Proc. Estonian Acad. Sci. Engng., 1997, 3, 2, 107–114
https://doi.org/10.3176/eng.1997.2.04

ADAPTIVE ESTIMATION SCHEME FOR LINEAR INTERCONNECTED SUBSYSTEMS

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Received 5 September 1996, revised 3 December 1996, accepted 17 March 1997

Abstract. This paper describes a decentralized state estimation scheme for linear discrete-time systems. It can also be used for recursive correction of possible unknown parameters, located in subsystems. The whole procedure is based on the repetitive real-time implementation of the well-known Kalman filter algorithm.

Key words: decentralized estimation, parameter identification.

1. INTRODUCTION

Recent years have seen growing interest towards decentralized control and estimation problems. For example, in most multi-layered planner-based autonomous robot architectures, the emergent behaviour is generated as an interaction between competing constituents [1]. On the same principles, the sensor information filtering and filter adaptation layer could be based.

Several papers discuss decentralized filtering. This paper analyzes a scheme of multi-level adaptive state and parameter estimation, based on the straight decomposition of the basic system of equations, defining the best centralized estimator [2]. The observable process is assumed to be a sampled-data Gaussian random process with parameters the values of which may be initially unknown. The actual parameter value is supposed to be one from the predefined set of values. To reduce the computational burden and to localize uncertainties, the decomposed form of filtering is used. As a result, we have at our disposal a set of Kalman filters, each for one subsystem, where the subsystem's state estimate is recursively generated, and the state estimates from all other subsystems are also

taken into consideration. In the case of subsystem uncertainties, simple competing adaptation is used. Both the decentralized state estimation and subsystem parameter identification processes allow for a real-time implementation.

2. PROBLEM STATEMENT

Consider a discrete-time linear dynamic system consisting of m, $m \geq 2$, subsystems in the form

$$x_i(k+1) = A_i(\alpha_i)x_i(k) + \sum_{j \neq i}^{m} A_{ij}x_j(k) + u_i(k),$$

$$y_i(k) = H_i x_i(k) + v_i(k), \qquad i = 1, 2, \ldots, m; \quad k = 1, 2, \ldots$$

The noises $u_i(k)$ and $v_i(k)$ are assumed to be zero-mean white Gaussian processes with covariance matrices $R_i(k)$ and $Q_i(k)$, respectively. Symbol α_i stands for an unknown parameter of the i-th subsystem, given by the set of possible values and by *a priori* probabilities of appearing. The unknown parameters are located in matrices A_i. They may reside as well in matrices H_i, R_i, and Q_i. State transition interconnection matrices A_{ij} represent the only allowed couplings between subsystems.

The problem is to generate the state estimations $\bar{x}(k)$, using measurement sequences

$$Y_{i,k} = [y_i(1), y_i(2), \ldots, y_i(k)], \qquad i = 1, 2, \ldots, m; \qquad k = 1, 2, \ldots$$

such that for the entire system, the mathematical expectation of the squared estimation error

$$J(k) = E\left[\left(x(k) - \bar{x}(k)\right)' W \left(x(k) - \bar{x}(k)\right) \right], \qquad W > 0,$$

takes its minimum. In addition, the value of the unknown parameter must be identified during the filtering process.

3. DECENTRALIZED STATE ESTIMATION

Let us assume now that all the parameters have already taken one value from the set of the possible ones, and during the estimation process they do not change. As is well known, the best estimator for the defined overall system after n measurements is represented by the two-point boundary value problem [2]:

$$\bar{x}_i(k+1|n) = A_i \bar{x}_i(k|n) + Q_i \lambda_i(k) + \sum_{i \neq j}^{m} A_{ij} \bar{x}_j(k|n),$$

$$\lambda_i(k) = A_i \lambda_i(k+1) + H_i' R_i^{-1} \left(y_i(k+1) - H_i \bar{x}_i(k+1|n) \right) + \sum_{j \neq i}^{m} A_{ji}' \lambda_j(k+1)$$

with boundary conditions

$$\lambda_i(n) = 0,$$

$$\bar{x}_i(0|n) = \bar{x}_i(0|0) + C_i(0) \left[A_i \lambda_i(0) + \sum_{j \neq i}^{m} A_{ji}' \lambda_j(0) \right],$$

where

$$\bar{x}_i(0|0) = \bar{m}_i + C_i(0) H_i' R_i^{-1} \left(y_i(0) - H_i \bar{m}_i \right), \qquad \bar{m}_i \text{ – given},$$

$$C_i(0) = \left(I + P_i(0) H_i' R_i^{-1} H_i \right) P_i(0), \qquad P_i(0) \text{ – given},$$

$$k = 0, 1, 2, \ldots, n-1; \qquad i = 1, 2, \ldots, m.$$

Using the Kalman filter derivation scheme, we can transform these equations into a partly recursive form:

$$\bar{x}_i(k|n) = \tilde{x}_i(k|k) + C_i(k) \left[A_i' \lambda_i(k) + \sum_{j \neq i}^{m} A_{ji}' \lambda_j(k) \right] + S_i^{-1}(k) \cdot \beta_i(k|n),$$

$\bar{x}_i(0|n)$ – given and $\lambda_i(n) = 0$,

where

$$\beta_i(k|n) = A_i S_i^{-1}(k-1) \cdot \beta_i(k-1|n) + \sum_{i \neq j}^{m} P_{ij}(k) \lambda_j(k-1) + \sum_{i \neq j}^{m} A_{ij} \bar{x}_j(k-1|n),$$

$\beta_i(0|n) = 0$,

and

$$S_i(k) = \left(I + P_i(k) H_i' R_i^{-1} H_i \right),$$

$$C_i(k) = S_i^{-1}(k) P_i(k),$$

$$P_i(k) = A_i C_i(k-1) A_i' + Q_i,$$

$$P_{ij}(k) = A_i C_i(k-1) A_{ji}'.$$

The term $\tilde{x}_i(k|k)$ is generated recursively

$$\tilde{x}_i(k|k) = A_i\tilde{x}_i(k-1|k-1) + C_i(k)H_i'(k)R_i^{-1}\left(y_i(k) - H_iA_i\tilde{x}_i(k-1|k-1)\right),$$

$$\tilde{x}_i(0|0) = \bar{x}_i(0|0), \quad k = 1, 2, \ldots, n; \quad i = 1, 2, \ldots, m.$$

If $A_{ij} = A_{ji} = 0$, all $\beta_i(k|n) = 0$, and we have m separate Kalman filters for noncoupled subsystems. If it is not so, an iteration process for the correction of $\beta_i(k|n)$ in each step k must be organized. The computational burden reduces significantly if we replace the term

$$\beta_i(n-1|n) \quad \text{by} \quad \beta_i(n-1|n-1).$$

In this case, we have an essentially recursive algorithm

$$\bar{x}_i(n|n) = \tilde{x}_i(n|n) + S_i^{-1}(n)\cdot\beta_i(n|n),$$

where

$$\beta_i(n|n) = A_iS_i^{n-1}(n-1)\cdot\beta_i(n-1|n-1) + \sum_{i\neq j}^{m} A_{ij}\bar{x}_j(n-1|n) + \sum_{i\neq j}^{m} P_{ij}(n)\lambda_j(n-1),$$

$$\beta_i(0|0) = 0,$$

and

$$\bar{x}_j(n-1|n) = \tilde{x}_j(n-1|n-1) + C_j(n-1)\left[A_j'\lambda_j(n-1) + \sum_{j\neq i}^{m} A_{ji}'\lambda_i(n-1)\right] +$$

$$+ S_j^{-1}(n-1)\cdot\beta_j(n-1|n-1),$$

$$\lambda_j(n-1) = H_j'R_j^{-1}\left(y_j(n) - H_j\bar{x}_j(n|n)\right), \quad n = 1, 2, \ldots.$$

Here n represents a current measurement step, denoted in recursive parts as k.

Now we must coordinate subsystem estimations only in the current step. It may be accomplished by a simple iteration, the initial value for $\bar{x}_j(n|n)$ taken as $\tilde{x}_j(n|n)$. As a result, the estimate of the state of the whole system is generated in a decentralized way by a set of subsystem Kalman filters, each for a subsystem. The estimates from all other subsystems are taken into consideration, but not in their full extent. In a given version, the new added measurement improves the sequence of estimates from other filters only in the two last steps. For example, for the n-th step, subsystem i uses estimates of all other subsystems as sequences

$$\left\{\bar{x}_j(0|0), \bar{x}_j(1|1), \ldots, \bar{x}_j(n-2|n-2), \bar{x}_j(n-1|n), \bar{x}_j(n|n)\right\}.$$

This fact makes the decentralized filter theoretically nonoptimal, but if n is large enough, the loss is not so significant. Moreover, from the point of view of asymptotic behaviour, the filter gains some good properties of sliding window filtering.

4. ADAPTIVE PARAMETER ESTIMATION

Any subsystem may include some parameters the values of which are considered as initially unknown. We assume that only matrices A_{ij} will not contain any uncertainties. If the unknown parameter or parameters are localized in some, say in the i-th subsystem, then we will try to identify its value by a set of filters of that subsystem, each tuned for one given vector of the values. The resulting state estimate of that subsystem is given as a weighted sum

$$\bar{x}_i(k) = \sum_{r=1}^{N} \bar{x}_i\left(\alpha_{i,r}, k|Y_k\right) \cdot p\left(\alpha_{i,r}|Y_k\right),$$

where N is the number of filters of subsystem i; $\bar{x}_i\left(\alpha_{i,r}, k|Y_k\right)$ is the state estimate of subsystem i at the given parameter value after k steps of measurements; $p\left(\alpha_{i,r}|Y_k\right)$ is the conditional probability of appearing that value, initially given. These probabilities are recursively updated as the new measurement is added to the sequence of measurements. It is already assumed that if one of the probabilities tends to one and all others to zero, then the value that corresponds to the maximal probability may be taken as a real value of that parameter.

It is well known from basic literature that

$$p\left(\alpha_{i,r}|Y_k\right) = \frac{p\left(\alpha_{i,r} \cdot Y_k\right)}{p\left(Y_k\right)} = \frac{p\left(\alpha_{i,r} \cdot y_k|Y_{k-1}\right)}{p\left(y(k)|Y_{k-1}\right)}.$$

The same is valid for all N filters of the i-th subsystem

$$p\left(\alpha_{i,r}|Y_k\right) = \frac{p\left(y_i(k)|Y_{k-1};\alpha_{i,r}\right) \cdot p\left(\alpha_{i,r}|Y_{k-1}\right)}{\sum_{r=1}^{N} p\left(y_i(k)|Y_{k-1};\alpha_{i,r}\right) \cdot p\left(\alpha_{i,r}|Y_{k-1}\right)},$$

$$p\left(\alpha_{i,r}|Y_0\right) = p\left(\alpha_{i,r}\right); \quad r = 1, 2, \ldots, N; \quad k = 1, 2, \ldots$$

Here $p\left(y_i(k)\middle|Y_{k-1};\alpha_{i,r}\right)$ is the conditional probability of $y_i(k)$ at fixed $\alpha_{i,r}$ and Y_{k-1}, evaluated at the measured level. The $y_i(k)$ is normal, with he conditional mean

$$\bar{y}_i\left(k\middle|Y_{k-1};\alpha_{i,r}\right)=H_i\bar{x}_i\left(k\middle|Y_{k-1};\alpha_{i,r}\right),$$

and with the covariance matrix

$$P_{y,i}\left(k\middle|Y_{k-1};\alpha_{i,r}\right)=H_iP_i\left(k\middle|Y_{k-1};\alpha_{i,r}\right)H_i'.$$

These terms are available from the set of subsystem filters.

Now we can write

$$p\left(\alpha_{i,r}\middle|Y_k\right)=\frac{\left|P_{y,i}\left(k\middle|Y_{k-1};\alpha_{i,r}\right)\right|^{1/2}\cdot\exp\left[-W_i\left(k\middle|\alpha_{i,r}\right)\right]\cdot p\left(\alpha_{i,r}\middle|Y_{k-1}\right)}{\sum_{r=1}^{N}\left|P_{y,i}\left(k\middle|Y_{k-1};\alpha_{i,r}\right)\right|^{1/2}\exp\left[-W_i\left(k\middle|\alpha_{i,r}\right)\right]\cdot p\left(\alpha_{i,r}\middle|Y_{k-1}\right)},$$

$$p\left(\alpha_{i,r}\middle|Y_0\right)=p\left(\alpha_{i,r}\right),\quad r=1,\ 2,...,N;\quad i=1,2,...,m;\quad k=1,2,...$$

Here k denotes the current measurement step, and

$$W_i\left(k\middle|\alpha_{i,r}\right)=\hat{y}_i\left(k\middle|Y_{k-1};\alpha_{i,r}\right)'P_{y,i}^{-1}\hat{y}_i\left(k\middle|Y_{k-1};\alpha_{i,r}\right),$$

$$\hat{y}_i\left(k\middle|Y_{k-1};\alpha_{i,r}\right)=y_i(k)-\bar{y}_i\left(k\middle|Y_{k-1};\alpha_{i,r}\right).$$

5. ILLUSTRATIVE EXAMPLE

Consider a system consisting of two subsystems with the state vector of dimension three. The same unknown parameter is located in all diagonal elements of the state transition matrix of the first subsystem $a_{ii}\cdot\alpha_{1,r}$ and is defined to have five possible values. For example, for

$$a_{ii}=0.88,\ a_{ij}=-0.1,\ R=Q=I,\ p\left(\alpha_{1,r}\right)=0.2,\ \alpha_{1,r}=0.7+0.1r,\ r=1,2,..,5$$

the value of increasing *a posteriori* probability $p\left(\alpha_{1,3}\middle|n\right)$ has taken values $p(1.0|5)=0.51$, $p(1.0|10)=0.74$, $p(1.0|15)=0.92$, $p(1.0|20)=0.98$. Thus, after fifteen measurements, the third parameter value could clearly be considered as the real value.

The ratio of traces of covariance matrices of the estimation error for the proposed decentralized scheme and the centralized optimal one increases with

the increase of couplings of the subsystems. For the given example, this ratio was 1.17. The decentralized state estimation step was accomplished with the best-tuned subsystem.

6. CONCLUDING REMARKS

The proposed decentralized scheme is essentially recursive, every new measurement vector is treated iteratively. Each iteration includes a step of subsystem parameter identification and a step of state estimation-coordination. The parameter estimation step could be implemented in parallel with the filters of one subsystem and the state estimation step by the local filters of all subsystems.

In a centralized adaptive estimation version, a set of filters of full state dimension would be required. The amount of filters grows exponentially with the number of value combinations the unknown parameters may take.

Decomposition results in a set of filters of much smaller dimensions, and in the case of identification much smaller amount of them, since the unknown parameters are localized in subsystems. But it is at the expense of some loss in estimation optimality and of the need for some coordinating iterations.

It must be pointed out that during the iterations, no feasibility constraints are violated, and the system could be implemented in real time. The adaptive identification procedure may be started and stopped in any subsystem in any time step when unknown parameter value is localized.

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SEOSTATUD ALAMSÜSTEEMIDE OLEKU JA PARAMEETRITE ADAPTIIVNE HINDAMINE

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On esitatud algoritm seostatud alamsüsteemide oleku osaliselt rekursiivseks hindamiseks. Alamsüsteemide parameetrite väärtused võivad olla ebatäpsed. Nende täpsustamine toimub laekuvate mõõteandmete rekursiivsel töötlemisel

samaaegselt oleku hindamisega. Esitatud adaptiivse hindamise algoritm võimaldab oluliselt vähendada vajalike filtrite arvu (ja nende dimensiooni) võrreldes klassikalise, tsentraliseeritud variandiga. Kõik protseduurid baseeruvad Kalmani filtri võrrandite korduvkasutamisel ning on realiseeritavad reaalaja kitsendustega süsteemides.