

LBE-MODELLING APPLIED TO REINFORCED CONCRETE T-SECTION–STEEL U-SECTION COMPOSITE BEAM

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Abstract. A composite beam consisting of a reinforced concrete T-section and a steel channel section concreted together is a novel type of a composite girder, in which the benefits of permanent shuttering techniques are successfully used. The steel section of the beam is initially a U-shaped channel, upon which profiled steel sheetings for composite slabs are connected to form a shuttering for the flooring. After pouring the concrete and its solidification, the final structure forms a very resistive composite system, where slabs and beams are principally of the same type. In Finland Rautaruukki has patented the flooring system described.

In order to have composite behaviour between the concrete T-section and the channel around its web-section, the inner surface of the channel is anchored to the concrete, and the paper discusses the behaviour of the beam as influenced by different types of load-slip characteristics in the steel-concrete interface.

To study the problem, layered beam elements (LBEs), are employed for considering the various aspects of the behaviour. The elements have been derived from the ordinary Timoshenko beam elements, which have two nodes, by separating the three degrees of freedom in a node to three nodes having only one degree of freedom each.

Examples of calculation results consider the effects of varying interface stiffness and strength, together with various structural details on the behaviour, and a comparison of the calculation results to some experimental results obtained at Tampere University of Technology for full-scale test specimens (simply supported and continuous beams) is also performed.

Key words: composite construction, shear connection, finite element method.

1. INTRODUCTION

Finite elements having non-linear properties are frequently required for problems of composite behaviour of layered flexural structures, and it may be natural to select plane elements for the structures in which nodal interconnection of springs is possible, when standard two-dimensional

2. LBE: TRANSFORMATIONS REQUIRED

In order to employ the beam elements in layered structures, the three degrees of freedom in one node have to be divided into three nodes, each having then only one degree of freedom, axial or transverse displacement (Fig. 1). The top and bottom nodes, denoted as 'ti' and 'bi', will take care of the longitudinal displacements, and the middle one, situated on the centroidal axis, will move transversely.

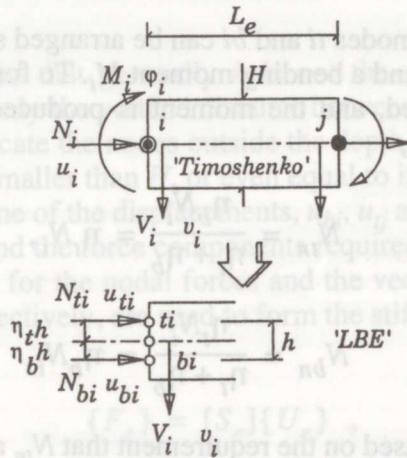


Fig. 1. Principle of transforming the basic degrees of freedom.

2.1. Derivation of stiffness coefficients for the LBE

The nodal force and displacement vectors, $\{F_e\}$ and $\{U_e\}$, respectively, are composed of the components shown in Fig. 1:

$$\begin{aligned} \{F_e\} &= \{N_{ti} \ N_{bi} \ V_i \ N_{tj} \ N_{bj} \ V_j\}, \\ \{U_e\} &= \{u_{ti} \ u_{bi} \ v_i \ u_{tj} \ u_{bj} \ v_j\}. \end{aligned} \quad (2)$$

There may be several ways to come to the stiffness matrix required, but the simplest and maybe the 'fool-proof' method is employed here, i.e. the introduction of a positive displacement on one node at a time while the others are kept zero, whereupon the force components of the ordinary element needed to produce this are considered.

The nodal displacement $u_{ti} > 0$ may be taken as an example, and the forces required to impose the state of Fig. 2 are therefore introduced. The displacement state is composed using axial displacement $u_i = \eta_b u_{ti}$ and rotation $\varphi_i = u_{ti}/h$ in the basic element. These are produced by the forces:

$$N_i = \frac{(EA)}{L_e} u_i = -N_j, \quad (3a)$$

$$V_i = \frac{6(EI)}{(1+\theta)L_e^2} \Phi_i, \quad (3b)$$

$$M_i = \frac{(4+\theta)(EI)}{(1+\theta)L_e} \Phi_i, \quad (3c)$$

$$M_j = \frac{(2-\theta)(EI)}{(1+\theta)L_e} \Phi_i. \quad (3d)$$

The axial forces on nodes ti and bi can be arranged so that these result in a total axial force N_i and a bending moment M_i . To form N_i , components N_{tn} and N_{bn} are required, and the moment is produced by the force couple $N_{tm} = -N_{bm} = M_i/h$,

$$N_{tn} = \frac{\eta_b N_i}{\eta_t + \eta_b} = \eta_t N_i, \quad (4a)$$

$$N_{bn} = \frac{\eta_t N_i}{\eta_t + \eta_b} = \eta_b N_i. \quad (4b)$$

Equations (4) are based on the requirement that N_{tn} and N_{bn} have to satisfy the condition of an axial deformation state. The resultant forces on the nodes of the i -end are then:

$$N_{ti} = N_{tn} + N_{tm} = \left(\eta_t \eta_b \frac{(EA)}{L_e} + \frac{(4+\theta)(EI)}{(1+\theta)L_e h^2} \right) u_{ti}, \quad (5a)$$

$$N_{bi} = N_{bn} + N_{bm} = \left(\eta_b^2 \frac{(EA)}{L_e} - \frac{(4+\theta)(EI)}{(1+\theta)L_e h^2} \right) u_{ti}. \quad (5b)$$

Accordingly, due to u_{ii} , for the j -end:

$$N_{tj} = \left(-\eta_t \eta_b \frac{(EA)}{L_e} + \frac{(2-\theta)(EI)}{(1+\theta)L_e h^2} \right) u_{ii}, \quad (6a)$$

$$N_{bj} = \left(-\eta_b^2 \frac{(EA)}{L_e} + \frac{(2-\theta)(EI)}{(1+\theta)L_e h^2} \right) u_{ii}. \quad (6b)$$

Transverse forces depend only on rotation, and due to u_{ii} they are written as:

$$V_i = -V_j = \frac{6(EI)}{(1+\theta)L_e^2 h} u_{ii}. \quad (7)$$

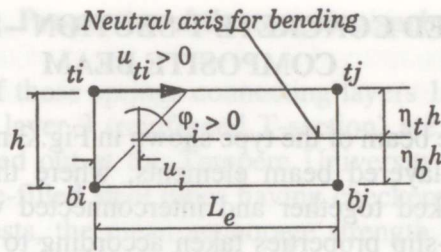


Fig. 2. Nodal displacement u_{ti} to form the rotation ϕ_i at the end face of the element (end face for nodes t_i and b_i).

It should be noted that generally h is not the depth of the element (Figs. 1 and 2), but only the vertical distance between t - and b -nodes and it is also possible to locate the nodes outside the depth of the element. Thus, h can be greater or smaller than H , or even equal to it.

Similarly to u_{ti} , one of the displacements, u_{bi} , u_{tj} and u_{bj} can be set to be non-zero at a time and the force components required are then considered. The resulting vector for the nodal forces and the vector of displacements, $\{F_e\}$ and $\{U_e\}$, respectively, are used to form the stiffness equation for the element:

$$\{F_e\} = [S_e]\{U_e\}, \quad (8)$$

$$[S_e] = \begin{bmatrix} [S_1] & [S_2] \\ [S_2]^T & [S_3] \end{bmatrix},$$

where the submatrices $[S_1]$, $[S_2]$ and $[S_3]$ are given in detail in [1, 2] and are not repeated here.

3. COUPLING OF ELEMENTS

Coupling springs are introduced to interconnect elements as layers, and nodes t_i and b_i can be connected to nodes of elements in other layers with horizontal springs which model the behaviour of composite connection between each structural layer. Moreover, vertically displacing nodes of each layer of elements have to be connected together with springs adequately stiff to make each layer deflect similarly. The stiffness of the horizontal springs is dependent on the interlayer slip capacity and strength of the connection.

Various systems may be generated, e.g. parallel and stacked. Parallel systems are formed when t -nodes of various layers are coupled, and stacked systems are those where t -nodes of a layer are connected to b -nodes of another layer. Thus, a variety of structures can be covered by making appropriate arrangements in the system matrix of the structure.

4. REINFORCED CONCRETE T-SECTION – STEEL CHANNEL COMPOSITE BEAM

The composite beam of the type shown in Fig. 3 may be discretized as a system of three-layered beam elements, where the layers are used as parallel and stacked together and interconnected with coupling springs which have load-slip properties taken according to push-out tests for the steel-concrete interface of the checkered channel inner surface.

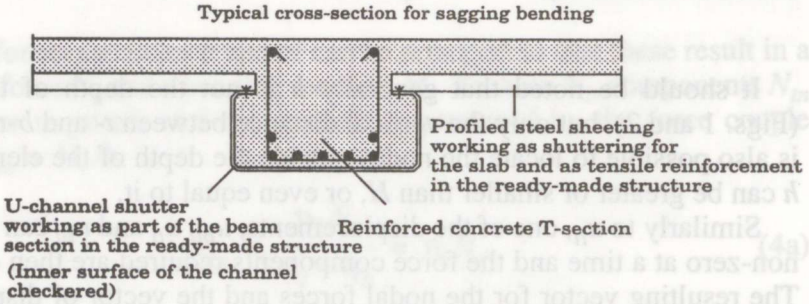


Fig. 3. Cross-section of the beam-slab flooring having integrated shuttering.

The element layers shown schematically in Fig. 4 are:

- (1) upper half of the U-channel section (= half of the web depth + lip),
- (2) lower half of the U-channel section (= half of the web + bottom flange),
- (3) reinforced T-section.

The location of horizontally deforming nodes in layer (3) is in top and bottom fibres of the web, into which top and bottom nodes of channel section elements are interconnected by interface 1 and 2 springs which represent the properties of the checkered inner surface interface. The halves of the channel section are connected together with stiff, non-flexible springs.

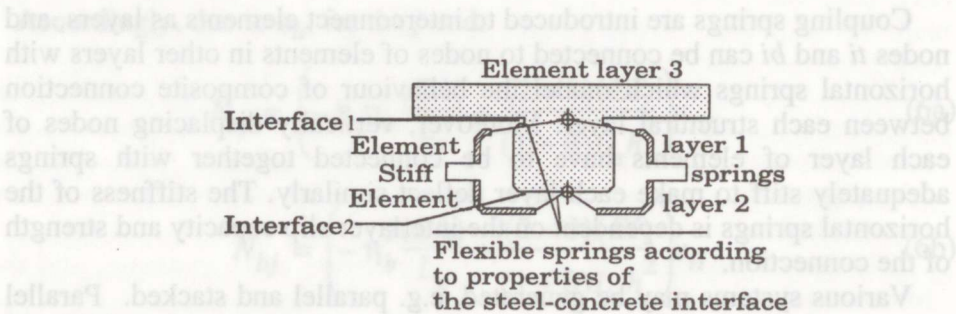


Fig. 4. Discretization of the beam section into layered beam elements and the system of interconnecting the horizontally deforming nodes of elements for composite behaviour. Nodes t and b for element layer 3 are shown with crossed circle markers.

4.1. Properties of the coupling springs

The properties of those springs connecting layers 1 and 2 (parts of the channel section) to layer 3 (reinforced T-section) are taken according to push-out tests carried out at the Tampere University of Technology for rectangular concrete-filled steel tubes having checkered inner surface [3]. According to the tests, the mean anchorage strength will depend on the geometry and size of the tube and on the strength of the concrete. Thus, a representative anchorage resistance per unit length of the connection may be calculated from the tests and the anchorage resistance will be distributed to the nodal connections of element layer 3 in a way that the upper interface will get approximately 1/3 of the total resistance.

It is most convenient to use 'unit curves' for the load-slip properties of the springs and scale the curve coordinates in the calculation with proper resistances and ultimate slip values. Figure 5 represents a typical unit curve derived from the tests [3].

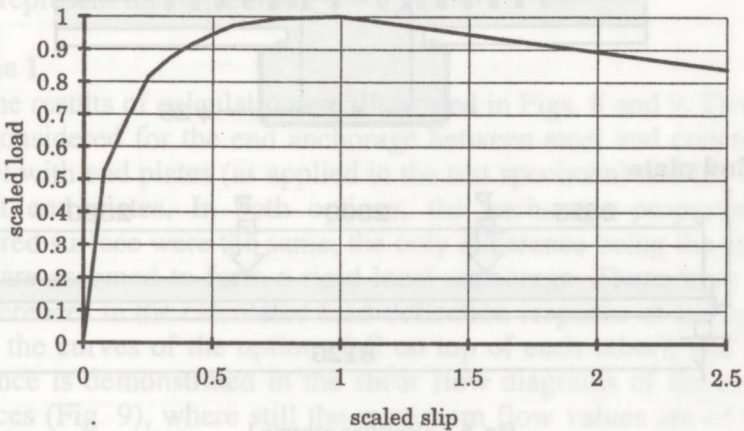


Fig. 5. Unit curve for the load-slip properties of the connection springs (courtesy of Rautaruukki Oy, Finland).

5. CALCULATION RESULTS

Two schemes, based on structural full-scale tests carried out at Tampere University of Technology, were employed for the calculation to demonstrate the capabilities of the LBE method to predict the structural behaviour correctly:

Scheme 1 includes a two-span continuous beam, which imitates the test [4], where span lengths were 8.125 m and the applied load consisted of two concentrated loads on both spans. In the calculation, this scheme was modeled as a single-span beam with one end simply supported and the other end clamped.

4. Scheme 2 includes a single span and a simply supported beam, which imitates the test [5], where the length of the span was 5.620 m and the applied load consisted of two concentrated loads in 1/3 and 2/3-points of the span.

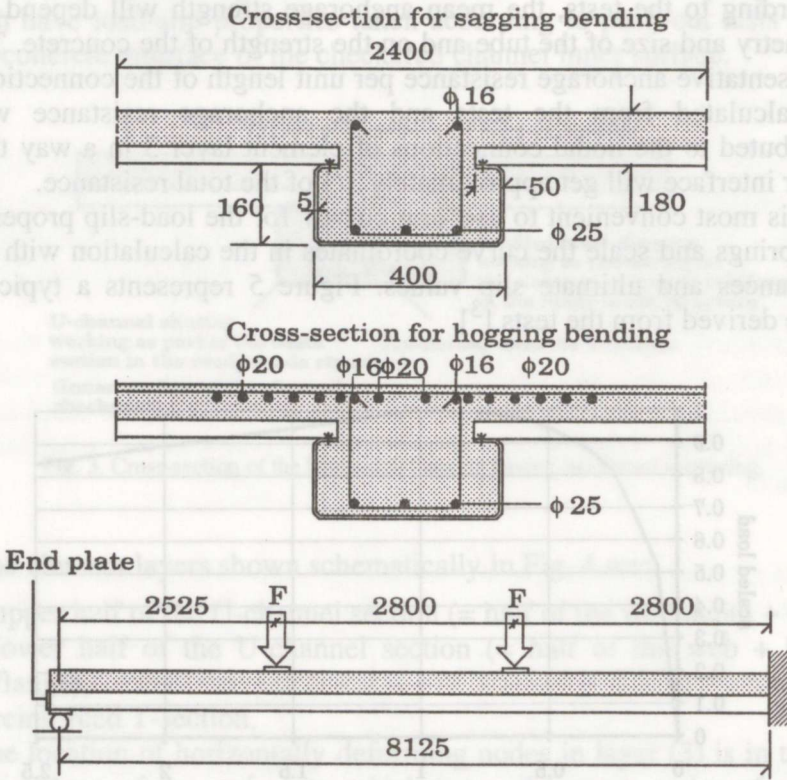


Fig. 6. Calculation scheme 1.

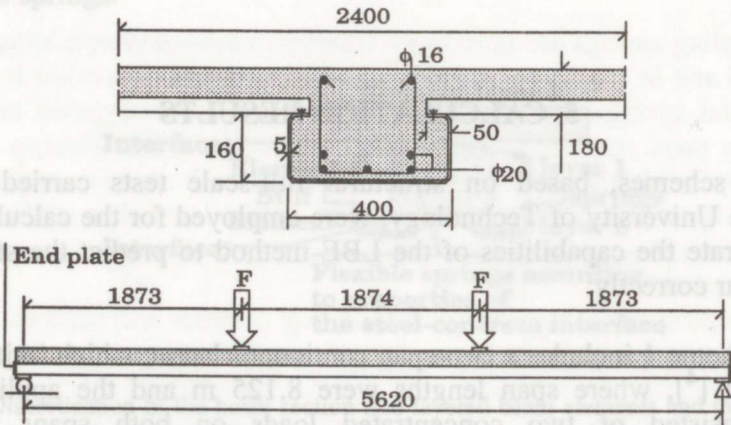


Fig. 7. Calculation scheme 2.

Similar channel and concrete sections were employed in both schemes, the only difference in the cross-section being the sets of reinforcement. The relevant sections are shown in Figs. 6 and 7. There were end plates welded into the channels of the tested structures, and the effect of these was considered in the calculation.

The grade of the concrete was C35/40 in scheme 1 and C40/45 in scheme 2 (grading according to Eurocodes 2 and 4). According to the coupon tests, the yield strength of the channel section is $f_y = 265$ MPa (nominal steel grade S235) and tensile strength is evaluated to be $f_u = 400$ to 420 MPa. The reinforcement grading was S500, $f_{sk} = 530$ MPa, according to the material tests (the ultimate strength is evaluated to be 650 to 700 MPa). The real stress-strain curves were simulated in the calculation. The form of the load-slip curve for the longitudinal shear interface springs was as shown in Fig. 5. Various resistance values for the springs were employed so as to see their effect on the behaviour. Best fit for the tested behaviour was obtained when only one third of the experimental strength of the connection was applied. The results shown below represent this selection.

Scheme 1

Some results of calculation are illustrated in Figs. 8 and 9. Two options were considered for the end anchorage between steel and concrete: (a) a channel with end plates (as applied in the test specimen) and (b) a channel without end plates. In both options, the anchorage properties of the checkered surface were the same, the only difference being the end plates, which are assumed to form a rigid local anchorage. There were virtually no differences in the calculated load-deflection response of the options (in Fig. 8, the curves of the options fall on top of each other), and the main difference is demonstrated in the shear flow diagrams of the connection interfaces (Fig. 9), where still the maximum flow values are of the same magnitude in both cases.

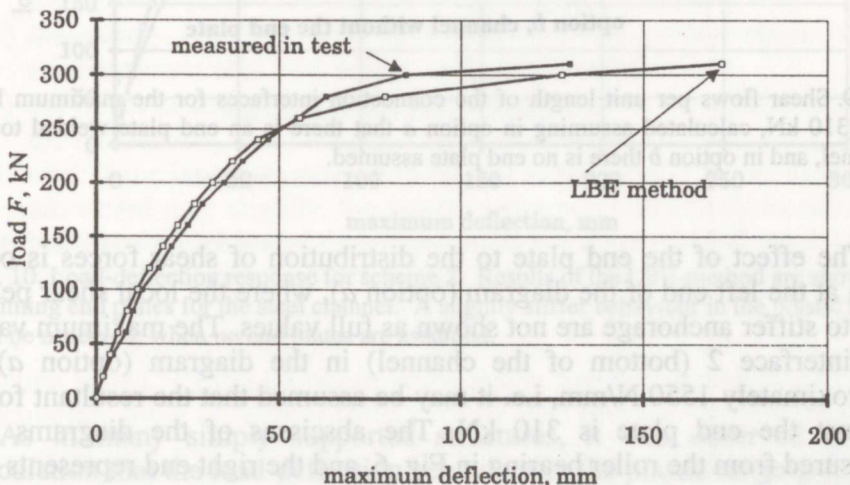
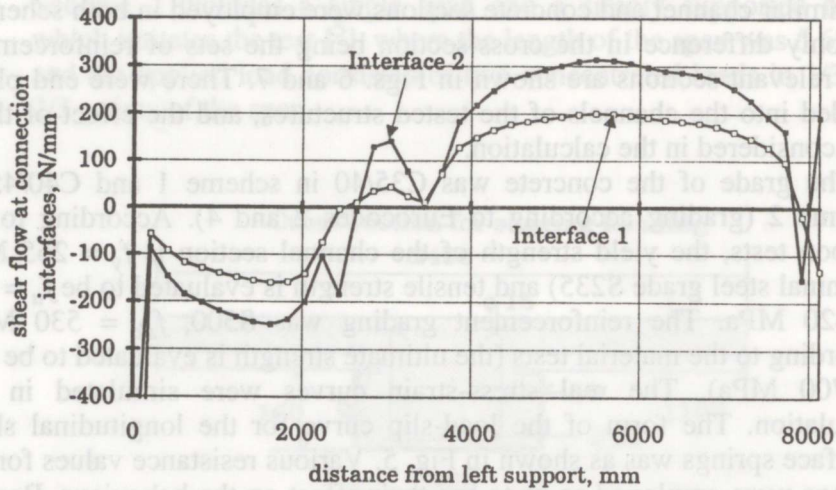
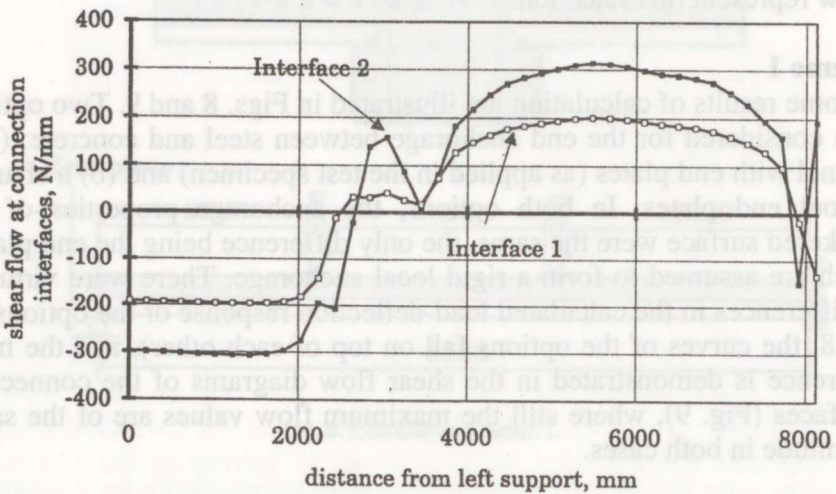


Fig. 8. Load-deflection response of scheme 1.



option *a*, channel with the end plate



option *b*, channel without the end plate

Fig. 9. Shear flows per unit length of the connection interfaces for the maximum load $F_u = 310$ kN, calculated assuming in option *a* that there is an end plate welded to the channel, and in option *b* there is no end plate assumed.

The effect of the end plate to the distribution of shear forces is only seen at the left end of the diagram (option *a*), where the local shear peaks due to stiffer anchorage are not shown as full values. The maximum value for interface 2 (bottom of the channel) in the diagram (option *a*) is approximately 1550 N/mm, i.e. it may be assumed that the resultant force against the end plate is 310 kN. The abscissas of the diagrams are measured from the roller bearing in Fig. 6, and the right end represents the clamped support.

The calculated deflection follows quite closely the measured values, but the ultimate deflection for the maximum load $F_{u.test} = 310$ kN is greater than the measured one. The difference may partly be explained by the material parameters, which cannot be adjusted exactly to be the same as for cold-worked steel material, i.e. at channel corners strain hardening due to cold forming should be considered for better coexistence. The explanation to the slightly stiffer 'elastic' behaviour in the calculation may be found in the shear-lag of the slab section, which is not considered in the calculation. It could be done by reducing slightly the design width, i.e. the flange width in the specimen, 2400 mm, is not exactly the effective width, although in test high compressive strains at the ultimate limit state were also observed near the edges of the flange.

Scheme 2

The results of the calculation are shown in Figs. 10 and 11. Similar options to scheme 1 were considered for the anchorage of the channel. In Fig. 10, the load-deflection curve for LBE-calculation was only drawn according to option *a*, together with the corresponding curve measured in the test. Figure 11 shows the differences in shear flows for the ultimate load of the test $F_{u.test} = 325$ kN. In option *a*, very stiff end plates are assumed, which means that very high pushing forces will be directed to the end plates. There is also a clear difference to option *b* in which no end plates for the anchorage are assumed, and hence, higher shear flows and end slips are induced.

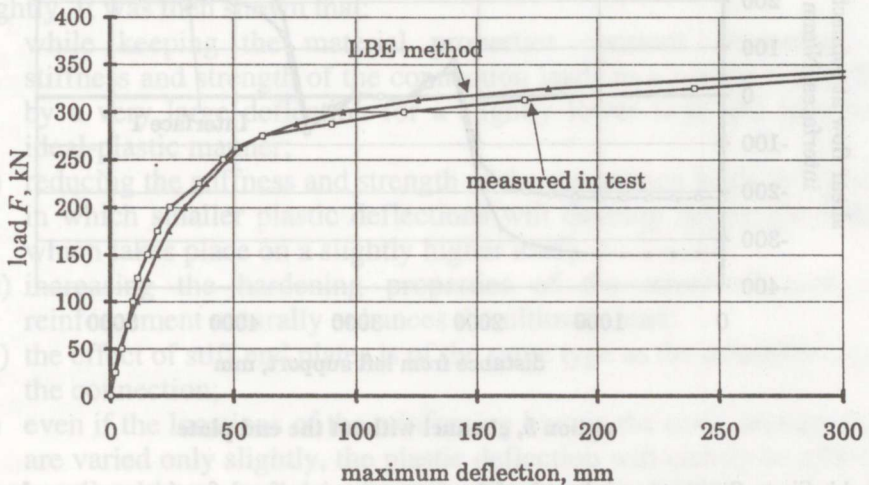
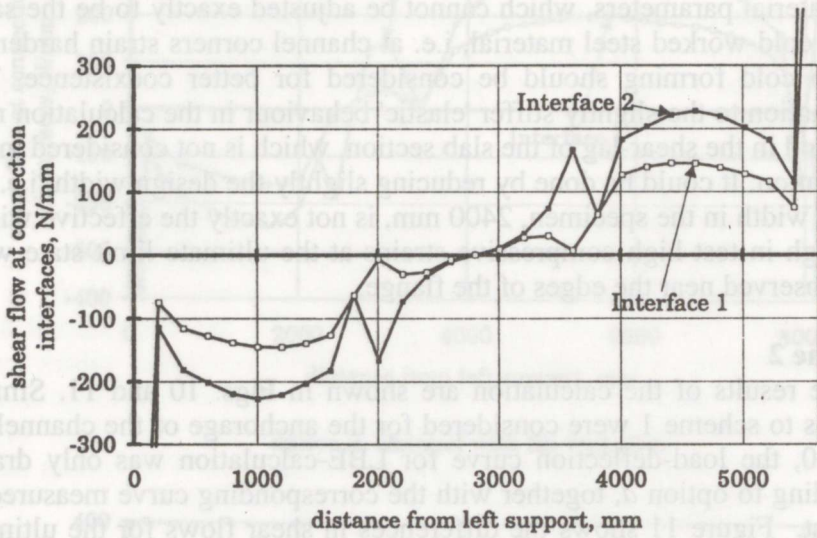


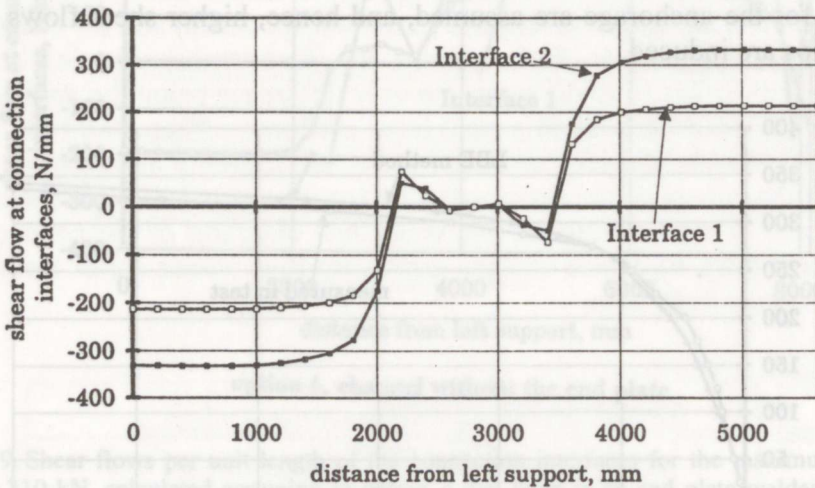
Fig. 10. Load-deflection response for scheme 2. Results of the LBE-method are shown as assuming end plates for the steel channel. A slightly stiffer behaviour in the plastic range will be observed, when no end plates are assumed.

As in many simply supported structures, it was observed in the calculation that the load-deflection behaviour in the plastic range is highly sensitive to the stress-strain properties of steel materials in hardening

range, which however, is not given in the test reports and could only be estimated by trial calculation.



option a, channel with the end plate



option b, channel without the end plate

Fig. 11. Shear flows per unit length of the connection interfaces for the maximum load in the calculation, $F_u = 350$ kN. The effect of end plates (option a) is seen here more clearly than in scheme 1.

6. DISCUSSION

This article has demonstrated the applicability of the LBE-method for examining the behaviour of a composite structure, which is quite similar to

composite slabs, and the procedure in discretizing the structure is also similar. Three layers of elements were used, and this was done mainly to facilitate appropriate connections of the steel section to the main structure consisting of reinforced concrete. It would be possible to have only two layers of elements, if only one connection for the steel-concrete interface was selected. The total number of the elements required in the problem is not quite high and can also be reduced by selecting the length of elements, L_e , appropriately. The properties of the continuous shear connection are 'lumped' in nodes, which is a normal procedure in all FE-calculations. The effect of varying the element length has not been considered in the problems above, where lengths $L_e \approx 200$ mm were employed. This means that the depth to length ratio, H/L_e is greater than one in main elements, but in other problems also it has been proved that the behaviour of the Timoshenko-type elements is quite stable and independent of the length on quite a large range.

In the examples above, the stress-strain curve for the concrete was adopted from [6] and the functions programmed are well compatible also for high strength concrete. The properties of steel materials, both structural and reinforcement, are more of a problem in a way that similar 'form functions' are employed for both, but in reality they may differ.

Although not expressed quantitatively by examples, the effect of material and structural selection on the plastic behaviour was studied in scheme 2 by varying the hardening properties of structural steel and reinforcement, and further by varying the flexibility and strength of the shear connection. Moreover, the location of reinforcing bars was varied slightly. It was then shown that:

- (i) while keeping the material properties constant, increasing the stiffness and strength of the connection leads to a system which fails by a very large deflection for a slightly lower load and in a more ideal-plastic manner;
- (ii) reducing the stiffness and strength of the connection leads to a system in which smaller plastic deflections will develop before the failure, which takes place on a slightly higher load;
- (iii) increasing the hardening properties of the structural steel and reinforcement naturally enhances the ultimate load;
- (iv) the effect of stiff end plates is of the same type as the strengthening of the connection;
- (v) even if the locations of the reinforcing bars in the cross-section depth are varied only slightly, the plastic deflection will clearly be affected, but the ultimate load, however, is affected only slightly.

By reference to comment (i) above, it is stated that, in fact, by applying the strength for the shear connection as proposed by the relevant push-out tests [3], a system behaving best according to the observations in the flexural tests [4, 5] should be obtained. While this was not justified by the calculation, there must be reasons why the real strength of the connection is smaller than that according to push-out tests. In the push-out tests, a concrete block cast inside a steel tube is forced to move with respect to steel, and this does not affect the elasticity of the tube. In flexural

members, however, the channel finally becomes plastic over a considerable length of the connection, and this affects the strength in those parts becoming plastic. This explains partly why the calculation with the reduced strength of the connection matches better to the real load-deflection response than does the calculation with the 'push-out' values.

Other applications

The LBE-method was first developed for ordinary steel-concrete composite beams [7], but has also been used intensively for structures, where prestressed hollow-core slab units are supported on beams and a compositely behaving system is automatically formed [8,9]. The importance of having proper analysis tools for these structures is understood by the fact that the vertical shear resistance of the slabs is greatly reduced when supported on beams [9]. The method then employs four various layers and one connection is used to model the behaviour of the webs in hollow-core slabs.

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T-RISTLÕIKEGA RAUBETOON- JA U-RISTLÕIKEGA TERASELEMENTIDEST KOOSTATUD KOMPOSIITTALADE KIHILISTE ELEMENTIDE MODELLEERIMINE

Matti V. LESKELÄ

T-ristlõikega raudbetoon- ja U-ristlõikega teraselementidest koosnevad komposiittalad on uut tüüpi komposiitkandjad, milles on edukalt kasutatud alalise raketise tehnika eeliseid. Tala terasosa on esialgselt U-kujuline kanal, millega ühendatakse profileeritud teraslehed ava komposiitkatte moodustamiseks. Pärast betooni kivistumist tekib komposiitsüsteem, milles plaadid ja talad on põhimõtteliselt sama tüüpi.

Betoonist T-kujulise ristlõikega osa ja selle seina ümber oleva U-kujulise teraskanali koostöö tagamiseks on kanali sisepind ankurdatud betooni. On vaadeldud tala käitumist terase ja betooni kokkupuutepinnal mõjuva koormuse ja nihke vaheliste seoste eri tüüpi karakteristikute korral. Selliste talade käitumise uurimiseks on kasutatud tala kihiliste elementide modelleerimise meetodit. Elemendid saadakse tavalise Timošenko tala kahesõlmelistest elementidest, eraldades kolme vabadusastmega sõlme kolmeks ühe vabadusastmega sõlmeks.

Näidetes on vaadeldud liitepinna jäikuse ja tugevuse muutumise ja erinevate struktuurielementide mõju tala käitumisele ning võrreldud arvutustulemusi tulemustega, mis on saadud Tampere Tehnoloogiaülikoolis mõningate täismõõduliste liht- ja jätkuvtalade katsetamisel.