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APPLICATION OF WAVELETS IN PAVEMENT PROFILE EVALUATION AND ASSESSMENT

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Abstract. This paper describes application of an advanced mathematical tool for analysing the measurements obtained on the profilograph, an instrument for measuring pavement smoothness profiles. This mathematical tool, wavelet, has the potential for detecting abnormal behavior in profile data, the presence of noise, and can produce a better statistical interpretation of the profilograph. Wavelet transformation is a linear operation that decomposes signal (profile) data into components that appear at different scales (or resolutions). The multiscale property and the structure of wavelet can lead to a method of analysis and display which highlights changes in profile data. The wavelet algorithm is demonstrated using actual profile data, and a numerical distortion measure is made on the wavelet.

Key words: pavement evaluation, pavement profile, profilograph, wavelet.

1. INTRODUCTION

Pavement smoothness or roughness can be described by the magnitude of profile irregularities and their distribution over measurement intervals. The surface smoothness, especially on a newly constructed pavement, is a major concern for the highway industry. This affects the road users directly. The smoothness, or riding comfort, is a measure of the quality of the newly constructed pavement [1].

Profilographs are basic instruments for characterization, evaluation, specification, and quality control of pavement smoothness/roughness during pavement construction. The profilograph measures the vertical deviations from a moving fixed length and reference plane. The procedure generates graphical charts (strips) known as profilograms. A process known as trace reduction [2] is used to derive a profile index. The profile index is a quantitative indication of the smoothness of the pavement. The profilograph belongs to class 2 in pavement roughness measuring equipment according to [3].

The objectives for smoothness measurement include:

– Maintenance of construction quality control.

– Location of abnormal changes in the pavement profile.

– Establishment of a basis for allocation of resources for road maintenance and rehabilitation.

– Determination of pavement roughness that can be used in pavement performance and deterioration modelling.

Study [4] lists problems regarding smoothness measures and the interpolation of test results. These include:

– Effect of the surface type.

– Trace reduction.

– Interpretation of traces (profile).

– Identification of grinding locations (maintenance spots).

A recent study [5] includes data from more than 200 pavement projects in 10 states. For most of the pavement types, a 25% increase in initial smoothness produced at least a 9% increase in life, and 50% increase in smoothness yielded a minimum 15% increase in pavement life.

The aim of this paper is to describe a refined wavelet technique for pavement profile evaluation and assessment. The method will be used only to analyse the profilograph, no attempt will be made to define the profile index. The wavelet algorithm will be used to assess and evaluate the following with reference to the profilograph:

– The denoising of the original profilograph.

– Identification of abnormal behaviour of the profilograph.

– Detection of the ageing trend (how the smoothness is changing with reference to the age of pavement).

– Multiscale feature detection.

2. BASIC IDEAS OF DISCRETE WAVELET ANALYSIS

Wavelets are a new family of localized basic functions that can be used to express and approximate other functions [6]. They are functions with a combination of powerful features such as orthonormality, locality in time frequency domain, different degrees of smoothness, fast implementation, and data compression.

A discrete wavelet transform (DWT) is a linear operation that decomposes a signal $F(x)$ into the weighted sum of basic functions $\psi_k(x)$

$$F(x) = \sum \sum c_{j,k} \psi_{j,k}(x), \qquad j,k \in Z \qquad (1)$$

where j is the dilation scale index, and k is the translational index. The empirical wavelet coefficient $c_{j,k}$ is found by projecting the data onto a wavelet basis $\psi_k(x)$. If j is large, the large features (i.e. low frequency characteristics) of the data are analysed; conversely, if j is small, small features (i.e. high frequency characteristics) of the data are analysed. Wavelets are functions that satisfy certain mathematical requirements and are used in representing data or other functions. In wavelet analysis, the scale used to look at the data plays a very important role. Wavelet analysis and algorithms process data at different resolutions or scales. When the observer views a signal or a function through a large "window," gross features are noticed. But when a signal is observed through a small "window," small features are noticed.

Wavelet transforms comprise an infinite set. The different wavelet families make different trade-offs between how compactly the basic functions are localized in space and smoothness. Within each family, wavelets have subclasses distinguished by the number of coefficients and by the level of iteration [6].

There are many kinds of wavelets. One can choose between smooth wavelets, compactly supported wavelets, or wavelets with simple mathematical expressions. A number of different wavelets are used to approximate any given function with each wavelet generated from one original wavelet, called mother wavelet or analysing wavelet. The new elements called daughter wavelets are simply scaled and translated mother wavelets. Scaling implies that the mother wavelet is either dilated or compressed and translation implies shifting of the mother wavelet [7]. Wavelet transforms, unlike Fourier transforms which utilize only the sine and cosine functions, have an infinite set of possible basic functions. Thus, wavelet analysis provides immediate access to information that can be obscured by other time-frequency methods such as Fourier analysis [6].

A wavelet, in the sense of DWT, is an orthogonal function which can be applied to a finite data. The DWT can be written as a matrix W operating on an input sequence

$$X_P = (x_1, x_2, \ldots, x_N)^T \equiv (x_{P,1}, x_{P,2}, \ldots, x_{P,N})^T,$$

where $N = 2^P$. Subscript P has been added to the original sequence to emphasize that it is of length 2^P, the original sequence is said to be of scale 1 [8]. The transformed sequence is denoted by

$$Z = WX_P, \tag{2}$$

$$X_P = W^{-1} Z = W. \tag{3}$$

The basic function $\Phi(x)$ is a set of functions defined by a recursive difference equation

$$\Phi(x) = \sum_{k=0}^{M-1} C_k \Phi(2x - k), \tag{4}$$

where M is a non-zero coefficient. The number of non-zero coefficients is arbitrary and referred to as the order of the wavelet [9,10]. The value of the coefficients is determined by constraints of orthogonality and normalization. The following condition must be satisfied:

$$\sum C_k = 2. \tag{5}$$

The orthogonality of Eq. (1) to its translation

$$\int \Phi(x)\Phi(x-k)\,\mathrm{d}x = 0$$

and equations which are orthogonal to its dilations, or scale, leads to

$$\int \psi(x)(2x-k)\,\mathrm{d}x = 0.$$

The function ψ is given by

$$\psi(x) = \sum_k (-1)^k C_{1-k}\Phi(2x-k), \tag{6}$$

which depends upon the value of $\Phi(x)$. Normalization requires that

$$\sum_k C_k C_{k-2m} = 2\delta_{0m}, \tag{7}$$

where δ is the delta function. Another important equation that can be derived is

$$\sum (-1)^k C_{1-k} C_{k-2m} = 0. \tag{8}$$

The coefficients $\{C_1, \ldots, C_n\}$ can be assumed to act as filters. A wavelet becomes smoother as its number of coefficients increases. The current x sequence is "filtered" through a low-pass filter $\{g_l\}$ to produce the set of scaling coefficients:

$$x_{m,k} = \sum_{l=0}^{L-1} g_l x_{m+1,1+2k-1}, \qquad k = 1,\ldots,2^m, \tag{9}$$

where L is the length of the filter. The output is a low-pass or smoothed version of the input. At each stage the current x-vector component is also filtered with another filter $\{h_l\}$. This is a high-pass filter to produce the next set of wavelet coefficients

$$d_{m,k} = \sum_{l=0}^{L-1} h_l x_{m+1,1+2k-1}, \qquad k = 1,\ldots,2^m, \tag{10}$$

Here $\{g_l\}$and $\{h_l\}$ are referred to as wavelet filters. These filters satisfy two equations

$$\Phi(t) = \sqrt{2}\sum_{l} g_l \Phi(2t-1), \tag{11}$$

$$\psi(t) = \sqrt{2}\sum_{l} h_l \Phi(2t-1), \tag{12}$$

where $\Phi(.)$ are the scaling functions and $\psi(.)$ are the wavelet functions.

The low-pass filter, sometimes called "the odd output," contains most of the information content of the original input. The high-pass filter, sometimes called "the even output," contains the difference between the true input and the value of the reconstructed input.

3. APPLICATION

Data analysis by wavelets involves four steps (Fig. 1).

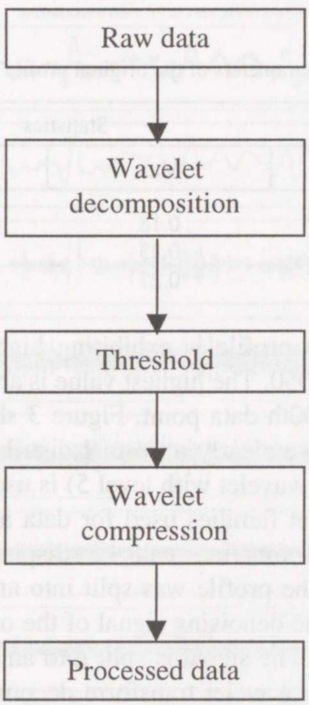

Fig. 1. Data analysis by wavelets.

The profile data used in this study was obtained from the Remote Sensing and Image Processing Laboratory of Louisiana State University. The numerical format of profile trace reduction was obtained through a digitizing process using APPARE (Automated Pavement Profile Analysis Evaluation) software [11]. In APPARE, digitizing is achieved through a three step process:

1) scanning of the profile,
2) editing of the scanned image,
3) extracting of the digitized profile trace from the scanned image.

The data created by APPARE can be used for roughness evaluation and analysis. The profile trace produced by a profilograph strip chart contains lots of noise due to the pavement texture, dirt and rocks on the road, mechanical vibrations, and defects of the equipment.

The APPARE uses the "midpoint extraction" procedure performed on scanned images to obtain a single valued function described in an *x–y* plane. The midpoint extraction method uses a moving slope threshold based on the assumption that the slope of the road from one data to the next cannot exceed 45°.

The data contains 12 153 discrete values. The elevation of the profile is sampled at every inch of the pavement. The unit for the elevation data is in inches at about zero mean. Table 1 shows the basic statistics of the original profile data. Only the figures for profile 1 are presented in the paper. All the profiles have the same standard deviation, but different means.

Table 1. Statistical parameters of the original profile data (data in inches)

Profile number	Statistics			
	Mean	Max	Min	Std Dev
1	$1.10e^{-0.005}$	0.18	0.17	0.05
2	$7.87e^{-0.006}$	0.33	–0.21	0.05
3	$-3.44e^{-0.006}$	0.27	–0.17	0.05

Figure 2 shows that the profile is exhibiting high peaks around data points 1 000, 5 000, 8 000, and 10 050. The highest value is around 1 100th data point and the lowest around the 11 000th data point. Figure 3 shows the distribution of the profile data points; they have clearly a normal distribution. The Matlab Wavelet Toolbox [12] (Daubechies-5 wavelet with level 5) is used. The Daubechies wavelet belongs to the set of wavelet families used for data analysis. The level indicates how detailed the wavelet function can be displayed. Figure 4 shows the decomposition at level 5. The profile was split into an approximation a_5 and five details d_1 to d_5. Here a_5 is the denoising signal of the original signal, d_1 to d_5 show the detail filter (low noise). The signal is split into an approximation and a detail. Figure 5 shows the discrete wavelet transform decomposition tree, showing also the high filtered a_i and low filter pass d_i. The discrete wavelet tree shows the possible ways of decomposing the original profile **s**. Figure 6 shows the profile

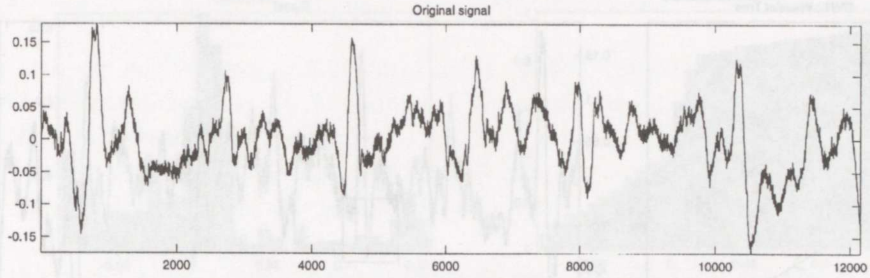

Fig. 2. Original profile data (x-axis: length of the pavement; y-axis: elevation in inches about zero mean).

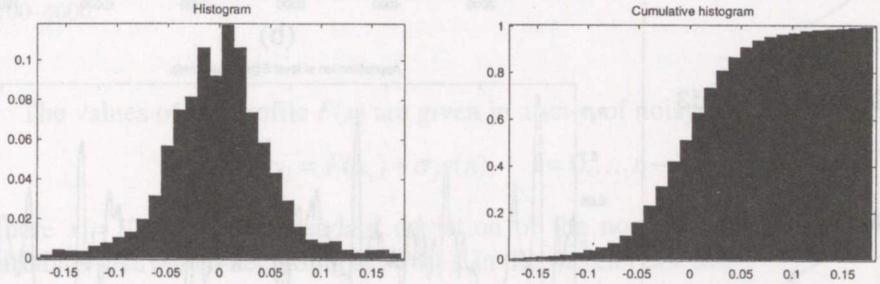

Fig. 3. Histogram and cumulative histogram of the distribution of the profile data.

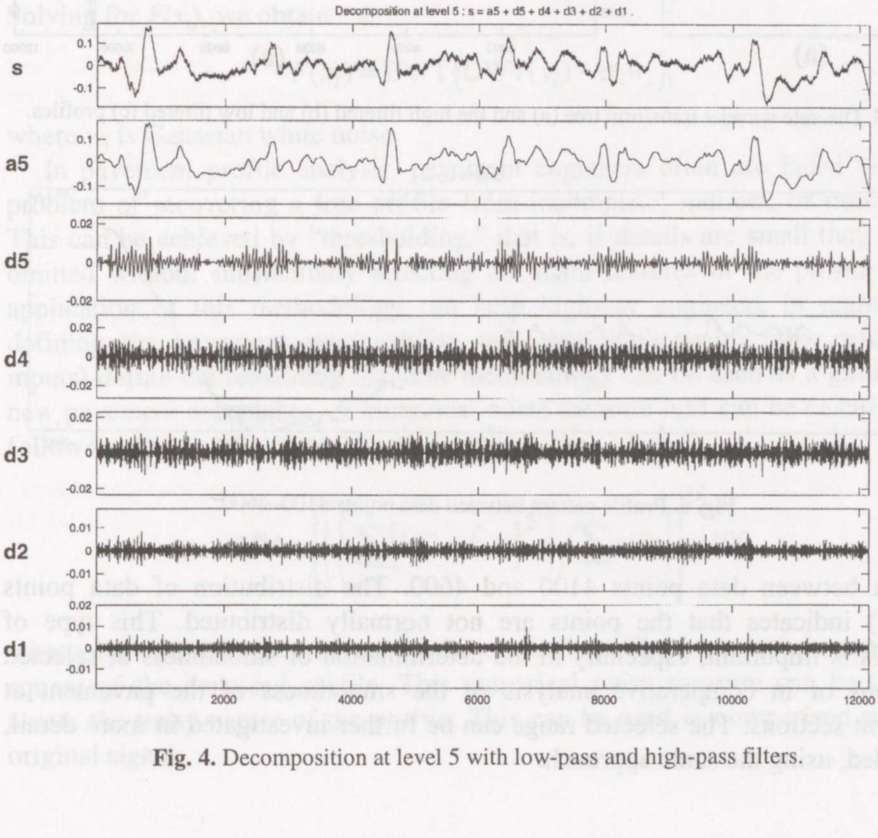

Fig. 4. Decomposition at level 5 with low-pass and high-pass filters.

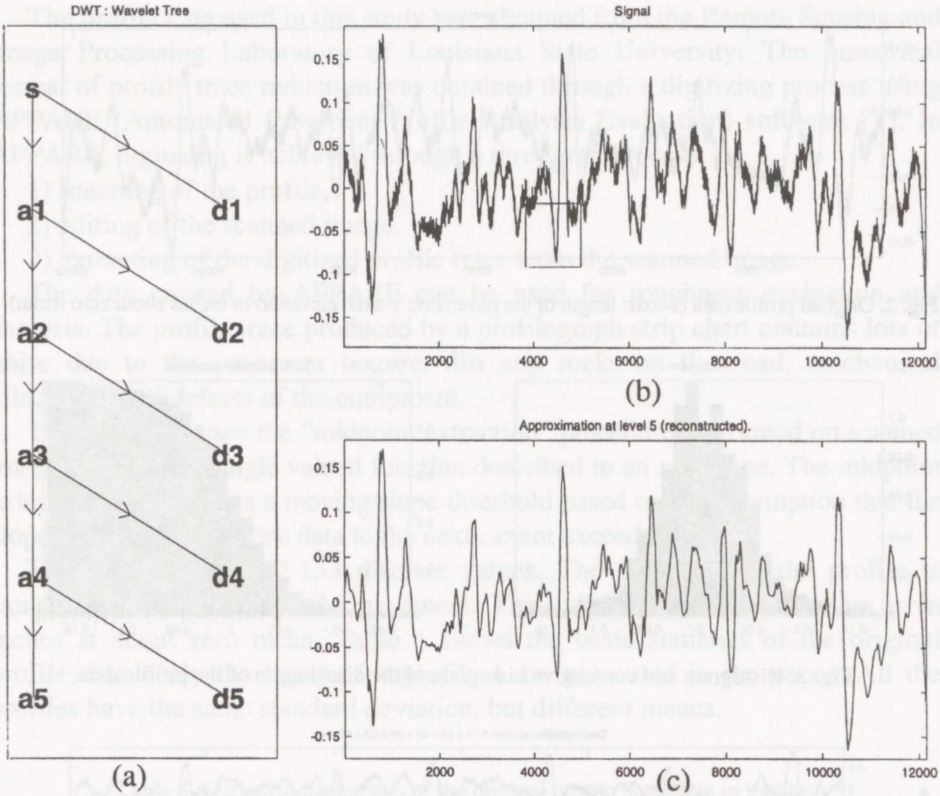

Fig. 5. Discrete wavelet transform tree (a) and the high filtered (b) and low filtered (c) profiles.

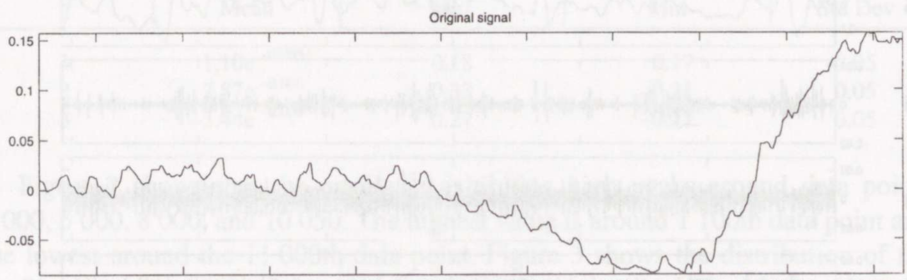

Fig. 6. Profile pattern between data points 4100–4600.

pattern between data points 4100 and 4600. The distribution of data points (Fig. 7) indicates that the points are not normally distributed. This type of analysis is important, especially in the determination of smoothness at selected locations or in comparative analysis of the smoothness of the pavement at different sections. The selected range can be further investigated in more detail, if needed, using the same approach.

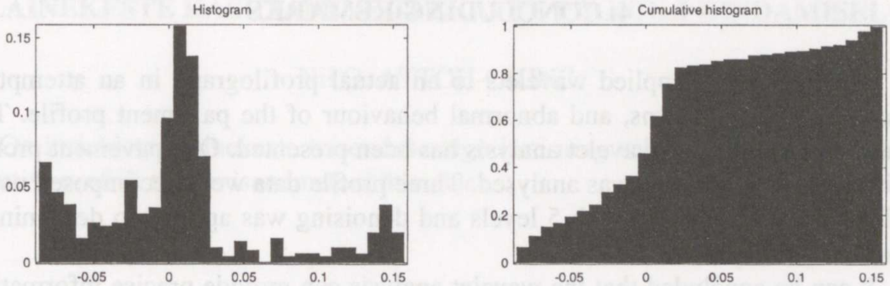

Fig. 7. Histogram and cumulative histogram of the distribution of the profile data for data points 4100–4600.

The values of the profile $F(x)$ are given in a set y_i of noisy observations

$$y_i = F(x_i) + \sigma_i e(n), \quad i = 0,\ldots,n-1, \tag{13}$$

where $x_i = i/n$, σ_i is the standard deviation of the noise, and $e(n)$ are random variables distributed according to $N(0,1)$. In the wavelet domain

$$\mathrm{DWT}(y_i) = \mathrm{DWT}\{F(x_i) + \sigma_i e(n)\} = \mathrm{DWT}\{F(x_i)\} + \sigma_i W(\sigma_i(n)). \tag{14}$$

Solving for $F(x_i)$, we obtain

$$F(x_i) = \mathrm{DWT}\{\mathrm{DWT}(y_i) - \sigma_i w_i\},$$

where w_i is Gaussian white noise.

In pavement profile analysis, pavement engineers often are faced with the problem of recovering a true profile from incomplete, indirect, or noisy data. This can be achieved by "thresholding," that is, if details are small they can be omitted without substantially affecting the main features of the profile. Right application of this methodology can help highway engineers in realistically defining the pavement serviceability and objectively (with other additional inputs) define the remaining life. The methodology can be used as a guide for a new pavement acceptance. A numerical noise measure NM can be calculated as follows:

$$\mathrm{NM} = \left[\left(\left(\sum_{i=1}^{n}\left[k(i) - \hat{k}(i)\right]^2\right) \Big/ \sum k(i)\right)\right]^{1/2} 100, \tag{15}$$

where $k(i)$ is root mean square of the original profile, and $\hat{k}(i)$ is root mean square of the denoised profile. This numerical noise measure can be used to assess the noisy nature of the profile. This can be used in comparison with the original signal.

4. CONCLUDING REMARKS

This paper has applied wavelets to an actual profilograph in an attempt to identify trends, patterns, and abnormal behaviour of the pavement profile. The basic idea of discrete wavelet analysis has been presented. One pavement profile with 12 153 data points was analysed. Three profile data were decomposed using a Daubechies-5 wavelet with 5 levels and denoising was applied to determine a more accurate profile.

It can be concluded that the wavelet analysis can provide precise information of the ranges that define the profile, and it can eliminate noise which is generally encountered in data collection and processing of original profile data. Furthermore, the numerical "noisy" measure can provide an insight of the original profile before it is used to define other parameters, such as the roughness index.

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LAINEKESTE KASUTAMINE TEEKATTE PROFIILI HINDAMISEL

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On kirjeldatud lainekeste teisenduse algoritmi ning selle kasutamist tegeliku sillutise profiili mõõtmisandmete töötlemisel.