Proc. Estonian Acad. Sci. Eng., 1999, **5**, 2, 142–154
https://doi.org/10.3176/eng.1999.2.05

MULTISTAGE FINITE-TIME OPTIMIZATION OF ENDOREVERSIBLE THERMAL MACHINES

Stanislaw SIENIUTYCZ

Faculty of Chemical Engineering, Warsaw Polytechnic University, 1 Waryńskiego Street, 00-645 Warsaw, Poland; sieniutycz@ichip.pw.edu.pl

Received 12 October 1998

Abstract. General optimization theory of multistage systems is applied to optimally controlled cascades composed of Curzon–Ahlborn–Novikov engines or heat pumps. The optimal cascades are those which work in the engine mode (process approaches equilibrium) and maximize the work production, or those which work in the heat-pump mode (process departures from equilibrium) and minimize the work consumption. A unified mathematical description is proposed which deals with multistage Curzon–Ahlborn–Novikov processes and with the limiting continuous processes. A kinetic extension of the Carnot theory shows deviations of the stage efficiency from the Carnot formula, caused by the process irreversibility. A relatively unknown discrete theory, with a Hamiltonian function constant along the optimal path, is used for the purpose of work optimization in both discrete and continuous cases. Nonlinear difference equations which follow from the energy balance and kinetics of heat transfer are constraints in the work optimization. The optimal discrete set is canonical and preserves most of the properties of the classical Pontryagin algorithm for continuous optimization. Generalized exergy (available energy) is obtained from the discrete functionals of work for both discrete and continuous cases. This exergy refers to finite time or finite size systems and simplifies to the classical exergy in the case of infinite duration. The important issue is that the bounds provided by this generalized exergy are stronger than classical thermostatic bounds and hence they are closer to reality.

Key words: multistage optimization, exergy, thermal engines, heat pumps.

1. INTRODUCTION

It has been recognized recently that approaches which use discrete formalisms of non-equilibrium thermodynamics are capable of providing quite realistic performance criteria and bounds for real processes occurring in finite time and for practical systems of finite size. Especially finite-time thermodynamics and

thermoeconomics, both related in spirit to an older field called *second law analysis*, have been developed to aid in the search for the optimum ways to operate machines and processes, either by finding best or optimum values of key parameters or by determining optimum pathways of operation associated with various thermal engines and operations of units in thermal and chemical plants. One of the reasons for the attractiveness of these newer branches of thermodynamics over older ones is that they admit explicitly that for given process requirements, operations must occur with finite intensities, and that inherent loss mechanisms need to be taken into account.

Thanks to the synthesizing nature of these approaches, as well as to their effectiveness, significant progress has been achieved in the design of new or improved thermal, separation, chemical and radiative systems. With the help of thermodynamic approaches, a variety of practical and industrial systems can now be modelled and optimized, including thermal and solar-driven engines, heat exchangers, diffusional separators, semiconductor devices, etc. These applications, on the one hand, result in a deeper understanding of the theory and, on the other, lead to further improvements in the design of practical devices. The unifying factor for these subjects follows from the way how most of them deal with thermodynamic systems which have some control that can be adjusted to achieve the best or extremal performance. Thus, in essence, these approaches focus primarily on the optimization of energy conversion and transmission systems using thermodynamic methods. Having developed formulations for evaluating system performance, design variables and costs, researchers can now address various problems of system control and optimization. Dynamic bounds can be determined which are usually functions of operational constraints consistent with finite rates of operation for the process. These bounds can be contrasted with the static bounds of classical thermodynamics pertinent to infinitely slow processes.

For optimization purposes, system models are being developed which contain, in general, control (decision) variables, state variables, and (usually) some uncontrolled or fixed parameters. These models incorporate diverse process characteristics such as finite heat conductance, semiconductor bandgap, diffusion and beam transfer channels, friction, heat loss, chemical resistance, and other factors which are essential in real energy conversion and transmission processes. Approaches from classical and finite-time thermodynamics and exergy analysis search for the best or optimum values of the principal parameters of various engines (thermal, solar, combustion, etc.) and unit operations or processes (distillation, chemical reactions, etc.) as well as of some combined structures operating in thermal or chemical plants under specified operational constraints. Optimization usually yields a few basic recommendations on how to run a practical system. Optimal paths and optimal controls (for instance, driving heat fluxes maximizing work produced) are determined. The role of mathematical programming and optimal control theory is essential when solving these problems.

2. BASIC FEATURES OF THE MATHEMATICAL MODEL

In a series of our recent papers [1–3] we have developed a general optimal control framework for a difficult class of problems of work maximization in multistage endoreversible processes [4–6] which yield mechanical work with finite rates and may be characterized by multiple (vectorial) efficiencies. Methods originally designed to processes with pure heat transfer [1–3] have then been extended to those with simultaneous heat and mass transfer [6]. The main purpose of the extension was to develop an analysis which would be a suitable starting point to investigate non-isothermal chemical engines. To date the only acknowledged theory is that dealing with isothermal chemical engines [7,8].

Equations of dynamics which follow from energy and matter balances and transfer equations are difference constraints for optimized work. Irreversibilities caused by the energy and mass transport are essential. Variation of efficiencies takes place due to finite heat and mass fluxes as natural control variables. With our modelling, enhanced bounds for the work released from an engine system or added to a heat-pump system can be evaluated. Lagrangians and Hamiltonians of work functionals and discrete canonical equations are effective; they reach their continuous counterparts in the limit of an infinite number of stages. Bellman's dynamic programming is an efficient method either to construct his recurrence equation or to arrive at a discrete maximum principle of the Pontryagin type, in which a Hamiltonian is maximized with respect to controls [9]. Both these algorithms are powerful computational tools which serve to maximize power output and evaluate optimal controls. For a finite-time passage of a resource fluid between two given thermodynamic states, an optimal process is shown to be irreversible. Its optimal intensity is characterized well by the Hamiltonian of the optimization problem. Characteristic functions which describe extremal work are found numerically in terms of final states, process duration, and number of stages [9].

An extension of the classical exergy and its underlying work potential I to thermal and separation systems with a finite number of stages and a finite holdup time of the resource fluid is one of the main results of such analyses [1–3]. This extended exergy simplifies to the classical thermal exergy in the limit of infinite duration and an infinite number of stages. The extended exergy exhibits a hysteretic property as a decrease of maximum work received from a multistage engine system and an increase of minimum work added to a heat-pump system, two properties which are particularly important in high-rate regimes. Our purpose here is, in particular, to apply the extended exergy and its underlying work potential to irreversible finite-period cycles in thermodynamic spaces.

The theory of infinite sequence of infinitesimal Curzon–Ahlborn–Novikov (CAN) processes has been applied [1–3] to describe the active (work producing) exchange of heat between two fluids, in particular fluid and bath, thus leading to the dissipative extension of the corresponding reversible problem. Both in the

irreversible problem and in its reversible prototype, a maximum of work from a finite resource is sought. In the case of the reversible process, the maximum work is the well-known exergy or available energy of the resource. In this work we show that the CAN sequence is the basic theoretical tool to define a duration-dependent work potential and related available energy (exergy) which generalizes the classical exergy for finite time processes with dissipation occurring in associated resistances. Earlier generalizations [10] have missed related functional statements and associated time evolution, and have not made distinction between the processes approaching and leaving the equilibrium. This distinction has been emphasized only recently, and the analysis has explained that the distinction becomes unnecessary only for the reversible quasistatic process, when the effect of resistances vanishes and the nonideal exergy simplifies to the classical reversible exergy, the latter being known from many textbooks. The classical thermal exergy can also be obtained in a less standard way, as the limiting work received from the sequence of a finite number of Carnot cycles at the limit when the number of these cycles tends to infinity. For the purpose of the standard exergy, the commencement of the theoretical scheme with a finite-stage model is unnecessary, though; the traditional model of infinitesimal stages is sufficient. Indeed, the presence of *reversible* cycles fixes automatically the first-law efficiency η of each infinitesimal stage at the Carnot level

$$\eta = 1 - T^e/T,$$

where T is the instantaneous temperature of the resource, and T^e is the temperature of the environment or an infinite bath. Since the unit mass of resource releases the heat

$$dQ = -cdT,$$

where c is the specific heat, the classical thermal exergy E_x follows by integration of the product

$$-c\eta dT = -c(1 - T^e/T)dT$$

between the limits T and T^e. The integration yields the well-known classical expression for the thermal exergy in quite simple way.

However, the approach to the exergy using sequential cycles becomes nontrivial in the irreversible (finite rate) situation, since in this case the instantaneous efficiencies η of the system differ from those of Carnot at each instant, and their prior evaluation is necessary before the maximizing and integration of a related work integral can be made. Moreover, these efficiencies have to be evaluated in a proper form, as certain functions of the process state T and the corresponding rate dT/dt, to assure the genuine functional property (path dependence) for the work

integral. In the case of more state variables (for example T, X) vectorial generalizations of the theory are to be taken into account [6]. The work integration must be preceded by an optimization procedure which maximizes the work integral and assures the choice of an optimal path.

Following our previous works [1–3], we now outline the derivation of the power flux and a related work functional for a steady process of work production in a sequence of infinitesimal CAN engines. We introduce the differential of the overall conductance γ associated with the overall transfer coefficient α'

$$d\gamma \equiv \frac{d\gamma_1 d\gamma_2}{d\gamma_1 + d\gamma_2} = \alpha' dA = \alpha' a_v F dx , \tag{1}$$

where A is total area of heat transfer, F is cross-sectional area of the fluid flow, and a_v is specific area of heat exchange.

We also introduce a spatial scale for the overall transfer process. It is identical with the *height of the heat transfer unit* H_{TU}

$$\frac{Gc}{\alpha' a_v F} = H_{TU} = \frac{\rho c}{\alpha' a_v} \nu \equiv \chi \nu . \tag{2}$$

Since unit of the mass flux G is mass per unit time and that of the volumetric heat transfer coefficient $\alpha' a_v$ is J/(m^{3}Ks), the unit of the quantity H_{TU} is length. In the above equation, H_{TU} is referred to the driving fluid (fluid 1). This equation also contains the quantity $\chi = \rho c / \alpha' a_v$, the ratio of the volumetric capacity ρc to $\alpha' a_v$. Its unit is time, and it plays the role of a time constant for the driving fluid flow. The quantity χ remains a finite constant at the limit of vanishing velocities ν since the transfer coefficients approach in this limiting case certain nonvanishing constant values governed by the fluid conductivity in absence of the convection. In the case of finite α', the quasistatic limit can be attained by pushing the resource fluid infinitely slowly through the system.

The process evolution is traced in terms of a nondimensional quantity τ, Eq. (3), related to t or x, which is defined as the ratio x/H_{TU}. It represents a nondimensional length which is known as the *number of transfer units*. It is the length x scaled in units of the H_{TU}. Since τ is a measure of both the extent of the system and of the fluid's contact time with the energy exchange area, it plays also the role of nondimensional time, and this is why it is designated by τ. The variable τ can be defined by several expressions

$$\tau \equiv \frac{x}{H_{TU}} = \frac{\alpha' a_v}{pc\nu} x = \frac{x}{\chi \nu} = \frac{t}{\chi} , \tag{3}$$

the latter being restricted to the case of constant velocity ν. Then it can be shown [1–3] that the heat balance for the driving fluid yields

$$\frac{dQ_1}{d\gamma} \equiv -u = -\frac{GcdT}{\alpha' dA} = -\chi\nu \frac{dT}{dx} = -\chi \frac{dT}{dt} = -\frac{dT}{d\tau}, \tag{4}$$

and the power differential in terms of the driving heat differential dQ_1 is

$$dW' = \left(1 - \frac{T^e}{T - dQ_1/d\gamma}\right) dQ_1 = -Gc\left(1 - \frac{T^e}{T+u}\right) dT. \tag{5}$$

Consequently, the corresponding power integral per unit mass flow or the specific work of the fluid at flow W is

$$W \equiv \frac{W'}{G} = -\int_{\tau^i}^{\tau^f} L d\tau = -\int_{\tau^i}^{\tau^f} c\left(1 - \frac{T^e}{T+u}\right) u d\tau. \tag{6}$$

Here u is the rate of the change of the temperature. This equation defines the process Lagrangian L or the integrand of the work functional for the case of pure heat transfer. A formula of this kind can also be found for an unsteady counterpart of the process with the active heat exchange between a body and the thermal reservoir.

When simultaneous transfer of heat and mass occurs, the form of L is much more involved. In this case, L depends not only on the rates of temperature change but also on the rates of change of all independent concentrations. For an active evaporation process the derivatives $u = dT/d\tau$ and $\nu = dX/d\tau$ can be accepted as controls [6]. These controls are related to fluxes of heat and mole numbers, q and n, by the expressions $q = -gcu$ and $n = -g\nu$.

The negative ratio of the one-stage power w to the mass conductance g defines the Lagrangian L of the problem in terms of the controls u and ν. In the present case it is more suitable to use L that has units of energy per mole. One may also use the quantity $f_o = -L$ which is the power output per unit molar conductance $g = G\theta$; then

$$dW/d\tau = w/g = w/(G\theta) \equiv f_o = -L. \tag{7}$$

To obtain the work produced per unit mole of the fluid, we integrate over τ the function $f_o = -L$ expressed in terms of controls and state. For simultaneous transfer of heat and mass satisfying the Lewis analogy we obtain

$$-L(u,v)=\frac{w}{g}=c_1u-c_1T_1v+\frac{g_2}{g}c_2T_2-\left(\frac{g_2}{g}c_2-c_pv\right)T_2$$

$$\times\frac{T_1+\left(c_1\dfrac{g_1}{g}+c_pv\right)^{-1}c_1u}{T_1+\dfrac{g_1}{g}\left(c_1\dfrac{g_1}{g}+c_pv\right)^{-1}c_1u}\left(\frac{(1+X_1)^{(1+X_1)}\left(X_1+\dfrac{gv}{g_1}\right)^{\left(X_1+\frac{gv}{g_1}\right)}}{X_1^{X_1}\left(1+X_1+\dfrac{gv}{g_1}\right)^{\left(1+X_1+\frac{gv}{g_1}\right)}}\right)^{\left(\frac{g_1R}{g_2c_2}\right)}$$

$$\times\left(\frac{(1+X_2)^{(1+X_2)}\left(X_2-\dfrac{gv}{g_2}\right)^{\left(X_2-\frac{gv}{g_2}\right)}}{X_2^{X_2}\left(1+X_2-\dfrac{gv}{g_2}\right)^{\left(1+X_2-\frac{gv}{g_2}\right)}}\right)^{\left(-\frac{g_2R}{g_2c_2}\right)} \tag{8}$$

This equation allows us to find a maximum for the cumulative mechanical work W when a finite-resource fluid changes its thermodynamic parameters in a finite time between two assumed states. It may be shown that this power formula reduces *exactly* to that of pure heat transfer when $n=0$. The methodology of the optimization approach is preserved even for more complicated Lagrangians.

It may be shown that the basic functional for the process with pure heat transfer, Eq. (6), can be written in a general form which explicitly contains the entropy production S_σ, i.e.

$$W=-\int_{T^i}^{T^f}c\left(1-\frac{T^e}{T}\right)dT-T^eS_\sigma=W^{rev}-T^eS_\sigma, \tag{9}$$

where W^{rev} is the work in the reversible process.

An analogous form holds for the mass transfer functional derived from the Lagrangian (8). The first or classical term of the work integral has the potential or path-independent property. Consequently, this term does not have any influence on the family of extremal trajectories obtained as solutions of corresponding Euler–Lagrange equations. It is the entropy production which causes the (second) non-potential component of the work integral. The equivalence of the work and the entropy production extremizations, stemming from Eq. (9), is shown here for the realm of functionals rather than functions,

thus representing the second law formulation of Guy and Stodola [2,9] in infinite-dimensional spaces.

Whenever a quasistatic limit is achieved for the process of pure heat transfer, in which case the rates $\dot{T}$ vanish, the work functional leads to the *classical thermal exergy*

$$E_x = -\int_T^{T^e} c\left(1-\frac{T^e}{T}\right)dT = c\left(T-T^e\right)-cT^e \ln\frac{T}{T^e}. \tag{10}$$

For a quasistatic process with simultaneous heat and mass transfer, a generalization of the limiting formula (10) involves the concentration of the active component, X. Again, in the quasistatic case the classical exergy follows.

3. FINITE-TIME BOUNDS DIFFERENT FOR WORK OF CREATION AND WORK OF DESTRUCTION

A spectrum of optimization methods can be used to solve the problem of extremum work. They include maximum principles [11–15] which apply a Hamiltonian, and canonical equations or dynamic programming algorithms which make use of Bellman's recurrence equation [16,17]. The connection between these methods is now well understood [15,18]. The results of the optimal work should be presented in the form of a generalized potential function which depends on the end states and duration. For the purpose of direct evaluation of the optimal work potential either the so-called Hamilton–Jacobi–Bellman equation, known from the optimal control theory, or the related Hamilton–Jacobi equation, known from the variational methods of classical mechanics, has to be found and then solved [9,15]. But, especially in the case of vectorial efficiencies, when the analytical solutions are difficult to find, discrete counterparts of these fundamental equations are necessary to find the optimal solutions by numerical methods. An approach which uses the dynamic programming remains then virtually the sole suitable method. The numerical solution of Bellman's equation is achieved with the help of a computer, due to the iterative nature of operations involved.

In the case of pure heat transfer, the extremal specific work can be found analytically for every process mode; the result is

$$\begin{aligned} I &\equiv c\left(T^i-T^f\right)-cT^e \ln\frac{T^i}{T^f}+cT^e\frac{\xi}{1+\xi}\ln\frac{T^i}{T^f} \\ &= c\left(T^i-T^f\right)-cT^e \ln\frac{T^i}{T^f}-cT^e\left(H/cT^2\right)^{1/2}\ln\frac{T^i}{T^f} \equiv -R, \end{aligned} \tag{11}$$

where ξ is the logarithmic intensity of the optimal process.

The principal function I, or its negative $R=-I$, constitute the solution of the Hamilton–Jacobi equation for the work optimization problem. The particular extremal work which describes the (generalized or classical) exergy should contain the environment temperature as one of the boundary states. The finite-time exergy is the maximal work $W_{\max}=I\left(T^i,\tau^i,T^f,\tau^f\right)$ with $T^i=T$ and $T^f=T^e$ for the engine mode, and the negative minimal work $-W_{\min}=R\left(T^i,t^i,T^f,\tau^f\right)$ with $T^i=T^e$ and $T^f=T$ for the heat-pump mode. For the vanishing intensities H or ξ the change of the classical thermal exergy is recovered.

A simple formula for the dissipative thermal exergy is obtained in terms of the Hamiltonian

$$E_x=c\left(T-T^e\right)-cT^e\ln\frac{T}{T^e}\pm cT^e\left(H/cT^e\right)^{1/2}\ln\frac{T}{T^e}=E_x\left(T,T^e,0\right)\pm T^eS_\sigma, \quad (12)$$

where $E_x\left(T,T^e,\infty\right)$ is the classical exergy, and $S_\sigma=\min S_\sigma$ is the minimal entropy production. The upper sign refers to the heat-pump mode and the lower sign to the engine mode.

In the case of simultaneous heat and mass transfer in an evaporation process, the following molar exergy was found [6]

$$E_x=\left(c_g+Xc_p\right)\left[\left(T-T^e\right)-T^e\ln\frac{T}{T^e}\right]+RT^e\left[X\ln\frac{X\left(1+X^e\right)}{\left(1+X\right)X^e}+\ln\frac{1+X^e}{1+X}\right]$$

$$\pm T^eS_\sigma\left(T,X,T^e,X^e,\tau^f\right). \quad (13)$$

This exergy also corresponds with underlying functions, $I\left(T,X,T^e,X^e,\tau^f\right)$ and $R\left(T,X,T^e,X^e,\tau^f\right)$, which are extensions of those described by Eq. (11). As it is very easy to find these functions from Eq. (13), we do not write down the respective formulae; yet, in analysis that follows, we assume that these formulae are known. Of the two parts of the finite-time exergy, the classical one is known from reversible thermodynamics. Thus one may use tables of E_x to evaluate the associated minimal entropy production for the process. These results are both in the spirit of the finite-time thermodynamics [1,4,5,10] and the entropy generation minimization [19–21]. Both steady-state processes and their unsteady counterparts can effectively be modelled [22].

It follows that the classical exergy suffices for an exact estimation of the extremal work for small H_{TU}, i.e., for the excellent transfer conditions, or for infinitely long times of the energy exchange. The generalized exergy E_x serves to explain the restrictive applicability of the classical thermodynamic bounds

when they are applied to real processes, and to show that these bounds should be replaced by stronger bounds obtained from non-equilibrium thermodynamics.

The classical exergy is the quantity which defines bounds on work delivered from (or supplied to) very slow reversible processes. These classical bounds are reversible in the sense that the magnitude of the work delivered during the reversible approaching of the system to equilibrium is equal to the magnitude of the work supplied, after the initial and final states are inverted, i.e. when the second process reverses to the initial state of the first. Our results (12) and (13) generalize the classical exergy for finite rate transitions.

Let us now discuss main physical properties of such transitions. During the approach to the equilibrium the *engine mode* of the system takes place in which the work is released, during the departure from the equilibrium the *heat-pump mode* occurs in which the work is supplied. The work W delivered in the engine mode is positive by assumption. In the heat-pump mode the work W is negative, which means that the positive work $(-W)$ must be supplied to the system. The optimization problems, which yield the generalized exergies and their underlying potential functions I and R, involve the maximum of the work delivered, $\max W$, and the minimum of the work supplied, $\min(-W)$. We have shown that while the reversibility property is lost for such exergy, its thermokinetic bounds are stronger and hence more useful than classical thermostatic bounds. This substantiates the role of the dissipative exergy for evaluation of energy limits in practical systems.

The reversibility of bounds in classical thermodynamics means that with the classical exergy, thermostatics simultaneously provides the lower bound on the real work which should be supplied to the system, and the upper bound on the work which can be released by the system. The work-producing process is the inversion of the work-consuming process (the final state of the second process is the initial state of the first, and conversely), and the duration of each process is infinitely long. In thermostatics the two bounds on the work coincide. From a more general viewpoint, the two processes (direct and inverse) represent the destruction and creation of a non-equilibrium structure. Thus, in classical thermostatics, the two bounds, the one on the structure destruction and the one on the structure creation, coincide.

However, such static limits are often too far from reality to be very useful. Whenever one takes into account the necessity of termination of the process in finite time and inherent role of resistances as dissipative parts of the system (in boundary layers, in particular), the finite-rate exergy provides stronger (lower and upper) bounds on the real work which should be supplied to the system or extracted from it. These bounds do not coincide, but they are more realistic than the quasistatic bounds of classical thermostatics. It is the hysteretic effect, caused by the dissipation, i.e., an increase of the exergy supplied in the pump mode of the system (work consumption) and a decrease of the exergy released in the engine mode of the system (work delivery), which makes thermokinetic

bounds stronger than thermostatic bounds. The divergence of the bounds proves that a large number of processes, which are permitted by classical thermostatics, are excluded by limitations inherent in the process kinetics. Also, from the more general viewpoint formulated above, the divergence of the bounds proves that processes of creation of a non-equilibrium structure require larger magnitudes of mechanical energy than processes of destruction of this structure. In the latter only a part of the available mechanical energy can be released in a finite time. In effect, the creation processes consume always more mechanical energy than the corresponding destruction processes with inverted thermodynamic states of the system.

Summing up, for a process and its inversion, the two bounds which coincide in thermostatics diverge in thermokinetics, and the divergence grows with the rate indices of the process. This means that, for sufficiently high rate indices, one can obtain quite high lower bound on the supplied work and even vanishing upper bound on the released work. The finite rate processes always increase the absolute value of the extremal work supplied in processes departing from the equilibrium, and decrease the corresponding work produced in processes approaching the equilibrium. These conclusions, along with the quantitative analysis presented in the series of works [1–3], provide a means for improved evaluation of the mechanical energy limits in practical systems.

In particular, we can apply the above general results along with Eqs. (11) to (13) and principal functions $I\left(T, X, T^e, X^e, \tau^f\right)$ and $R\left(T, X, T^e, X^e, \tau^f\right)$ to the important problem of irreversible cycles undergoing with finite periods. These cycles can be described in standard thermodynamic spaces or in extended spaces including the physical time or its monotonic measure (such as, for example, our τ or R) as an additional variable.

The irreversible cycles constitute closed loops in thermodynamic spaces. However, in the extended spaces which contain time as an extra coordinate, these cycles are no longer represented by closed loops, but by screwlines. Thermodynamic cycles are projections of these screwlines into the space of thermodynamic parameters. As this work shows, not all of the mathematically possible screwlines are admissible from the viewpoint of the second law. In other words, Nature admits only definite types of screwlines in the space-time $T - X - \tau$. Their basic property is determined by the requirement that in a real cycle parts corresponding to work consumption must assure larger magnitude of work than the parts corresponding to work production.

In conclusion, enhanced bounds are formed by the generalized exergy for the work released (destruction) and consumed (creation) in real processes, in terms of the residual generated entropy as the rate penalty. The hysteretic properties of the generalized exergy as a finite-time work function are important. They are associated with different values of the extremal work obtained when the process which leaves the equilibrium is compared with its inverse, which approaches the equilibrium. While in the classical reversible thermodynamics these two modes

are accomplished with exactly the same magnitude of work, in the dissipative theory the work of creation and work of destruction are no longer equal. A significant decrease of the maximal work, received from the destruction of a system, and an increase of the minimal work that has to be added to create a system, is shown in the high-rate regimes and for short durations of thermodynamic processes. This property makes it possible to exclude definite cyclic evolutions as forbidden by the second law, which is a result important for the theory of irreversible processes.

ACKNOWLEDGEMENT

The author acknowledges a support from the grant T09C06316 of the Polish Committee of National Research.

REFERENCES

1. Sieniutycz, S. Irreversible Carnot problem of maximum work in a fnite time via Hamilton–Jacobi–Bellman theory. *J. Non-Equilib. Thermodyn.*, 1997, **22**, 3, 260–284.
2. Sieniutycz, S. Hamilton–Jacobi–Bellman theory of dissipative thermal availability. *Phys. Rev. E*, 1997, **56**, 5, 5051–5064.
3. Sieniutycz, S. Nonlinear thermokinetics of maximum work in finite time. *Int. J. Eng. Sci.*, 1998, **36**, 5/6, 577–597.
4. de Vos, A. *Endoreversible Thermodynamics of Solar Energy Conversion*. Clarendon, Oxford, 1992.
5. Curzon, F. L. and Ahlborn, B. Efficiency of Carnot engine at maximum power output. *Am. J. Phys.*, 1975, **43**, 1, 22–24.
6. Sieniutycz, S. Optimal control for multistage endoreversible engines with heat and mass transfer. *J. Non-Equilib. Thermodyn*, 1999, **23** (in print).
7. Gordon, J. M. and Orlov, V. Performance characteristics of endoreversible chemical engines. *J. Appl. Phys.*, 1993, **74**, 9, 5303–5309.
8. Chen, L., Sun, F., Wu, Ch., and Wu, J. Performance characteristic of endoreversible chemical engines. *Energy Convers. Manage.*, 1997, **38**, 18, 1841–1846.
9. Sieniutycz, S. *Optimization in Process Engineering*. Wydawn. Naukowo Techniczne, Warszawa, 1978.
10. Andresen, B. *Finite-Time Thermodynamics*. University of Copenhagen, Copenhagen, 1983.
11. Sieniutycz, S. and Szwast, Z. A discrete algorithm for optimization with a constant Hamiltonian and its application to chemical engineering. *Int. J. Chem. Eng.*, 1983, **23**, 2, 155–166.
12. Fan, L. T. and Wang, C. S. *The Discrete Maximum Principle: A Study of Multistage System Optimization*. Wiley, New York, 1964.
13. Boltyanskij, V. G. *Optimal Control of Discrete Systems*. Nauka, Moscow, 1973 (in Russian).
14. Fan, L. T. *The Continuous Maximum Principle: A Study of Complex System Optimization*. Wiley, New York, 1966.
15. Findeisen, W., Szymanowski, J., and Wierzbicki, A. *Theory and Computational Methods of Optimization*. Państwowe Wydawn. Naukowe, Warszawa, 1980.
16. Bellman, R. E. *Adaptive Control Processes: A Guided Tour*. Princeton Univ. Press, Princeton, 1961.
17. Aris, R. *Discrete Dynamic Programming*. Blaisdell, New York, 1964.

18. Boltyanskij, V. G. *Mathematical Methods of Optimal Control.* Nauka, Moscow, 1969 (in Russian).
19. Bejan, A. Models of power plants that generate minimum entropy while operating at maximum power. *Am. J. Phys.*, 1996, **64**, 8, 1054–1059.
20. Bejan, A. Entropy generation minimization: The new thermodynamics of finite-size devices and finite-time processes. *J. Appl. Phys.*, 1996, **79**, 3, 1191–1218.
21. Bejan, A. *Entropy Generation Minimization: The Method of Thermodynamic Optimization of Finite-Time Systems.* CRC Press, Boca Raton, 1995.
22. Sieniutycz, S. Non-equilibrium and classical thermodynamics for practical systems: Today closer together than ever before. *Int. J. Appl. Thermodyn.*, 1998, **1**, 1, 45–59.

ENDOPÖÖRATAVATE SOOJUSMASINATE MITMEASTMELINE OPTIMEERIMINE LÕPLIKU AJA JOOKSUL

Stanislaw SIENIUTYCZ

Üldist mitmeastmelise optimeerimise teooriat on rakendatud Curzoni–Ahlborni–Novikovi masinatest või soojuspumpadest koosnevate süsteemide uurimisel. Optimeerimiseks on koostatud süsteemide mudelid, mis sisaldavad juhtimis- ja olekumuutujaid ning mittejuhitavaid või fikseeritud parameetreid. Lähtudes klassikalisest ja lõpliku aja termodünaamikast ning energia analüüsist on optimeeritud erinevate masinate (soojus-, päikese-, sisepõlemismasinate) ja üksikoperatsioonide (destilleerimise, keemiliste reaktsioonide) juhtimine.