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Investigations of prestressed cable structures at Tallinn Technical University

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Abstract. The paper presents a summary of research activities of the Department of Structural Design at TTU in the field of suspension structures against the background of the world developments in this field.

Key words: cable, cable network, girder-stiffened structure, hanging roof, hypar-network.

1. INTRODUCTION

The modern era of suspension bridges has lasted for nearly two centuries while the first contemporary suspension roof structures appeared only about 50 years ago. Investigation of suspension structures at TTU began in the end of 1950s. The methods elaborated for the calculation of different suspension structures are based on general equations for the determination of relative displacements, taking into account actual boundary conditions. Suitability of using hypar-networks inside an elliptical contour beam, lying on three or four plane supports which permit free deformation of the contour beam in horizontal directions, has been verified.

Some outstanding prestressed suspension structures will be analysed below. The main attention is paid to girder-stiffened suspension bridges and plain and spatial suspension roof structures, prestressed by means of contact loads between the carrying and the stretching cables. The references include a number of proceedings of international conferences devoted to the problems of suspension roof and bridge structures. Historical development of suspension bridges is illustrated by a diagram demonstrating the increase of the covered spans. The drawings of some outstanding bridge and roof structures are presented.

Description of the cable networks for the acoustic screens erected in Estonia is illustrated by their schematic drawings. The problems related to the design of the self-anchored suspension bridge for the strait crossing to the Saaremaa Island are discussed.

2. DEVELOPMENT OF LONG-SPAN CABLE SUPPORTED BRIDGES AND SUSPENSION ROOFS

Erection of the Menai bridge in UK in the 1830s with a span of 177 m may be considered as the beginning of the modern era of design and construction of suspension bridges. Development of bridge building continued mainly in the USA. The 1000 m boundary was exceeded by George Washington bridge with the span of 1067 m in 1931. In 1981 the Humber bridge in UK (span 1410 m) and in 1998 the East bridge over the Great Belt in Denmark (span 1624 m) became record-makers. In the 1990s the main arena of construction of long span bridges went over to Asia. Bridges with the main span over 1000 m may be mentioned as examples: Tsing Ma in Hong Kong, Yangtse in China, and Kudos Kurushima in Japan. The span record with 1990 m belongs to Akashi Kaikyo bridge in Japan (Fig. 1) which was opened in 1999. Development of record span suspension bridges is illustrated by a diagram in Fig. 2. Erection of cable-stayed bridges has a shorter history than of the suspension ones. Some prominent samples belong to the 1950s; in the following decades symmetrical and non-symmetrical cable-stayed bridges became very popular. The bridges with the longest span (856 m) were erected in the 1990s in France (Normandia bridge) and Japan (Tatara bridge, 890 m). Self-anchored suspension bridges combine the advantages of suspension and cable-stayed bridges.

Erection of suspension structures for roofs has a 50 years history. Most popular types of suspension roof structures are: 1) prestressed truss-type plane structures, 2) prestressed structures with doubled radial cables inside of a circular or elliptical contour beam, and 3) saddle-formed prestressed cable networks with closed contour structures. The first contemporary suspension roof structures were

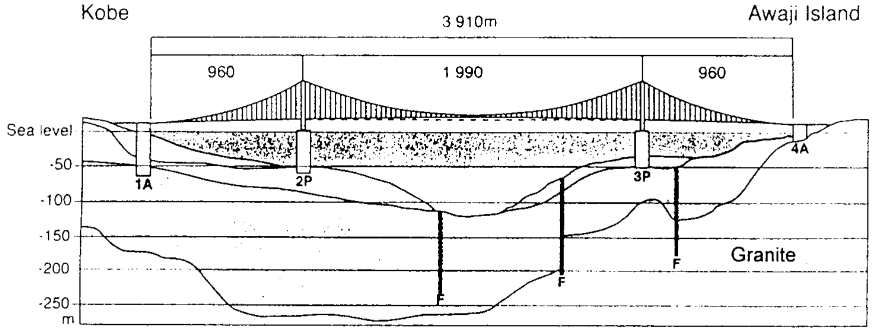


Fig. 1. Schema of the Akashi Kaikyo bridge in Japan.

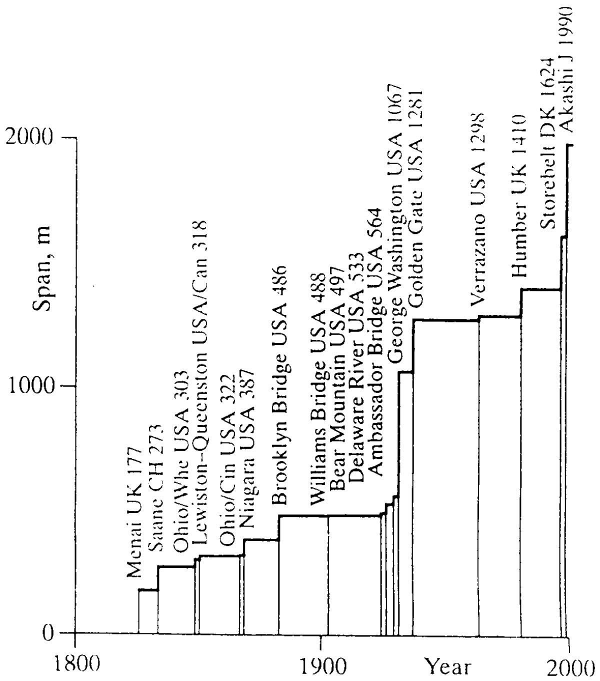


Fig. 2. Record span suspension bridges.

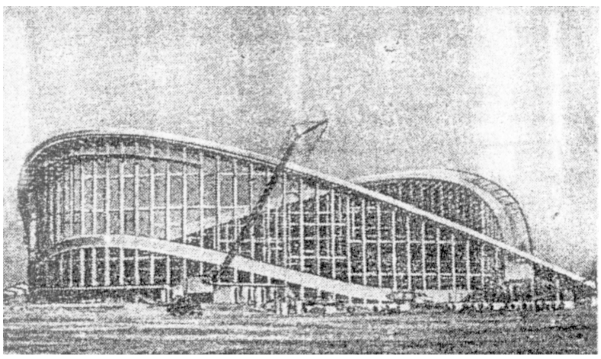


Fig. 3. Saddle-shaped roof of the Raleigh Arena, USA.

built in 1950s. The best known of them is the saddle-shaped roof for the Raleigh Arena in the USA (Fig. 3). In the 1950s a number of roofs with cable-anchored suspension trusses and with radial cables were also built.

Experience of suspension structures obtained in Estonia includes special types of roof networks and a few suspension bridges with relatively small spans. Two cable networks were built as acoustic screen structures with timber coverings for song festival tribunes. The first of them was erected in 1960 in Tallinn as a network inside the inclined contour beam of two plane arches, supported by massive counterforts (Fig. 4). The second one, erected in 1994 in Tartu, is an inclined hypar-network with an elliptical contour beam (Fig. 5). The contour beam lies on three plane supports, enabling its free deformation in horizontal directions. The historical suspension footbridge in Viljandi was reconstructed in the 1990s by means of a replacement of the handrails by truss-beams (Fig. 6). New suspension bridges with spans of 60 and 50 m were erected in Võru (1998) and Kurgja (2001).

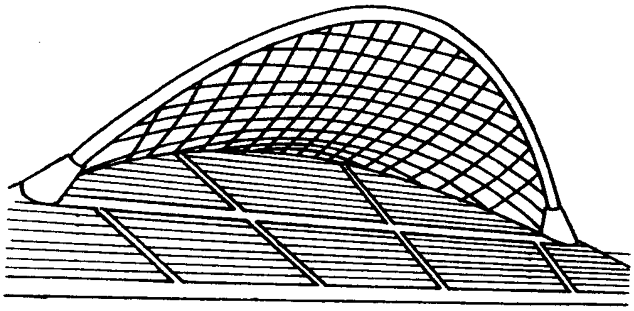


Fig. 4. Schema of the Tallinn song festival tribune.

Fig. 5. Erection of the song festival tribune in Tartu.

Fig. 6. Suspension footbridge in Viljandi.

3. A REVIEW OF INTERNATIONAL EXPERIENCE OF CABLE STRUCTURE ANALYSIS

The equations for elastic or rigid cables have been used as the basis for calculation of suspension structures. Suspension bridges with a stiffening girder were usually calculated as statically indeterminate structures. For solving the problem of the behaviour of a geometrically non-linear model, a cubic equation relative to the horizontal cable force was often used. Transferring the equations to prestressed compound structures led to very complicated calculation models. First remarkable works for calculation of different cable systems were published in the 1950s [1–3]. Some monographs were published in the 1960s [4–10]. A number of conference reports about suspension structures were published in different proceedings and collections [11–22]. A summary of the main studies, dealing with the analysis of suspension structures before 1960s, is presented in [7]. It is worth to mention that the methods of analysis, presented in this monograph, are complicated and difficult to use. No attention was paid to the problem of interaction of the network and the contour beam. In spite of that, the description of different suspension structures and their behaviour is of great interest.

In the field of spatial suspension roof structures, several publications deal with the determination of the initial form of the network [8,10,23]. However, we have found some inaccuracies in the postulates used. For instance, the assumptions about the coincidence of the cable form with the geodetic lines of the chosen roof surface or about determination of the roof surface by equations of a membrane or as minimum surfaces, are not correct.

In the 1970s and 1980s a number of conferences, devoted to suspension roof structures, were organized by the International Association for Shell and Spatial

Structures and other organizations [16–20]. Much attention was paid to prestressed suspension networks [24–28]. During the last decades of the 20th century an extensive erection of long-span cable-stayed and suspension bridges took place, especially in Japan and other Asian countries. A number of international conferences dealing with problems of bridge design and construction were organized [21,22]. The topics were primarily related to the description of erected structures; in the field of calculations the finite element method predominated. In these conferences much attention was paid to long-span bridges, erected in Asian countries, Denmark, and Korea.

Let us mention also interesting papers devoted to different problems related to suspension structures [29–35].

4. STUDIES OF PRESTRESSED CABLE STRUCTURES AT TTU

Investigations of suspension structures at TTU were provoked by the problems of design and erection of the acoustic screen for the song festival stage in Tallinn in the period 1958–1960 [36–38]. The list of publications up to 1982 is presented in the booklet [39]. This booklet includes 105 publications in the field of suspension roof structures. The main attention was paid to different saddle-formed suspension structures [40–52]. Problems of determination of the initial form of networks were also studied [53,54]. Some problems of dynamic behaviour of structures were discussed in [55–57]. Both continuous and discrete calculation models were used. Continuous modelling was mainly applied to hypar-networks with elliptical contour beams. It was proved that a simple approximation of the deflection function gives results, very close to the exact ones. For the case of one-side snow loading, dividing of the total load into symmetrical and antisymmetrical parts with successive calculation under these loads was recommended. In this regard the possibility and conditions for application of the principle of superposition of stresses for geometrically non-linear systems was verified [58]. Specific features of these investigations are presenting of the basic equations with the relative deflection as an unknown and consideration of the displacements of the contour beam. Besides theoretical investigations, a lot of attention has been paid to experimental investigations on great scale models (Figs. 7 and 8).

Together with the hypar-networks, the behaviour of a single cable has also been determined by a cubic equation regarding the relative deflection [59].

Relative deflection of the cable $\zeta_0 = w_0 / f$ for all load cases may be determined from the cubic equation

$$\zeta_0^3 + 3\zeta_0^2 + (2 + p_0^*)\zeta_0 = p^*,$$

where p^* is the loading factor shown in Table 1, $p_0^* = p^* p_0 / p$ is initial loading factor, p_0 and p are initial and additional loads of the cable, respectively.

Deflection forms and inner forces of cables for different loading schemas are presented in Table 1.

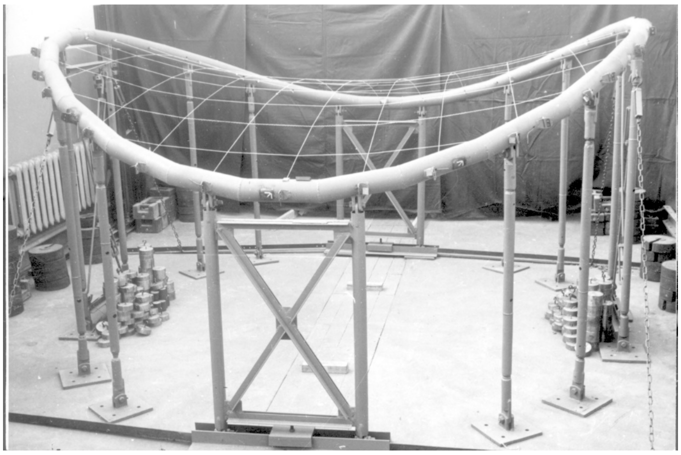


Fig. 7. Model of a hypar-network with elliptical layout.

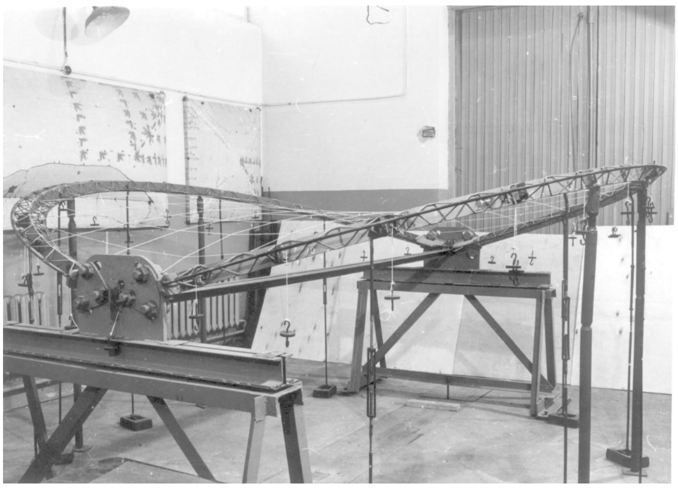
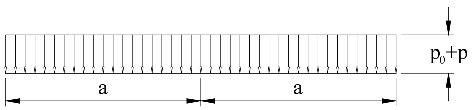
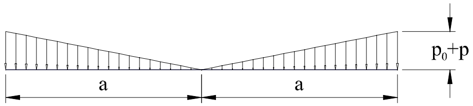
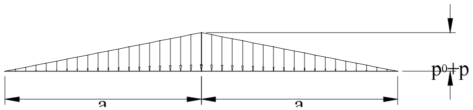
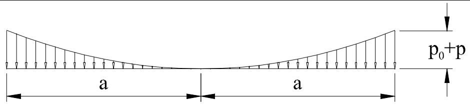
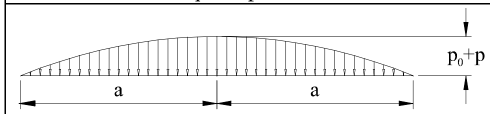
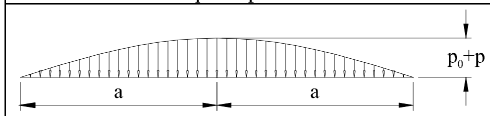


Fig. 8. Model of a cable network surrounded by two plane arches.

Later the method of continuous analysis was adapted for the calculation of prestressed cable-stiffened structures [60], cable-stayed mast cranes [61,62], and girder-stiffened cable structures [63]. For suspension bridges, the cases both with straight and loaded anchor cables have been investigated [64,65]. Several publications deal with self-anchored bridges [66,67]. A new method for discrete analysis of suspension bridges was presented in [68]. The formulae for cable-stiffened structures were also adapted for the suspension roof structures with radial cables [69,70]. Unified equations for the determination of deflections and inner forces in different cable structures are presented in Table 2.

Table 1. The values of the load parameters and the cable forces for different load cases

Load type	$z(x)$	$w(x)$	p^*	H
	$f\frac{x^2}{a^2}$	$w_0\left(\frac{x^2}{a^2}-1\right)$	$\frac{3pa^4}{4EAf^3}\left(EA\frac{\bar{u}}{a}+1+2\frac{f^2}{a^2}+\frac{6}{5}\frac{f^4}{a^4}-\frac{4}{7}\frac{f^6}{a^6}+\dots\right)$	$\frac{\left(p-\frac{w_0}{f}p_0\right)a^2}{2f\left(1+\frac{w_0}{f}\right)}$
	$f\frac{x^3}{a^3}$	$w_0\left(\frac{x^3}{a^3}-1\right)$	$\frac{5pa^4}{27EAf^3}\left(EA\frac{\bar{u}}{a}+1-\frac{27f^2}{10a^2}+\frac{27}{8}-\frac{729f^6}{208a^6}+\dots\right)$	$\frac{\left(p-\frac{w_0}{f}p_0\right)a^2}{6f\left(1+\frac{w_0}{f}\right)}$
	$\frac{fx^2}{2a^2}\left(3-\frac{x}{a}\right)$	$w_0\left(\frac{3x^2}{2a^2}-\frac{x^3}{2a^3}-1\right)$	$\frac{5pa^4}{9EAf^3}\left(EA\frac{\bar{u}}{a}+1+\frac{9f^2}{5a^2}+\frac{27f^4}{35a^4}-\frac{243f^6}{1001a^6}+\dots\right)$	$\frac{\left(p-\frac{w_0}{f}p_0\right)a^2}{3f\left(1+\frac{w_0}{f}\right)}$
 Square parabola	$f\frac{x^4}{a^4}$	$w_0\left(\frac{x^4}{a^4}-1\right)$	$\frac{7pa^4}{96EAf^3}\left(EA\frac{\bar{u}}{a}+1+\frac{24f^2}{7a^2}+\frac{96f^4}{13a^4}-\frac{256f^6}{19a^6}+\dots\right)$	$\frac{\left(p-\frac{w_0}{f}p_0\right)a^2}{12f\left(1+\frac{w_0}{f}\right)}$
 Square parabola	$\frac{fx^2}{5a^2}\left(6-\frac{x^2}{a^2}\right)$	$w_0\left(\frac{6x^2}{5a^2}-\frac{x}{5a^4}-1\right)$	$\frac{4375pa^4}{96EAf^3}\left(EA\frac{\bar{u}}{a}+1+1{,}865\frac{f^2}{a^2}+0{,}883\frac{f^4}{a^4}-1{,}485\frac{f^6}{a^6}+\dots\right)$	$\frac{5\left(p-\frac{w_0}{f}p_0\right)a^2}{12f\left(1+\frac{w_0}{f}\right)}$
 Sinusoid	$f\left(1-\cos\frac{\pi x}{2a}\right)$	$-w_0\cos\frac{\pi x}{2a}$	$\frac{64pa^4}{\pi^4EAf^3}\left(EA\frac{\bar{u}}{a}+1+1{,}85\frac{f^2}{a^2}+0{,}853\frac{f^4}{a^4}-0{,}292\frac{f^6}{a^6}+\dots\right)$	$\frac{4\left(p-\frac{w_0}{f}p_0\right)a^2}{\pi^2f\left(1+\frac{w_0}{f}\right)}$

Notations: $z = z(x)$ – initial ordinate of the cable; f – initial sag; a – half-span; w_0 – maximum deflection of the cable under the load $p = p(x)$; H – horizontal cable force after loading; EA – rigidity of the cable in tension; $\bar{u}$ – horizontal displacement of the supporting node of the cable under action of the unit load.

Table 2. Relative deflections and cable forces for different suspension structures

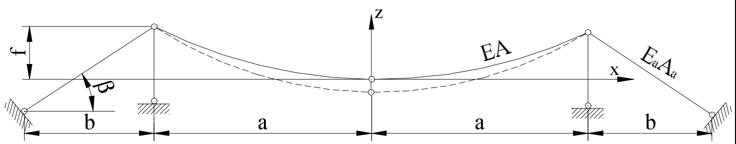
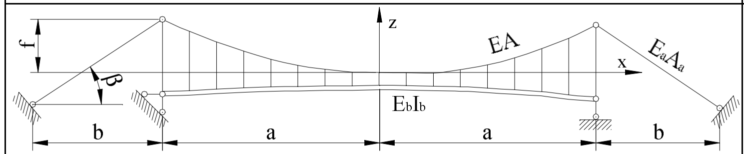
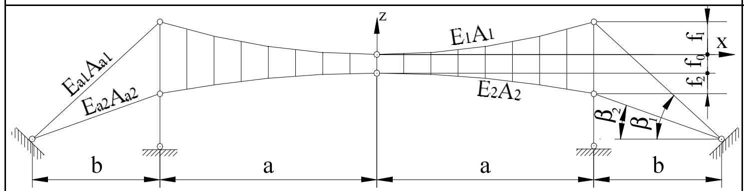
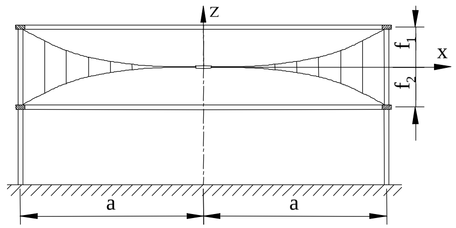
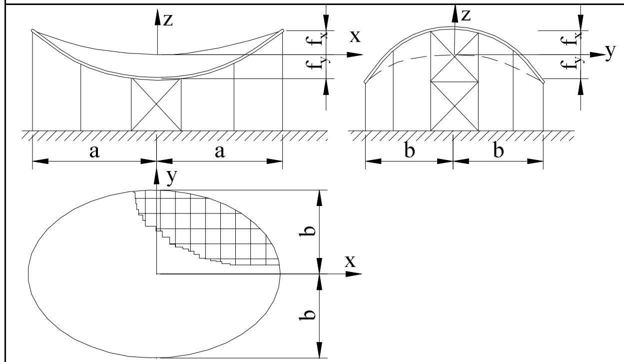
Scheme of suspension structure	Cubic equation for determination of cable relative deflection	Change of cable force $kN(kN/m)$	Cable stiffness $kN(kN/m)$	Factors of stiffness
	$\zeta_0^3+3\zeta_0^2+\left(2+p_0^*\right)\zeta_0=p^*$ $\zeta_0=\frac{w_0}{f}\quad \delta=\frac{f}{a}$ $p_0^*=\frac{H_0}{\Phi}\quad p^*=\frac{P}{\Phi}\quad P=\frac{pa^2}{2f}$	$H-H_0=\Phi\zeta_0\left(2+\zeta_0\right)$	$\Phi=\frac{2EA\delta^2}{3(1+\kappa+\vartheta)}$	$\vartheta=\frac{EAb}{E_aA_a a\cos^3\beta}$
	$\zeta_0^3+3\zeta_0^2+\left(2+\rho+p_0^*\right)\zeta_0=p^*$ $\zeta_0=\frac{w_0}{f}\quad \delta=\frac{f}{a}$ $p_0^*=\frac{H_0}{\Phi}\quad p^*=\frac{P}{\Phi}\quad P=\frac{pa^2}{2f}$	$H-H_0=\Phi\zeta_0\left(2+\zeta_0\right)$	$\Phi=\frac{2EA\delta^2}{3(1+\kappa+\vartheta)}$	$\rho=\frac{8}{3}\frac{E_bI_b}{\Phi a^2}$ $\vartheta=\frac{EAb}{E_aA_a a\cos^3\beta}$
	$(1+\psi)\zeta_0^3+3(1-\alpha\psi)\zeta_0^2+$ $+\left[2\left(1+\alpha^2\psi\right)+(1+1/\alpha)p_0^*\right]\zeta_0=p^*$ $\zeta_0=\frac{w_0}{f_1}\quad \delta_1=\frac{f_1}{a}$ $p_0^*=\frac{H_0}{\Phi}\quad p^*=\frac{P}{\Phi}\quad P=\frac{pa^2}{2f_1}$	$H_1-H_{01}=\Phi\zeta_0\left(2+\zeta_0\right)$ $H_2-H_{02}=-\psi\Phi\zeta_0\left(2\alpha-\zeta_0\right)$	$\Phi=\frac{2E_1A_1\delta_1^2}{3\left(1+\kappa_1+\vartheta_1\right)}$	$\psi=\frac{E_2A_2\left(1+\kappa_1+\vartheta_1\right)}{E_1A_1\left(1+\kappa_2+\vartheta_2\right)}$ $\vartheta_1=\frac{E_1A_1b}{E_{a1}A_{a1}a\cos^3\beta_1}$ $\vartheta_2=\frac{E_2A_2b}{E_{a2}A_{a2}a\cos^3\beta_2}$

Table 2 continued

Scheme of suspension structure	Cubic equation for determination of cable relative deflection	Change of cable force $kN(kN/m)$	Cable stiffness $kN(kN/m)$	Factors of stiffness
	$(1+\psi)\zeta_0^3+3(1-\alpha\psi)\zeta_0^2+$ $+[2(1+\alpha^2\psi)+p_0^*(1+1/\alpha)]\zeta_0=p^*$ $\zeta_0=\frac{w_0}{f_1}$ $\delta_1=\frac{f_1}{a}$ $p_0^*=\frac{H_0}{\Phi}$ $p^*=\frac{P}{\Phi}$ $P=\frac{pa^2}{6f_1}$	$H_1-H_{01}=\Phi\zeta_0(2+\zeta_0)$ $H_2-H_{02}=-\psi\Phi\zeta_0(2\alpha-\zeta_0)$	$\Phi=\frac{9E_1A_1\delta_1^2}{10(1+\kappa_1)}$	$\psi=\frac{E_2A_2(1+\kappa_1)}{E_1A_1(1+\kappa_2)}$
	$(1+\psi+4\xi)\zeta_0^3+3[(1-\alpha\psi)+$ $+2(1-\alpha)\xi]\zeta_0^2+\{2[(1+\alpha^2\psi)+$ $+(1-\alpha)^2\xi]+p_0^*(1+1/\alpha)\}\zeta_0=p^*$ $\zeta_0=\frac{w_0}{f_x}$ $\delta_x=\frac{f_x}{a}$ $p_0^*=\frac{H_0}{\Phi}$ $p^*=\frac{P}{\Phi}$ $P=\frac{p_1a^2}{2f_x}$ $\mu=1+1/\psi$	$H_x-H_{0x}=\Phi\zeta_0[(2+\zeta_0)+$ $+2(1-\alpha+\zeta_0)\xi]$ $H_y-H_{0y}=-\beta^2\Phi\zeta_0[(2\alpha-\zeta_0)\psi-$ $-2(1-\alpha+\zeta_0)\xi]$	$\Phi=\frac{5Et_x\delta_x^2}{9(1+\kappa_x)(1+\mu\xi)}$	$\psi=\frac{Et_ya^4(1+\kappa_x)}{Et_xb^4(1+\kappa_y)}$ $\xi=\frac{5Et_ya^3\sqrt{a/b}}{72E_cI_c(1+\kappa_x)}$

Notations: $\zeta_0 = w_0/f$, w_0/f_1, w_0/f_x – relative deflections of the structure; f, f_1, f_x – initial sags of the carrying cable; f_2, f_y – initial sags of the stretching cable; H, H_1, H_x – horizontal forces of the carrying cable; H_2, H_y – horizontal forces of the stretching cable; A, A_1 – cross-sectional areas of the carrying cable; A_2 – cross-sectional area of the stretching cable; E_aA_a, $E_{a1}A_{a1}$, $E_{a2}A_{a2}$, – rigidities of anchor cables in tension; t_x, t_y – effective thicknesses of the family of carrying and stretching cables, respectively; E_bI_b, E_cI_c – rigidity of the stiffening girder and the contour beam in bending.

Application of discrete analysis to prestressed cable networks was based on spatial vector diagrams for the initial and loaded states. Main attention was paid to structures prestressed in conditions of free mutual sliding of the cables (self-formed network) and to orthogonal networks. The initial form for the latter may be found from a system of linear equations regarding the nodal ordinates. Discrete analysis of a self-formed network under the action of external loads leads to a system of non-linear equations, consisting of three conditions of equilibrium and two conditions of deformation compatibility of cable sections for every node of the network. The system of equations for an orthogonal network under the action of vertical loads consists of a condition of equilibrium for every node and an equation of deformation compatibility for every cable. Displacements of contour nodes of the network are taken into account in the equations of deformation compatibility. It is worth to mention that the system of discrete equations derived for an orthogonal network may be also obtained by means of application of the method of finite differences to the differential equations for continuous analysis of the network.

Besides publications cited above, research results in the field of different cable structures are presented also in [71–83].

5. ON THE FUTURE DEVELOPMENT OF SUSPENSION STRUCTURES

The main trend of bridge development is further enlargement of spans. Spans of cable-stayed bridges increase over 1000 m. In the field of suspension bridges preliminary proposals of spans up to 3000 and even 5000 m have been considered. As examples, fixed links between the Sicilia Island and the Peninsula of Apennines (the main span about 3000 m), between Gibraltar and Marocco (main spans 3×3500 or 2×5000 m) and across the Strait of Bering (a number of spans about 5000 m) may be mentioned. In case of extremely long spans, combination of suspension cables and stay-cables at the pylons and also spatial structures for pylons have been recommended. Dynamic behaviour may become a serious problem for super-span bridges. To increase the lateral rigidity, the V-formed pylons may be used. Recommendations for constructing the stiffening girder in two parallel streamline parts (twin-box cross-section) have been made. For spans over 3000 m, the weight of the suspension cables may exceed the weight of the stiffening girder. Therefore the use of the hybrid cable system, composed of synthetics and steel, may serve the purpose.

As for the roofs of buildings, usually spans up to 200 m or roof diameters of 200 to 300 m satisfy present needs for assembly or sports halls. The main attention is to be paid to the effective interaction between the cable system and the supporting structures. Collation of different network structures demonstrates remarkable advantages of hypar-forms: even distribution of network deflections and inner forces, small required prestressing forces, and small bending moments of the contour beam.

Fig. 9. Elevation of the supposed suspension structure of the bridge between Estonian mainland and the Saaremaa Island.

In connection with the preliminary design of the fixed link between the Saaremaa Island and the Estonian mainland, erection of a self-anchored suspension bridge for the navigable span of the crossing is being considered. The main issue is the stability of the bended and compressed stiffening girder. Both theoretical [67] and experimental [84] investigations have demonstrated sufficient stability of the relatively slender stiffening girder and have also shown advantages of the self-anchored structure with straight anchor cables. An elevation of the suspension structure for the main span of the supposed bridge is presented in Fig. 9. Economical comparison of different bridge structures for the navigable and approach spans is a task of the near future.

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Eelpingestatud kaabelkonstruktsioonide uurimisest Tallinna Tehnikaülikoolis

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On esitatud ülevaade kaabelkonstruktsioonide arengust maailmas ning nende teoreetilisest ja eksperimentaalsest uurimisest Tallinna Tehnikaülikoolis. On kirjeldatud olulisemaid Eestis püstitatud kaabelkonstruktsioone.