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THERMOELASTIC-PLASTIC ANALYSIS OF SOLIDS AT FINITE DEFORMATIONS

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Abstract. A method to describe the complex material behaviour of solids at finite strain is proposed. The method is based on the concept of the dual variables and on the assumption of intermediate configurations. Kinematically correct additive decomposition of the strain is presented. Geometric linear constitutive models are generalized for the finite strain theory. An application and an example are given for thermoelastic-plastic analysis.

Key words: finite deformations, plasticity, deformation decomposition, finite element.

1. INTRODUCTION

Solids usually show complex material behaviour. The forming processes of engineering materials consist of elastic, plastic, viscoelastic, creep and other deformations. If the deformation is large, the mechanical model is complicated. In the following we propose a method for modelling materials at finite deformations.

The method is a generalization of the multiplicative decomposition of the deformation gradient for the elastic and plastic parts proposed by Lee and Liu [1]. Hartmann and Haupt [2] and Lührs [3] developed similar procedures for the special cases of plasticity and viscoplasticity.

In this work we divide the deformation gradient into an arbitrary number of components. The strain and stress are calculated using the concept of the dual variables introduced by Haupt and Tsakmakis [4].

The paper is organized as follows. First, the basic relations are given, then we show the decomposition of the deformation. A general form of the constitutive relation for each deformation component is presented. Finally, an application and a numerical example are given to illustrate the results.

2. BASIC RELATIONS

The deformation gradient tensor is [5]

$$\mathbf{F} = \frac{\partial \mathbf{x}}{\partial \mathbf{X}}, \tag{1}$$

where $\mathbf{x}$ and $\mathbf{X}$ are the position vectors of a material point in the current and reference configurations, respectively.

In the reference configuration the Lagrange strain tensor $\mathbf{E}$ and the 2nd Piola–Kirchhoff stress tensor $\mathbf{S}$ are chosen as *conjugate variables* [6]

$$\mathbf{E} = \frac{1}{2}\left(\mathbf{F}^T\mathbf{F} - \mathbf{1}\right), \qquad \mathbf{S} = \det(\mathbf{F})\mathbf{F}^{-1}\boldsymbol{\sigma}\mathbf{F}^{-T} = \mathbf{F}^{-1}\boldsymbol{\tau}\mathbf{F}^{-T} \tag{2}$$

where $\boldsymbol{\sigma}$ is the Cauchy (true) stress, $\boldsymbol{\tau}$ is the Kirchhoff (weighted Cauchy) stress tensor and $\mathbf{1}$ denotes the second order identity tensor.

3. KINEMATICS

We assume n deformation components, therefore the total deformation gradient tensor is given by the product

$$\mathbf{F} = \mathbf{F}_n\mathbf{F}_{n-1}\cdots\mathbf{F}_k\cdots\mathbf{F}_2\mathbf{F}_1, \tag{3}$$

with $\det(\mathbf{F}_i) > 0$. The decomposition results in diverse fictitious intermediate configurations beside the reference and current configurations.

Using the multiplicative decomposition of the deformation gradient, an additive decomposition of the Lagrange strain tensor $\mathbf{E}$ can be derived. The component $\mathbf{E}_i$ is the deformation between two configurations next to each other:

$$\mathbf{E} = \sum_{i=1}^{n}\mathbf{E}_i, \quad \text{with} \quad \mathbf{E}_i = \frac{1}{2}\left(\boldsymbol{\Psi}_i^T\boldsymbol{\Psi}_i - \boldsymbol{\Psi}_{i-1}^T\boldsymbol{\Psi}_{i-1}\right), \tag{4}$$

where the tensor $\boldsymbol{\Psi}_k$ is defined as

$$\boldsymbol{\Psi}_k = \begin{cases} \mathbf{1}, & \text{for } k = 0, \\ \mathbf{F}_k\cdots\mathbf{F}_1, & \text{for } 0 < k < n, \\ \mathbf{F}, & \text{for } k = n. \end{cases} \tag{5}$$

In the kth configuration the strain $\boldsymbol{\Gamma}^k$, the ith strain component $\boldsymbol{\Gamma}_i^k$, and the stress $\mathbf{T}^k$ are calculated using the concept of the dual variables:

$$\boldsymbol{\Gamma}^k = \boldsymbol{\Psi}_k^{-T}\mathbf{E}\boldsymbol{\Psi}_k^{-1}, \qquad \boldsymbol{\Gamma}_i^k = \boldsymbol{\Psi}_k^{-T}\mathbf{E}_i\boldsymbol{\Psi}_k^{-1}, \qquad \mathbf{T}^k = \boldsymbol{\Psi}_k\mathbf{S}\boldsymbol{\Psi}_k^T. \tag{6}$$

The rates of the strain, the strain components and the stress, denoted by $\overset{\triangle}{\boldsymbol{\Gamma}}{}^k$, $\overset{\triangle}{\boldsymbol{\Gamma}}{}^k_i$ and $\overset{\triangledown}{\mathbf{T}}{}^k$, respectively, are Oldroyd derivatives and are given also by way of dual variables

$$\begin{aligned}
\overset{\triangle}{\boldsymbol{\Gamma}}{}^k &= \boldsymbol{\Psi}_k^{-T}\dot{\mathbf{E}}\boldsymbol{\Psi}_k^{-1} = \dot{\boldsymbol{\Gamma}}^k + \mathbf{L}^{kT}\boldsymbol{\Gamma}^k + \boldsymbol{\Gamma}^k\mathbf{L}^k, \\
\overset{\triangle}{\boldsymbol{\Gamma}}{}^k_i &= \boldsymbol{\Psi}_k^{-T}\dot{\mathbf{E}}_i\boldsymbol{\Psi}_k^{-1} = \dot{\boldsymbol{\Gamma}}^k_i + \mathbf{L}^{kT}\boldsymbol{\Gamma}^k_i + \boldsymbol{\Gamma}^k_i\mathbf{L}^k, \\
\overset{\triangledown}{\mathbf{T}}{}^k &= \boldsymbol{\Psi}_k\dot{\mathbf{S}}\boldsymbol{\Psi}_k^T = \dot{\mathbf{T}}^k - \mathbf{L}^k\mathbf{T}^k - \mathbf{T}^k\mathbf{L}^{kT}.
\end{aligned} \tag{7}$$

The velocity gradient in the kth configuration is defined as $\mathbf{L}^k = \dot{\boldsymbol{\Psi}}_k\boldsymbol{\Psi}_k^{-1}$.

In the reference configuration ($k = 0$) the dual variables give the Lagrange strain $\boldsymbol{\Gamma}^0 = \mathbf{E}$, $\boldsymbol{\Gamma}^0_i = \mathbf{E}_i$, and the 2nd Piola–Kirchhoff stress $\mathbf{T}^0 = \mathbf{S}$; the strain and stress rates are simply equal to the time derivatives. In the current configuration ($k = n$) the dual variables give the Almansi strain tensor $\boldsymbol{\Gamma}^n = \mathbf{a}$, Almansi strain components $\boldsymbol{\Gamma}^n_i = \mathbf{a}_i$, and the Kirchhoff stress tensor $\mathbf{T}^k = \boldsymbol{\tau}$. The strain and stress rates can be expressed in terms of the velocity gradient $\mathbf{L}^n = \mathbf{l} = \dot{\mathbf{F}}\mathbf{F}^{-1}$: $\overset{\triangle}{\boldsymbol{\Gamma}}{}^n = \overset{\triangle}{\mathbf{a}} = \frac{1}{2}\left(\mathbf{l} + \mathbf{l}^T\right)$, $\overset{\triangle}{\boldsymbol{\Gamma}}{}^n_i = \overset{\triangle}{\mathbf{a}}_i = \dot{\mathbf{a}}_i + \mathbf{l}^T\mathbf{a}_i + \mathbf{a}_i\mathbf{l}$ and $\overset{\triangledown}{\mathbf{T}}{}^n = \overset{\triangledown}{\boldsymbol{\tau}} = \dot{\boldsymbol{\tau}} - \mathbf{l}\boldsymbol{\tau} - \boldsymbol{\tau}\mathbf{l}^T$.

4. CONSTITUTIVE RELATIONS

A generalization of the geometric linear constitutive model for the finite strain is proposed. The method of generalization is the replacing of the stress and the linearized strain with dual variables in the geometric linear relation for each deformation component.

The constitutive relation for each deformation component is written in terms of an isotropic functional. The constitutive relation for the jth deformation component given in the kth configuration is

$$\mathcal{F}^k_j\left\{\mathbf{T}^k, \boldsymbol{\Gamma}^k_1, \cdots, \boldsymbol{\Gamma}^k_n\right\} = \mathbf{0}. \tag{8}$$

The functional contains the stress and one or more strain components. If the configuration is a *natural configuration* of the constitutive relation, the functional contains only one deformation component

$$\mathcal{F}^{j-1}_j\left\{\mathbf{T}^{j-1}, \boldsymbol{\Gamma}^{j-1}_j\right\} = \mathbf{0}, \quad \text{or} \quad \mathcal{F}^j_j\left\{\mathbf{T}^j, \boldsymbol{\Gamma}^j_j\right\} = \mathbf{0}. \tag{9}$$

The constitutive relation is objective and can be transformed between configurations without loosing objectivity, using the strain components only. For more details see Appendix A.

5. APPLICATION

The theory is now applied to a deformation process which consists of elastic, plastic, and thermal expansion parts. In this case the multiplicative decomposition of the deformation gradient is

$$\mathbf{F} = \mathbf{F}_3\mathbf{F}_2\mathbf{F}_1 \equiv \mathbf{F}_e\mathbf{F}_\theta\mathbf{F}_p, \tag{10}$$

where $\det(\mathbf{F}_e) > 0$, $\det(\mathbf{F}_\theta) > 0$, and $\det(\mathbf{F}_p) > 0$. (Exactly this decomposition was used by Idesman and Levitas [7], and the strain decomposition of Simo and Miehe [8] can also be understood to be similar to our method.) Beside the reference configuration B_0 and the current configuration B_t, the plastic intermediate configuration B_t^p and the thermal intermediate configuration B_t^θ also result from the decomposition.

The constitutive equations are formulated in the intermediate configurations and then transformed to the reference and current configurations, therefore all strain tensors in the different configurations are needed. The dual variables for this decomposition are presented in Appendix B. The constitutive relation can be formulated by means of a set of differential equations instead of functionals [9].

The constitutive model is a generalization of the geometric linear constitutive model of thermoplasticity for finite deformation. The elastic deformation is formulated in the thermal intermediate configuration and follows *Hooke*'s law

$$\mathbf{T}^\theta = 2\mu \left[\boldsymbol{\Gamma}_e^\theta + \frac{\nu}{1-2\nu} \operatorname{tr}\left(\boldsymbol{\Gamma}_\mathrm{e}^\theta\right) \mathbf{1} \right], \tag{11}$$

where μ is shear modulus and ν is Poisson's ratio.

The relation between thermal strain and temperature is defined in the plastic intermediate configuration as

$$\boldsymbol{\Gamma}_\theta^p = \alpha\,(\theta - \theta_0)\,\mathbf{1}, \tag{12}$$

where α is the thermal expansion coefficient. If the thermal expansion part of the deformation gradient is written in the form $\mathbf{F}_\theta = J_\theta^{1/3}\,\mathbf{1}$, the thermal volumetric expansion is $J_\theta = (2\alpha\,(\theta - \theta_0) + 1)^{3/2}$.

The plasticity model is formulated in terms of the plastic intermediate configuration. To define the elastic domain a *von Mises* yield function F is used

$$F = \frac{1}{2}\mathrm{dev}\mathbf{T}^\mathrm{p} : \mathrm{dev}\mathbf{T}^\mathrm{p} - \frac{1}{3}\sigma_y(s)^2, \tag{13}$$

where $\mathrm{dev}(.) = (.) - \frac{1}{3}\mathrm{tr}(.)\mathbf{1}$, ":" denotes the inner product of two second order tensors [5], and $\sigma_y(s)$ represents the yield stress function for the isotropic hardening material

$$\sigma_y(s) = \sigma_{y0} + Hs, \qquad \dot{s} = \sqrt{\frac{2}{3}\,\overset{\triangle}{\boldsymbol{\Gamma}}{}_p^p : \overset{\triangle}{\boldsymbol{\Gamma}}{}_p^p}. \tag{14}$$

Here σ_{y0} is the initial yield stress, H is the linear hardening modulus, and s is the accumulated inelastic strain defined by its time derivative. The associative flow rule is

$$\overset{\triangle}{\mathbf{\Gamma}}{}^{p}_{p} = \begin{cases} \lambda \dfrac{\partial F}{\partial \mathbf{T}^{\mathrm{p}}} = \lambda \operatorname{dev} \mathbf{T}^{\mathrm{p}}, & \text{for } F = 0 \text{ and loading in the plastic range,} \\ \mathbf{0}, & \text{for all other cases.} \end{cases} \tag{15}$$

The plastic deformation is isochoric if and only if

$$\frac{\mathrm{d}}{\mathrm{d}t} \det(\mathbf{F}_p) = \det(\mathbf{F}_p) \operatorname{tr} \mathbf{\Gamma}^{\mathrm{p}}_{\mathrm{p}} = 0 \longrightarrow \operatorname{tr} \mathbf{\Gamma}^{\mathrm{p}}_{\mathrm{p}} = 0. \tag{16}$$

Because of the deviatoric tensor $\operatorname{dev}(\mathbf{T}^{\mathrm{p}})$, the flow rule satisfies the condition above.

The set of the constitutive equations is transformed to other configurations and is given in a general form in Table 1.

Table 1. General form of the constitutive equations

Elasticity condition:	$\tilde{\sigma} = 2\mu \left[\tilde{\mathbf{g}}_\theta^{-1} \tilde{\varepsilon}_e \tilde{\mathbf{g}}_\theta^{-1} + \dfrac{\nu}{1-2\nu} \left(\tilde{\mathbf{g}}_\theta^{-1} : \tilde{\varepsilon}_e \right) \tilde{\mathbf{g}}_\theta^{-1} \right]$
Thermal expansion:	$\tilde{\varepsilon}_\theta = \alpha \left(\theta - \theta_0 \right) \tilde{\mathbf{g}}_p$
Yield condition:	$F = \dfrac{1}{2} \left[\left(\tilde{\mathbf{g}}_p \tilde{\sigma} \tilde{\mathbf{g}}_p \right) : \tilde{\sigma} - \dfrac{1}{3} \left(\tilde{\mathbf{g}}_p \, \tilde{\sigma} \right)^2 \right] - \dfrac{1}{3} \sigma_y^2$
Hardening:	$\sigma_y = \sigma_{y0} + Hs$
Plastic arc-length:	$\dot{s} = \sqrt{\dfrac{2}{3} \left(\tilde{\mathbf{g}}_p^{-1} \, \overset{\triangle}{\tilde{\varepsilon}}_p \, \tilde{\mathbf{g}}_p^{-1} \right) : \overset{\triangle}{\tilde{\varepsilon}}_p}$
Flow rule:	$\overset{\triangle}{\tilde{\varepsilon}}_p = \begin{cases} \lambda \left[\tilde{\mathbf{g}}_p \tilde{\sigma} \tilde{\mathbf{g}}_p - \dfrac{1}{3} \left(\tilde{\mathbf{g}}_p : \tilde{\sigma} \right) \tilde{\mathbf{g}}_p \right], \\ 0. \end{cases}$

Table 2. Configuration-dependent tensors of the constitutive relation

Configuration	$\tilde{\varepsilon}_e$	$\tilde{\varepsilon}_\theta$	$\overset{\triangle}{\tilde{\varepsilon}}_p$	$\tilde{\sigma}$	$\tilde{\mathbf{g}}_p$	$\tilde{\mathbf{g}}_\theta$
B_0	$\mathbf{E}_e$	$\mathbf{E}_\theta$	$\dot{\mathbf{E}}_p$	$\mathbf{S}$	$2\mathbf{E}_p + \mathbf{1}$	$2\mathbf{E}_p + 2\mathbf{E}_\theta + \mathbf{1}$
B_t^p	$\mathbf{\Gamma}_e^p$	$\mathbf{\Gamma}_\theta^p$	$\overset{\triangle}{\mathbf{\Gamma}}{}_p^p$	$\mathbf{T}^p$	$\mathbf{1}$	$2\mathbf{\Gamma}_\theta^p + \mathbf{1}$
B_t^θ	$\mathbf{\Gamma}_e^\theta$	$\mathbf{\Gamma}_\theta^\theta$	$\overset{\triangle}{\mathbf{\Gamma}}{}_p^\theta$	$\mathbf{T}^\theta$	$\mathbf{1} - 2\mathbf{\Gamma}_\theta^\theta$	$\mathbf{1}$
B_t	$\mathbf{a}_e$	$\mathbf{a}_\theta$	$\overset{\triangle}{\mathbf{a}}_p$	τ	$\mathbf{1} - 2\mathbf{a}_\theta - 2\mathbf{a}_e$	$\mathbf{1} - 2\mathbf{a}_e$

The meaning of the quantities in the equations are summarized for different configurations in Table 2. We note that the tensors $\tilde{\mathbf{g}}_p$ and $\tilde{\mathbf{g}}_\theta$ are the metric tensors on the plastic and the thermal intermediate configurations, respectively.

In the case of metals we can assume strain components to be small. This assumption results in a simplification of the numerical procedures. The quantities in the constitutive relations are presented for different cases in the current configuration in Table 3, where ε_e, ε_θ and ε_p are the linearized elastic, thermal, and plastic strain tensor components.

Table 3. Tensor quantities in case of small strain components

	$\tilde{\varepsilon}_e$	$\tilde{\varepsilon}_\theta$	$\overset{\triangle}{\tilde{\varepsilon}}_p$	$\tilde{\sigma}$	$\tilde{\mathbf{g}}_p$	$\tilde{\mathbf{g}}_\theta$
Elastic	$\mathbf{a}_e$	$\mathbf{a}_\theta$	$\overset{\triangle}{\mathbf{a}}_p$	τ	$\mathbf{1} - 2\mathbf{a}_\theta$	$\mathbf{1}$
Elastic, thermal	$\mathbf{a}_e$	$\mathbf{a}_\theta$	$\overset{\triangle}{\mathbf{a}}_p$	τ	$\mathbf{1}$	$\mathbf{1}$
Elastic, thermal, plastic	ε_e	ε_θ	$\dot{\varepsilon}_p$	σ	$\mathbf{1}$	$\mathbf{1}$

6. EXAMPLE

A thick-walled tube in plane strain under internal pressure $p = 400$ MPa with outer radius $r_o = 100$ mm and inner radius $r_i = 10$ mm is considered. The temperature at the surfaces is $\theta_o = 0$ °C and $\theta_i = 100$ °C.

The constants for the constitutive model are $\mu = 77.9$ GPa and $\nu = 0.3$ for the elasticity relation and $\alpha = 10^{-5}$ 1/°C for the thermal expansion. In the plastic region $\sigma_{yo} = 100$ MPa and $H = 2$ GPa.

We performed the numerical calculations with the FE-system MARC. The thermoelastic-plastic constitutive relation was implemented using the user subroutine hypela2.f. We assumed small elastic and small thermal deformation components and large plastic deformations. The temperature field and the thermal expansion are calculated with built in routines of MARC.

The tube was thermally loaded, then the internal pressure was applied. Figure 1 shows the equivalent Cauchy stress on the inner and outer surfaces as a function of the applied pressure after the thermal load was applied.

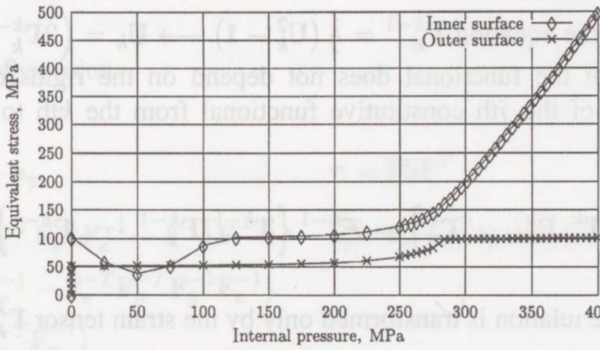

Fig. 1. Equivalent stress at the inner and outer surfaces.

7. CONCLUSIONS

A method for solving the problem of complex material behaviour at finite deformations has been developed. An additive decomposition of the strain, based on the multiplicative decomposition of the deformation gradient, has been derived using the concept of the dual variables. In the application of the method and in the example, a simple thermoelastic-plastic problem has been solved.

With this method various constitutive models of elasticity, viscoelasticity, plasticity and viscoplasticity may in a simple manner be generalized for the theory of finite deformations.

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APPENDIX

A. TRANSFORMATION OF THE CONSTITUTIVE RELATION

First, we transform the constitutive relation between two configurations next to each other. The strain and the stress in the kth configuration expressed in terms of the $(k-1)$th configuration are

$$\begin{aligned}\mathbf{\Gamma}_i^k &= \mathbf{F}_k^{-T}\mathbf{\Gamma}_i^{k-1}\mathbf{F}_k^{-1} = \mathbf{R}_k\mathbf{U}_k^{-1}\mathbf{\Gamma}_i^{k-1}\mathbf{U}_k^{-1}\mathbf{R}_k^T, \\ \mathbf{T}^k &= \mathbf{F}_k\mathbf{T}^{k-1}\mathbf{F}_k^T = \mathbf{R}_k\mathbf{U}_k\mathbf{T}^{k-1}\mathbf{U}_k\mathbf{R}_k^T,\end{aligned} \tag{17}$$

where the local polar decomposition $\mathbf{F}_k = \mathbf{R}_k\mathbf{U}_k$ ($\mathbf{U}_k = \mathbf{U}_k^T$ and $\mathbf{R}_k^{-1} = \mathbf{R}_k^T$) was used. Replacing the transformed variables in the functional of the constitutive relation, using the equation $\mathbf{\Gamma}_k^{k-1} = \frac{1}{2}\left(\mathbf{U}_k^2 - \mathbf{1}\right) \longrightarrow \mathbf{U}_k = \left(2\mathbf{\Gamma}_k^{k-1} + \mathbf{1}\right)^{1/2}$, and considering that the functional does not depend on the rigid rotation $\mathbf{R}_k$, the transformation of the jth constitutive functional from the kth to the $(k-1)$th configuration is

$$\mathcal{F}_j^k\left\{\mathbf{T}^k, \mathbf{\Gamma}_1^k, \cdots, \mathbf{\Gamma}_n^k\right\} = \mathcal{F}_j^{k-1}\left\{\mathbf{T}^{k-1}, \mathbf{\Gamma}_1^{k-1}, \cdots, \mathbf{\Gamma}_n^{k-1}\right\} = \mathbf{0}. \tag{18}$$

The constitutive relation is transformed only by the strain tensor $\mathbf{\Gamma}_k^{k-1}$.

B. DUAL VARIABLES IN DIFFERENT CONFIGURATIONS

In this appendix, the dual strain and stress measures are given for the deformation components.

Reference configuration:

$$
\begin{aligned}
&\mathbf{E} = \mathbf{E}_p + \mathbf{E}_\theta + \mathbf{E}_e && \mathbf{S} \\
&\mathbf{E}_p = \tfrac{1}{2}\left(\mathbf{F}_p^T\mathbf{F}_p - \mathbf{1}\right) && \dot{\mathbf{E}}_p = \tfrac{1}{2}(\dot{\mathbf{F}}_p^T\mathbf{F}_p + \mathbf{F}_p^T\dot{\mathbf{F}}_p) \\
&\mathbf{E}_\theta = \tfrac{1}{2}\left(\mathbf{F}_p^T\mathbf{F}_\theta^T\mathbf{F}_\theta\mathbf{F}_p - \mathbf{F}_p^T\mathbf{F}_p\right) \\
&\mathbf{E}_e = \tfrac{1}{2}\left(\mathbf{F}^T\mathbf{F} - \mathbf{F}_p^T\mathbf{F}_\theta^T\mathbf{F}_\theta\mathbf{F}_p\right)
\end{aligned} \tag{19}
$$

Plastic intermediate configuration:

$$
\begin{aligned}
&\mathbf{\Gamma}^p = \mathbf{\Gamma}^p_p + \mathbf{\Gamma}^p_\theta + \mathbf{\Gamma}^p_e && \mathbf{T}^p = \mathbf{F}_p\mathbf{S}\mathbf{F}_p^T \\
&\mathbf{\Gamma}^p_p = \tfrac{1}{2}\left(\mathbf{1} - \mathbf{F}_p^{-T}\mathbf{F}_p^{-1}\right) && \overset{\triangle}{\mathbf{\Gamma}}{}^p_p = \tfrac{1}{2}(\mathbf{L}^p + \mathbf{L}^{pT}) \\
&\mathbf{\Gamma}^p_\theta = \tfrac{1}{2}\left(\mathbf{F}_\theta^T\mathbf{F}_\theta - \mathbf{1}\right) \\
&\mathbf{\Gamma}^p_e = \tfrac{1}{2}\left(\mathbf{F}_\theta^T\mathbf{F}_e^T\mathbf{F}_e\mathbf{F}_\theta - \mathbf{F}_\theta^T\mathbf{F}_\theta\right)
\end{aligned} \tag{20}
$$

Thermal intermediate configuration:

$$
\begin{aligned}
&\mathbf{\Gamma}^\theta = \mathbf{\Gamma}^\theta_p + \mathbf{\Gamma}^\theta_\theta + \mathbf{\Gamma}^\theta_e && \mathbf{T}^p = \mathbf{F}_\theta\mathbf{F}_p\mathbf{S}\mathbf{F}_p^T\mathbf{F}_\theta^T \\
&\mathbf{\Gamma}^\theta_p = \tfrac{1}{2}(\mathbf{F}_\theta^{-T}\mathbf{F}_\theta^{-1} - \mathbf{F}_\theta^{-T}\mathbf{F}_p^{-T}\mathbf{F}_p^{-1}\mathbf{F}_\theta^{-1}) && \overset{\triangle}{\mathbf{\Gamma}}{}^\theta_p = \dot{\mathbf{\Gamma}}{}^\theta_p + \mathbf{L}^{\theta T}\mathbf{\Gamma}^\theta_p + \mathbf{\Gamma}^\theta_p\mathbf{L}^\theta \\
&\mathbf{\Gamma}^\theta_\theta = \tfrac{1}{2}(\mathbf{1} - \mathbf{F}_\theta^{-T}\mathbf{F}_\theta^{-1}) \\
&\mathbf{\Gamma}^\theta_e = \tfrac{1}{2}(\mathbf{F}_e^T\mathbf{F}_e - \mathbf{1})
\end{aligned} \tag{21}
$$

Current configuration:

$$
\begin{aligned}
&\mathbf{a} = \mathbf{a}_p + \mathbf{a}_\theta + \mathbf{a}_e && \boldsymbol{\tau} = \mathbf{F}\mathbf{S}\mathbf{F}^T \\
&\mathbf{a}_p = \tfrac{1}{2}(\mathbf{F}_e^{-T}\mathbf{F}_\theta^{-T}\mathbf{F}_\theta^{-1}\mathbf{F}_e^{-1} - \mathbf{F}^{-T}\mathbf{F}^{-1}) && \overset{\triangle}{\mathbf{a}}_p = \dot{\mathbf{a}}_p + \mathbf{l}^T\mathbf{a}_p + \mathbf{a}_p\mathbf{l} \\
&\mathbf{a}_\theta = \tfrac{1}{2}(\mathbf{F}_e^{-T}\mathbf{F}_e^{-1} - \mathbf{F}_e^{-T}\mathbf{F}_\theta^{-T}\mathbf{F}_\theta^{-1}\mathbf{F}_e^{-1}) \\
&\mathbf{a}_e = \tfrac{1}{2}(\mathbf{1} - \mathbf{F}_e^{-T}\mathbf{F}_e^{-1})
\end{aligned} \tag{22}
$$

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TAHKETE KEHADE TERMOELASTNE-PLASTNE ANALÜÜS LÕPLIKEL DEFORMATSIOONIDEL

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On esitatud meetod, mis võimaldab kirjeldada tahkete kehade keerukat käitumist lõplikel deformatsioonidel. Meetod baseerub duaalsete muutujate kontseptsioonil ja vahepealse deformeerunud oleku paradigmal. On tehtud kinemaatiliselt korrektne deformatsioonitensori aditiivne dekompositsioon. Geomeetriliselt lineaarsed olekumudelid on üldistatud lõplike deformatsioonide juhule. Teooriat on rakendatud termoelastse-plastse deformatsiooniprotsessi kirjeldamiseks ning toodud numbriline näide.