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A STUDY ON CONSTITUTIVE RELATIONS OF COPPER USING THE ACCELERATION WAVES AND THE THEORY OF DYNAMIC SYSTEMS

Dedicated to Professor Robert A. Heller on the occasion of his 70th birthday

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Abstract. Important part of the governing system of equations of moving solids consists of the constitutive relations. The latter involve usually four tensorial variables. One possible way to find the form of such a relation for copper is to assume the existence of an acceleration wave. Then the constitutive relations contain only two tensorial variables. This is shown for a constitutive relation obtained as a generalization from the first author's experiment. While the results of such investigations depend on the highest order derivatives appearing in the constitutive relation, we also study the solid body as a dynamic system. In such a way we can obtain more information including also the lower order terms.

Key words: acceleration wave, Poisson bracket, perturbation, bifurcation.

1. INTRODUCTION

The equations of solid mechanics are the equation of motion, the kinematic equation, and the constitutive equation. The equation of motion and kinematic equation are first order partial differential equations, thus it is expedient to look for constitutive equations in the form of a partial differential equation, too. Suppose that an acceleration wave propagates in a solid. Then the system of partial differential equations should satisfy the integrability condition. Further properties of solids and the general system of partial differential equations are based on the first author's experiment with the uniaxial state of stress [1,2] and his suggestions concerning the generalization of the experimental result for the three-axial state of stress [3].

If the system of the basic equations is written in the form of an evolution equation and the initial and boundary conditions are also taken into consideration, a dynamic system can be defined [4,5]. In the final part of the paper such a concept will be applied for the uniaxial constitutive equation of Section 4. Dynamic systems play an important role in the stability theory [6,7]. Here the connection between stability conditions and the form of the constitutive relation is studied.

2. BASIC EQUATIONS AND COMPATIBILITY CONDITIONS

The basic equations of solids for small deformations are the following: the equation of motion

$$\sigma^{ij}_{;j} + q^i = \rho \dot{v}^i \tag{1}$$

together with the condition

$$\sigma^{ij} = \sigma^{ji},$$

the kinematic equation

$$2\dot{\epsilon}_{ij} = v_{i,j} + v_{j,i}, \tag{2}$$

and the supposed constitutive equation

$$f_{ij}\left(\sigma_{pq;\hat{k}}, \epsilon_{mn;\hat{\ell}}, \sigma_{pq}, \epsilon_{mn}, x^{\hat{p}}\right) = 0. \tag{3}$$

The constitutive function f_{ij} is symmetric: $f_{ij} = f_{ji}$.

The notations in Eqs. (1)–(3) are: σ^{ij} – stress tensor, ϵ_{ij} – strain tensor, q^i – density of the body force, ρ – mass density, v_i – particle velocity, $\dot{v}_i$ – acceleration of the particle. The indices $i, j, m, n = 1, 2, 3$ indicate the spatial coordinates of tensors or vectors, $\hat{k}, \ldots, \hat{p} = 1, 2, 3, 4$, and $x^4 = x_4 = t$ is the time. The upper index marks the contravariant and the lower one covariant derivative of a tensor or vector. For example, $\sigma_{pq;k}$ is the covariant derivative of the covariant stress tensor σ_{pq}. Cartesian rectangular coordinates will be introduced but the upper and lower indices will be used for indication of summation when the upper index is equal to the lower one.

Let us consider the acceleration wave. The wavefront is $\varphi\left(x^{\hat{q}}\right) = 0$. Derivative of φ will be denoted by

$$\varphi_{\hat{q}} = \frac{\partial \varphi}{\partial x^{\hat{q}}}.$$

The unit normal vector of the wavefront is

$$n_q = \frac{\varphi_q}{\sqrt{\varphi_p \varphi^p}}$$

and the wave speed c is [8]

$$c = -\frac{\varphi_4}{\sqrt{\varphi_p \varphi^p}}.$$

It is known that the speed of wave propagation c can be obtained from the following equation

$$\varphi_{\hat{p}} dx^{\hat{p}} = 0. \tag{4}$$

The compatibility conditions [9] are:
– the dynamic compatibility condition

$$\mu_i{}^k \varphi_k = \rho \varphi_4 \nu_i, \tag{5}$$

$$\mu_{ij} = \mu_{ji},$$

– the kinematic compatibility condition

$$2\kappa_{ij}\varphi_4 = \nu_i \varphi_j + \nu_j \varphi_i, \tag{6}$$

– the constitutive compatibility condition [3]

$$f_{ij}\left(\overset{o}{\sigma}_{pq,\hat{k}} + \mu_{pq}\varphi_{\hat{k}}, \overset{o}{\epsilon}_{mn,\hat{\ell}} + \kappa_{mn}\varphi_{\hat{\ell}}, \sigma_{pq}, \epsilon_{pq}, x^{\hat{p}}\right)$$

$$-f_{ij}\left(\overset{o}{\sigma}_{pq,\hat{k}}, \overset{o}{\epsilon}_{mn,\hat{\ell}}, \sigma_{pq}, \epsilon_{pq}, x^{\hat{p}}\right) = 0, \tag{7}$$

where μ_i^k or $\mu_{pq}, \kappa_{ij}, \nu_i$ are the wave amplitudes of stress, strain, and acceleration, respectively; $\overset{o}{\sigma}_{pq,\hat{k}}$ and $\overset{o}{\epsilon}_{mn,\hat{\ell}}$ are the stress and strain derivatives before the wavefront. Equation (7) is a system of partial differential equations for the function φ. The characteristic equations of the condition (7) are

$$dx^p \Big/ \partial f_{ij}/\partial \varphi_p = dt \Big/ \partial f_{ij}/\partial \varphi_4, \tag{8}$$

where f_{ij} and φ_p have fixed indices.

From Eqs. (4) and (8) we get

$$M_{ij}{}^{\hat{p}} \varphi_{\hat{p}} = 0, \tag{9}$$

where

$$M_{ij}{}^{\hat{p}} = \frac{\partial f_{ij}}{\partial \sigma_{k\ell,\hat{p}}}\mu_{k\ell} + \frac{\partial f_{ij}}{\partial \epsilon_{k\ell,\hat{p}}}\kappa_{k\ell}.$$

Indices $\alpha, \beta, \gamma, \ldots$ will be used for a better view of the essence of what follows. The new index α is a function of former indices i, j, that is,

$$\alpha = \begin{cases} i & \text{if} \quad i = j, \\ i + j + 1 & \text{if} \quad i \neq j. \end{cases}$$

Now $M_{ij}^{\ \hat{p}}$ is expressed as $M_\alpha^{\ \hat{p}}$, or more precisely

$$M_\alpha^{\ \hat{p}} \equiv \frac{\partial f_\alpha}{\partial \sigma_{\beta\hat{p}}}\mu_\beta + \frac{\partial f_\alpha}{\partial \epsilon_{\gamma\hat{p}}}\kappa_\gamma.$$

Following new notations

$$S_\alpha^{\ \beta\hat{p}} = \frac{\partial f_\alpha}{\partial \sigma_{\beta\hat{p}}}, E_\alpha^{\ \gamma\hat{p}} = \frac{\partial f_\alpha}{\partial \epsilon_{\gamma\hat{p}}}, s_\alpha^{\ \vartheta} = \frac{\partial f_\alpha}{\partial \sigma_\vartheta}, e_\alpha^{\ \gamma} = \frac{\partial f_\alpha}{\partial \epsilon_\gamma} \tag{10}$$

are introduced for the next section.

3. THE EXISTENCE OF ACCELERATION WAVES

The acceleration wave exists, that is, the system of partial differential equations (7) satisfies the integrability condition, when the Poisson bracket of f_α and f_β is zero [9,10]

$$(f_\alpha, f_\beta) \equiv \frac{\partial f_\alpha}{\partial \varphi_{\hat{p}}}\frac{\partial f_\beta}{\partial x^{\hat{p}}} - \frac{\partial f_\alpha}{\partial x^{\hat{p}}}\frac{\partial f_\beta}{\partial \varphi_{\hat{p}}} = 0. \tag{11}$$

Equation (11) takes then the following form [11]

$$\begin{aligned}(f_\alpha, f_\beta) \equiv M_\alpha^{\ \hat{p}} \Big[\Big(S_\beta^{\ \vartheta\hat{i}}\mu_{\vartheta\hat{p}} + E_\beta^{\ \gamma\hat{i}}\kappa_{\gamma\hat{p}} \Big)\varphi_{\hat{i}} + \Big(s_\beta^{\ \vartheta}\mu_\vartheta + e_\beta^{\ \delta}\kappa_\delta \Big)\varphi_{\hat{p}} + N_{\beta\hat{p}} \Big] \\ - M_\beta^{\ \hat{p}} \Big[\Big(S_\alpha^{\ \vartheta\hat{i}}\mu_{\vartheta\hat{p}} + E_\alpha^{\ \gamma\hat{i}}\kappa_{\gamma\hat{p}} \Big)\varphi_{\hat{i}} + \Big(s_\alpha^{\ \vartheta}\mu_\vartheta + e_\alpha^{\ \delta}\kappa_\delta \Big)\varphi_{\hat{p}} + N_{\alpha\hat{p}} \Big] = 0,\end{aligned} \tag{12}$$

where $\mu_{\vartheta\hat{p}}$ and $\kappa_{\gamma\hat{p}}$ are $\partial\mu_\theta/\partial x^{\hat{p}}$ and $\partial\kappa_\gamma/\partial x^{\hat{p}}$, respectively, $M_\alpha^{\ \hat{p}}\varphi_{\hat{p}} = 0$ because of Eq. (9), and

$$M_\alpha^{\ \hat{p}} \equiv \frac{\partial f_\alpha}{\partial \varphi_{\hat{p}}}.$$

Finally [3],

$$N_{\alpha\hat{p}} \equiv \frac{\partial f_\alpha}{\partial x^{\hat{p}}} = \Big(S_\alpha^{\ \beta\hat{i}}\mu_{\beta\hat{p}} + E_\alpha^{\ \gamma\hat{i}}\kappa_{\gamma\hat{p}} \Big)\varphi_{\hat{i}} + \Big(s_\alpha^{\ \vartheta}\mu_\vartheta + e_\alpha^{\ \delta}\kappa_\delta \Big)\varphi_{\hat{p}} + f_{\alpha\hat{p}}.$$

Equation (12) is satisfied in case of arbitrary $\varphi_{\hat{i}}$, μ_ϑ, and κ_γ when the following equations are satisfied:

$$\begin{aligned}\Big(S_\alpha^{\ \vartheta\hat{p}} S_\beta^{\ \gamma\hat{i}} - S_\beta^{\ \vartheta\hat{p}} S_\alpha^{\ \gamma\hat{i}} \Big)\mu_{\gamma\hat{p}} + \Big(S_\alpha^{\ \vartheta\hat{p}} E_\beta^{\ \gamma\hat{i}} - S_\beta^{\ \vartheta\hat{p}} E_\alpha^{\ \gamma\hat{i}} \Big)\kappa_{\gamma\hat{p}} = 0, \\ \Big(E_\alpha^{\ \vartheta\hat{p}} S_\beta^{\ \gamma\hat{i}} - E_\beta^{\ \vartheta\hat{p}} S_\alpha^{\ \gamma\hat{i}} \Big)\mu_{\gamma\hat{p}} + \Big(E_\alpha^{\ \vartheta\hat{p}} E_\beta^{\ \gamma\hat{i}} - E_\beta^{\ \vartheta\hat{p}} E_\alpha^{\ \gamma\hat{i}} \Big)\kappa_{\gamma\hat{p}} = 0,\end{aligned} \tag{13}$$

$$M_\alpha^{\ \hat{p}} N_{\beta\hat{p}} - M_\beta^{\ \hat{p}} N_{\alpha\hat{p}} = 0. \tag{14}$$

Equations (13) will definitely be satisfied if [3]

$$E_{\alpha}^{\gamma\hat{i}} = H^{\gamma\hat{i}}_{\ \ \delta\hat{j}} S_{\alpha}^{\ \delta\hat{j}}, \tag{15}$$

that is,

$$\frac{\partial f_\alpha}{\partial \epsilon_{\gamma\hat{i}}} - H^{\gamma\hat{i}}_{\ \ \delta\hat{j}} \frac{\partial f_\alpha}{\partial \sigma_{\delta\hat{j}}} = 0,$$

and

$$g_{\hat{j}}^{\ \hat{i}} \mu_{\gamma\hat{p}} + H^{\omega\hat{i}}_{\ \ \gamma\hat{j}} \kappa_{\omega\hat{p}} = 0.$$

One of the solutions of Eq. (15) is

$$f_\alpha \left(\Omega_{\delta\hat{j}}, \sigma_\vartheta + h_\vartheta^{\ \gamma} \epsilon_\gamma, x^{\hat{p}} \right) = 0, \tag{16}$$

where

$$\Omega_{\delta\hat{j}} \equiv \sigma_{\delta\hat{j}} + H^{\gamma\hat{i}}_{\ \ \delta\hat{j}} \epsilon_{\gamma\hat{i}}.$$

Constitutive equation $f_\alpha = 0$ contains two tensor variables $\Omega_{\delta\hat{j}}, \sigma_\vartheta + h_\vartheta^{\ \gamma}\epsilon_\gamma$ and four coordinates x^1, x^2, x^3, t. It does not contain four coordinates if the solid is homogeneous and independent of time.

Equations (14) are definitely satisfied if [3]

$$N_{\alpha\hat{p}} = 0. \tag{17}$$

The wave propagation condition can be derived from Eqs. (5), (6), (9) (with indices $\alpha, \beta, \ldots$), and (15). This equation is

$$\left[2\rho \left(S_{\alpha}^{\ \gamma 4} c^3 - S_{\alpha}^{\ \gamma p} n_p c^2 \right) + S_{\alpha}^{\ \delta\hat{j}} H^{\eta 4}_{\ \ \delta\hat{j}} n_\eta^{\ \gamma} c - S_{\alpha}^{\ \delta\hat{j}} H^{\eta p}_{\ \ \delta\hat{j}} n_\eta^{\ \gamma} n_p \right] \mu_\gamma = 0. \tag{18}$$

Since $\mu_\gamma \neq 0$, we have

$$\det \left[2\rho \left(S_{\alpha}^{\ \gamma 4} c^3 - S_{\alpha}^{\ \gamma p} n_p c^2 \right) + S_{\alpha}^{\ \delta\hat{j}} H^{\eta 4}_{\ \ \delta\hat{j}} n_\eta^{\ \gamma} c - S_{\alpha}^{\ \delta\hat{j}} H^{\eta p}_{\ \ \delta\hat{j}} n_\eta^{\ \gamma} n_p \right] = 0. \tag{19}$$

This is called the wave speed equation, being an 18th order algebraic equation for c. In Eq. (19) $n_{\eta(ij)}^{\ \gamma(pq)} \equiv \left(g_i^{\ q} n^p n_j + g_j^{\ q} n^p n_i \right)$ [11] where $g_i^{\ q}$ is the Kronecker delta.

4. A SIMPLER CONSTITUTIVE EQUATION

The first author's experimental and theoretical investigations (uniaxial dynamic tensile stress) have shown that the constitutive equation of copper can be taken in the following form [1,2]

$$f\left(\sigma_t, \epsilon_t, \epsilon_x, \sigma, \epsilon\right) = 0, \tag{20}$$

where

$$\sigma_t = \frac{\partial \sigma}{\partial t}, \quad \epsilon_t = \frac{\partial \epsilon}{\partial t}, \quad \text{and} \quad \epsilon_x = \frac{\partial \epsilon}{\partial x}.$$

The wave propagation condition is

$$\left(\rho \frac{\partial f}{\partial \sigma_t} c^3 + \frac{\partial f}{\partial \epsilon_t} c - \frac{\partial f}{\partial \epsilon_x} \right) \mu = 0. \tag{21}$$

Again, as $\mu \neq 0$, we have

$$\rho \frac{\partial f}{\partial \sigma_t} c^3 + \frac{\partial f}{\partial \epsilon_t} c - \frac{\partial f}{\partial \epsilon_x} = 0.$$

However, there is no general method to obtain constitutive relations for the three-axial case starting from the uniaxial case. By comparing the coefficients of c^1, c^2, c^3 in Eqs. (21) and (18), a formal generalization can be suggested. If Eq. (21) does not contain second power of the speed c, then assume that Eq. (18) does not contain it either. Thus f_α is a function of the same types of variables as f in (20).

Now the wave propagation condition is

$$\left[S_\alpha^{\ \vartheta 4} \left(2\rho g_\vartheta^{\ \omega} c^3 + H^{\eta 4}_{\ \ \vartheta 4} n_\eta^{\ \omega} c - H^{\eta p}_{\ \ \vartheta 4} n_\eta^{\ \omega} n_p \right) \right] \mu_\omega = 0, \tag{22}$$

and the final equation reads

$$\det \left(2\rho g_\vartheta^{\ \omega} c^3 + H^{\eta 4}_{\ \ \vartheta 4} n_\eta^{\ \omega} c - H^{\eta p}_{\ \ \vartheta 4} n_\eta^{\ \omega} n_p \right) = 0, \tag{23}$$

since

$$\det \left(S_\alpha^{\ \vartheta 4} \right) \neq 0.$$

Using ν_q, which can be expressed by Eqs. (5) and (6), instead of μ_ω, Eq. (23) can be written in a more convenient form

$$\det \left(\rho g^q_{\ i} c^3 - H^{kq}_{\ \ ij} n_k n^j c + H^{kqp}_{\ \ \ ij} n_k n^j n_p \right) = 0, \tag{24}$$

where the index 4 is omitted. This equation is a 9th order algebraic equation for the speed of propagation c. Equation (24) has got at least two positive and two negative real roots.

5. CONTINUUM AS A DYNAMIC SYSTEM

Let us return to uniaxial cases and study the instability problem of a ribbon of length L with homogeneous boundary conditions as a stability problem of a dynamic system formed by Eqs. (1), (2), (3), and the boundary conditions [5]. Assume that constitutive equation (20) has the form

$$\dot{\sigma} = d_1 \dot{\epsilon} + d_2 \frac{\partial \epsilon}{\partial x} + d_3 \epsilon + d_4 \sigma, \tag{25}$$

i.e.,

$$\frac{\partial f}{\partial \sigma_t} \neq 0, \quad d_1 = \frac{\partial f}{\partial \epsilon_t} \Big/ \frac{\partial f}{\partial \sigma_t},$$

$$d_2 = \frac{\partial f}{\partial \epsilon_x} \Big/ \frac{\partial f}{\partial \sigma_t}, \quad d_3 = \frac{\partial f}{\partial \epsilon} \Big/ \frac{\partial f}{\partial \sigma_t}, \quad d_4 = \frac{\partial f}{\partial \sigma} \Big/ \frac{\partial f}{\partial \sigma_t}.$$

First, taking

$$\frac{\partial f}{\partial \sigma} = 0,$$

we have $d_4 = 0$.

By following [4,5], Eqs. (1), (2), and (25) can be transformed into the governing equation

$$\rho \dddot{v} = d_1 \frac{\partial^2}{\partial x^2} \dot{v} - d_3 \frac{\partial^2}{\partial x^2} v - d_2 \frac{\partial^3}{\partial x^3} v = 0. \tag{26}$$

Assume that v^0 is a stationary solution of (26) satisfying the boundary conditions. By introducing new variables for the small perturbations

$$y_1 = v - v^0, \quad y_2 = \dot{v} - \dot{v}^0, \quad y_3 = \ddot{v} - \ddot{v}^0,$$

Eq. (26) can be transformed into system

$$\begin{aligned} \dot{y}_1 &= y_2, \\ \dot{y}_2 &= y_3, \\ \dot{y}_3 &= \frac{1}{\rho} \left(\left(d_3 \frac{\partial^2}{\partial x^2} - d_2 \frac{\partial^3}{\partial x^3} \right) y_1 + d_1 \frac{\partial^2}{\partial x^2} y_2 \right). \end{aligned} \tag{27}$$

The characteristic equation of the differential operator defined by the right hand side of (27) is

$$\begin{aligned} \lambda y_1 &= y_2, \\ \lambda y_2 &= y_3, \\ \lambda y_3 &= \frac{1}{\rho} \left(\left(d_3 \frac{\partial^2}{\partial x^2} - d_2 \frac{\partial^3}{\partial x^3} \right) y_1 + d_1 \frac{\partial^2}{\partial x^2} y_2 \right), \end{aligned}$$

or in a more compact form

$$\lambda^3 \rho y_1 - \lambda d_1 \frac{\partial^2}{\partial x^2} y_1 - d_3 \frac{\partial^2}{\partial x^2} y_1 - d_2 \frac{\partial^3}{\partial x^3} y_1 = 0. \tag{28}$$

In the theory of dynamic systems the sign of the real part of the eigenvalues of the characteristic equation is an indicator of stability. If they are all negative, then the state of the material is stable. If they change sign, then there is a loss of stability. It may be a static bifurcation (a real eigenvalue becomes positive), or a dynamic one (at the loss of stability the eigenvalue is purely imaginary).

For homogeneous boundary conditions

$$y_1 = e^{i\psi_k x}, \quad \psi_k = \frac{k\pi}{L}, \quad k = 1, 2, \ldots$$

a complex equation

$$\lambda^3 \rho + \lambda d_1 \psi_k^2 + d_3 \psi_k^2 + i d_2 \psi_k^3 = 0 \tag{29}$$

is obtained.

The necessary condition for a static bifurcation is

$$d_2 = d_3 = 0.$$

Then

$$\lambda_{1,k} = 0,$$

and from (29) we obtain

$$\lambda_{2,3,k} = \pm \psi_k \sqrt{d_1}.$$

While $d_1 > 0$ (see [12]), these are all real values and half of them are positive, that is, this case is not a stability boundary.

For dynamic bifurcation the necessary condition is

$$d_3 = 0.$$

Assume now that $\lambda = \xi + i\omega$. Then for $\xi = 0$, the imaginary parts in (29) are governed by

$$-\omega^3 \rho + \omega d_1 \psi_k^2 + d_2 \psi_k^3 = 0. \tag{30}$$

Equation (30) should always have at least one real solution $\omega = \omega(\psi_k)$. Thus, for this kind of material only the dynamic bifurcation is possible.

Now let us study the case when in (25) instead of ϵ the stress σ is present:

$$\frac{\partial f}{\partial \epsilon} = 0, \quad \frac{\partial f}{\partial \sigma} \neq 0.$$

Then $d_3 = 0,\ d_4 \neq 0$ and the characteristic equation (29) with homogeneous boundary conditions is

$$\lambda^3 \rho - d_4 \lambda^2 + \lambda d_1 \psi_k^2 + i d_2 \psi_k^2 = 0. \tag{31}$$

The necessary condition for a static bifurcation, from (31), is

$$d_2 = 0,$$

and for the other (nonzero) eigenvalues

$$\lambda^2\rho - d_4\lambda + d_1\psi_k^2 = 0. \tag{32}$$

The solutions of (32) are

$$\lambda_{2,3,k} = \frac{d_4 \pm \sqrt{d_4^2 - 4\psi_k^2 d_1}}{2}. \tag{33}$$

In (33) the sign of d_4 has a great importance. If it is negative, then there is a static bifurcation, but in case of $d_4 > 0$ it does not exist.

For the dynamic bifurcation the condition is

$$d_4 = 0,$$

we have Eq. (30) and the same results for the eigenvalues as before.

As we have already seen, the analysis of uniaxial static bifurcation requires a strain-independent constitutive equation. To extend this result for the three-axial case let us assume that f_α does not depend on ϵ^δ. Then we can use a scalar function U^*

$$U^* = \sigma^\delta\gamma_\delta + W(\sigma_{\eta 4}, \epsilon_{\vartheta\hat{p}}, \epsilon_\delta - \gamma_\delta).$$

Let f_α be of the form

$$f_\alpha = \left.\frac{\partial U^*}{\partial \gamma_\alpha}\right|_{\gamma_\alpha = \epsilon_\alpha} = 0,$$

i.e.,

$$f_\alpha = \sigma_\alpha - \left.\frac{\partial W}{\partial \epsilon^\alpha}\right|_{\gamma^\delta = \epsilon^\delta} = 0, \tag{34}$$

where

$$\frac{\partial W}{\partial \gamma^\alpha} = -\frac{\partial W}{\partial \epsilon^\alpha}.$$

It means that, finally, from (34) follows

$$\sigma_\alpha = \frac{\partial W(\sigma_{\eta 4}, \epsilon_{\vartheta,\hat{p}}, 0)}{\partial \epsilon^\alpha}.$$

Instead of six equations $f_\alpha = 0$, only one scalar function U^* (or W) should be found to obtain constitutive relations.

6. CONCLUSIONS

By assuming the existence and finite speed of the acceleration wave, the paper shows that the wave propagation condition can be used for determining the constitutive equations. The main results can be summarized as follows.

1. Acceleration waves do exist, if the constitutive equation has the form (16) and the wave speed equation (Eq. (24) or (19)) has two positive and two negative roots.

2. When the constitutive equation of a solid does not depend on the stress gradient (like copper [1,2]), the wave speed equation is determined by a 3×3 matrix (Eq. (24)).

3. From the dynamic bifurcation conditions we get that at least one of $\partial f / \partial \sigma$ and $\partial f / \partial \epsilon$ should have non-zero values.

4. The condition for the loss of stability of a static bifurcation is

$$\frac{\partial f}{\partial \epsilon} \neq 0.$$

Thus, in the analysis of a static bifurcation (analysis of shear banding, or neck forming instabilities), the form of a possible constitutive equation is

$$f(\sigma, \sigma_t, \epsilon_t, \epsilon_x) = 0.$$

5. When the constitutive equation does not depend on the strain, a scalar function can be used to describe material properties.

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VASE OLEKUVÕRRANDITE TULETAMINE KIIRENDUSLAINETE JA DÜNAAMILISTE SÜSTEEMIDE TEOORIA ABIL

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On näidatud, et füüsikalisi tingimusi, mis määravad kiirenduslainete eksisteerimise, on võimalik kasutada olekuvõrrandite funktsionaalse kuju määramiseks. Näitena on toodud esimese autori eksperimendil põhinev vase pinge ja deformatsiooni seose määramine ühedimensioonilisel juhul. Meetod lubab üldistamist keerulisematele pingeolukordadele.