\alpha,\tau)$ *may be expressed as a multiplication of two independent ambiguity functions*

$$\Psi_k(\alpha,\tau)=\Psi_k(\alpha)\Psi_k(\tau),$$

where $\Psi_k(\alpha)$ is defined by the aerial direction diagram and $\Psi_k(\tau)$ is defined by the signal. At the same time we have to take into account that $\Psi_k(\alpha)=\Psi_p(\alpha)$ when direction is fixed, and $\Psi_k(\tau)=\Psi_d(\tau)$ because the processing of the input realization in channels does not depend on the direction.

We now consider the case

$$k=d+(p-1)D,\quad k=1,\dots,M,\quad p=1,\dots,P,$$
$$d=1,\dots,D,\ M=PD,$$

where p is discrete direction, $\alpha_k\equiv\alpha_p$, and d is discrete delay, $\tau_k\equiv\tau_d$.

Equation (3) may be written as

$$W_{pd}(y|A,\alpha,\tau,\varphi_p)$$
$$=C_W\exp\left(-\frac{A^2T}{4\sigma^2}\Psi_p^2(d)\Psi_d^2(\tau)+\frac{A}{\sigma^2}\Psi_p(\alpha)\Psi_d(\tau)R_{pd}\cos(\phi_{pd}+\varphi_p)\right).\quad(4)$$

Probability multiplication theorem gives

$$W_p(y|A,\alpha,\tau,\varphi_p)=\prod_{d=1}^{D}W_{pd}(y|A,\alpha,\tau,\varphi_p)$$
$$=C_W^D\exp\left(-\frac{A^2T}{4\sigma^2}\Psi_p^2(\alpha)\sum_{d=1}^{D}\Psi_d^2(\tau)+\frac{A}{\sigma^2}\Psi_p(\alpha)\sum_{d=1}^{D}\Psi_d(\tau)R_{pd}\cos(\phi_{pd}+\varphi_p)\right).\quad(5)$$

Introducing new quadrature components

$$Z_{pc}=\sum_{d=1}^{D}\Psi_d(\tau)Z_{kc},\quad Z_{ps}=\sum_{d=1}^{D}\Psi_d(\tau)Z_{ks},$$
$$R_p=(Z_{pc}^2+Z_{ps}^2)^{\frac{1}{2}},\quad \phi_p=\arctan\frac{Z_{ps}}{Z_{pc}},$$

we obtain

$$W_p(y|A,\alpha,\tau,\varphi_p)$$
$$=C_W^D\exp\left(-\frac{A^2T}{4\sigma^2}\Psi_p^2(\alpha)\sum_{d=1}^{D}\Psi_d^2(\tau)+\frac{A}{\sigma^2}\Psi_p(\alpha)R_p\cos(\phi_p+\varphi_p)\right).\quad(6)$$

Here φ_p may have any value between 0 and 2π with equal probability, and

$$W(\varphi_p)=\begin{cases}\dfrac{1}{2\pi}, & 0\le\varphi\le 2\pi,\\ 0\,, & 0>\varphi>2\pi,\end{cases}$$

$$W_p\big(y\big|A,\alpha,\tau\big)=\frac{1}{2\pi}\int_0^{2\pi}W_p(y|A,\alpha,\tau,\varphi_p)\,\mathrm{d}\varphi_p$$

$$=C_W^D\exp\left(-\frac{A^2T}{4\sigma^2}\Psi_p^2(\alpha)\sum_{d=1}^{D}\Psi_d^2(\tau)\right)I_0\left(\frac{A}{\sigma^2}\Psi_p(\alpha)R_p\right). \qquad (7)$$

The probability functional for the synthesis of the algorithm for primary radar post-processing is

$$W(y|A,\alpha,\tau)=\prod_{p=1}^{P}W_p(y|A,\alpha,\tau)$$

$$=C_W^M\exp\left(-\frac{A^2T}{4\sigma^2}\sum_{p=1}^{P}\Psi_p^2(\alpha)\sum_{d=1}^{D}\Psi_d^2(\tau)\right)\prod_{p=1}^{P}I_0\left(\frac{A}{\sigma^2}\Psi_p(\alpha)R_p\right). \qquad (8)$$

The results are reliable, if during the estimation process signal-to-noise power ratio is considerably greater than one

$$\frac{A^2}{\sigma^2}>>1.$$

Then

$$\ln I_0\left(\frac{A}{\sigma^2}\Psi_p(\alpha)R_p\right)\approx\frac{A}{\sigma^2}\Psi_p(\alpha)R_p$$

and optimal estimates $\hat{A},\hat{\alpha},\hat{\tau}$ can be found using functional $L(A,\alpha,\tau)$

$$L(A,\alpha,\tau)=\ln\left(\frac{1}{C_W^M}W(y|A,\alpha,\tau)\right)$$

$$=-\frac{A^2T}{4\sigma^2}\sum_{p=1}^{P}\Psi_p^2(\alpha)\sum_{d=1}^{D}\Psi_d^2(\tau)+\frac{A}{\sigma^2}\sum_{p=1}^{P}\Psi_p(\alpha)R_p. \qquad (9)$$

4. ESTIMATION OF THE OBJECT PARAMETERS

In order to get the amplitude estimate, we calculate

$$\frac{\partial}{\partial A}L(A,\alpha,\tau)=-\frac{AT}{2\sigma^2}\sum_{p=1}^{P}\Psi_p^2(\alpha)\sum_{d=1}^{D}\Psi_d^2(\tau)+\frac{1}{\sigma^2}\sum_{p=1}^{P}\Psi_p(\alpha)R_p=0,$$

$$\hat{A}_{\text{opt}}=\frac{2}{T}\frac{\sum_{p=1}^{P}\Psi_p(\alpha)R_p}{\sum_{p=1}^{P}\Psi_p^2(\alpha)\sum_{d=1}^{D}\Psi_d^2(\tau)}, \tag{10}$$

where $\Psi_p(\alpha)$ and $\Psi_d(\tau)$ are specific to every radar. Generally algorithms for estimating α and τ are not very productive because they can not be derived directly from probability equations. Instead of finding direct estimates for α' and τ', we are looking for their deviations in respect to α_k and τ_k. Assuming that $\alpha \ll 1$, $\tau \ll 1$, we use in Eq. (9) instead of $\Psi_p(\alpha)$ and $\Psi_d(\tau)$ their developments into Maclaurin sequence

$$\Psi_p(\alpha)=\Psi_p(0)+\frac{\partial}{\partial\alpha}\Psi_p(\alpha)\Big|_{\alpha=0}\alpha+...,$$

$$\Psi_d(\tau)=\Psi_d(0)+\frac{\partial}{\partial\tau}\Psi_d(\tau)\Big|_{\tau=0}\tau+... .$$

Taking into account only linear members of the last equations, we get

$$\Psi_p(\alpha)=1+a_p\alpha,$$
$$\Psi_d(\tau)=1+b_d\tau,$$

where

$$a_p=\frac{\partial}{\partial\alpha}\Psi_p(\alpha)\Big|_{\alpha=0},$$

$$b_d=\frac{\partial}{\partial\tau}\Psi_d(\tau)\Big|_{\tau=0}.$$

Now we may write

$$L(A,\alpha,\tau)=-\frac{A^2T}{4\sigma^2}\left(M+\alpha^2D\sum_{p=1}^{P}a_p^2+\tau^2P\sum_{d=1}^{D}b_d^2\right)+\frac{A}{\sigma^2}\left(\sum_{p=1}^{P}R_p+\alpha\sum_{p=1}^{P}a_pR_p\right),$$

$$R_p=\left((\overline{Z}_{pc}+\tau\overline{U}_{pc})^2+(\overline{Z}_{ps}+\tau\overline{U}_{ps})^2\right)^{\frac{1}{2}},$$

where

$$\overline{Z}_{pc}=\sum_{d=1}^{D} Z_{kc},\quad \overline{Z}_{ps}=\sum_{d=1}^{D} Z_{ks},$$

$$\overline{U}_{pc}=\sum_{d=1}^{D} b_d Z_{kc},\quad \overline{U}_{ps}=\sum_{d=1}^{D} b_d Z_{ks}.$$

Here we have assumed that a_p and b_d fulfill the following conditions

$$\sum_{p=1}^{P} a_p=0,\quad \sum_{d=1}^{D} b_d=0.$$

Thus

$$\frac{\partial}{\partial\alpha}L(A,\alpha,\tau)=-\alpha\frac{A^2T}{2\sigma^2}D\sum_{p=1}^{D} a_p^2+\frac{A}{\sigma^2}\sum_{p=1}^{D} a_p R_p=0,$$

$$\hat{\alpha}_{\text{opt}}=\frac{2}{T}\frac{\sum_{p=1}^{P} a_p R_p}{AD\sum_{p=1}^{P} a_p^2}$$

If we substitute A by its estimate $\hat{A}_{\text{opt}}\approx\frac{2}{TM}\sum_{p=1}^{P} R_p$, then

$$\hat{\alpha}_{\text{opt}}=\frac{P\sum_{p=1}^{P} a_p R_p}{\sum_{p=1}^{P} R_p\sum_{p=1}^{P} a_p^2}. \tag{11}$$

Analogically we obtain

$$\hat{\tau}_{\text{opt}}=\frac{D\sum_{p=1}^{P}\left(Z_{pc}U_{pc}+Z_{ps}U_{ps}\right)/R_p}{\sum_{p=1}^{P} R_p\sum_{d=1}^{D} b_d^2-D\sum_{p=1}^{P}\left(\overline{U}_{pc}^2+\overline{U}_{ps}^2\right)/R_p} \tag{12}$$

5. CONCLUSIONS

The algorithms for estimating the azimuth and delay period of the object for primary scanning radar are simple and have the same interference resistance as tracking algorithms. Notice, that results are correct if all estimates have been calculated using the same initial data.

In order to achieve better signal-to-noise ratio (*S/N*) we have used summing in our algorithms. Summing over d, the received initial data are bipolar. That gives increase in signal-to-noise ratio D times at the moment of estimation, compared with the estimation of the coordinates of the maximum value of the ambiguity function using differentiation. Additional increase in the *S/N* ratio is achieved by summing over p. But here we need to take into account that the process is noncoherent and the upper limit for it is P times.

If $D >> 1$, then Z_{pc} and Z_{ps} are defined using D times greater *S/N* ratio compared with the estimation based on a single point. When $D >> 1$, the summing over the values of p gives an increase in *S/N* nearly P times.

Derived algorithms give estimations of $\hat{\alpha}_{\text{opt}}$ and $\hat{\tau}_{\text{opt}}$ based on the $M \sim PD$ times greater *S/N* ratio. It means that estimates for the object azimuth and delay time will have about M times smaller dispersion compared to the dispersion of the estimates reached using only the maximum value of the ambiguity function. Application of these algorithms gives the possibility to discover objects which have M times smaller effective surface, or we can reduce by M times the output power transmitted to the object.

The derived algorithms give only one pair of azimuth and delay time estimates for every discovered object, which was our main goal.

Algorithms (10)–(12) are realizable, if $P \geq 2$ and $D \geq 2$, i.e., $M \geq 4$. Since dispersion of the estimated parameters decreases M times compared with estimates computed at the maximum point, it is practical to take into account all initial data filtered by the ambiguity function.

The derived algorithms are reliable if during the estimation process the squared signal-to-noise ratio is considerably greater than one

$$\frac{A^2}{\sigma^2} >> 1.$$

Generally algorithms for estimating α and τ are not very productive because they can not be derived directly from the probability equations [5,6]. Instead of finding direct estimates for α' and τ' we have been searching for their deviations with respect to α_k and τ_k.

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PRIMAARRADARI DIGITAALSETE SIGNAALIDE OPTIMAALNE TÖÖTLEMINE

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Primaarradari juures on põhiprobleemiks asjaolu, et antenni suunadiagrammi asendi muutumisel pealehe ulatuses toimub kümneid sondeerimisi, kusjuures üks ja sama objekt tekitab iga kord avastatava signaali. Selle järgi saadakse teatud arv objekti suuna ja kauguse hinnangu paare, mis mürade tõttu on erinevad. On sünteesitud objekti koordinaatide ekstraktor, mis võtab arvesse konkreetse radari tehnilisi andmeid selliselt, et iga avastatud objekt on kirjeldatud ainult ühe suuna ja ühe kaugusega. Viimane asjaolu võimaldab radari signaale otseselt seostada nõutava avastatavate objektide maksimaalse arvuga vaatlusruumis ning taandada andmevood väärtusteni, mis on tavalised digitaalsides.