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HEAT TRANSFER IN THE THERMALLY DEVELOPING REGION FOR PULSATING TUBE FLOW

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Abstract. The problem of thermally developing pulsating flow in a tube with constant heat flux at wall was studied with a simple mathematical model. Intervals of the frequency at which the Nusselt number either increases or decreases, are determined. Some experimental results are given. The phenomenon of cutting off the peaks of temperature pulsating curves is explained.

Key words: heat transfer in a tube, pulsating flow, IR-thermography.

1. INTRODUCTION

The heat transfer problem of pulsating flow in tubes and ducts has been considered in several publications [1–7]. The interest in this problem is due to the possible applications, mainly in industry, by increasing heat exchange efficiency, and in biomechanics.

The heat transfer problem for developing pulsating laminar flow in duct was studied in [1]. The slug-flow assumption was used. The dependence of heat transfer on pulsation frequency at uniform wall temperature and constant heat flux at the wall was demonstrated. The same problem with finite difference method was considered in [2]. The importance of the entry region for heat transfer and for its development was indicated.

Results of numerical studies of heat transfer characteristics of a pulsating flow in the pipe at uniform temperature at the wall are given in [3]. Complete laminar boundary-layer equations were solved. It was found that in the downstream fully established region the Nusselt number either increases or decreases in comparison with the steady-flow value, depending on the frequency. The same problem in a channel was numerically studied in [4]. The change in the Nusselt number due to pulsation in the entrance region was pronounced.

The problem of pulsating laminar flow in a tube with constant heat flux at the wall, when temperature becomes a linear function of downstream direction, was considered in [5]. A range of moderate values of the frequency is indicated when the Nusselt number increases. Earlier the same problem for turbulent flow was dealt with in [6]. The problem with sinusoidal wall temperature distribution has also been studied [7].

In the present paper we investigate the problem of thermally developing pulsating flow with constant heat flux at the tube wall by means of a simple theoretical model. Some experimental results are given to prove the usefulness of the latter.

2. THEORETICAL ANALYSIS

The problem of pulsating flow with constant heat flux at the tube wall is governed by the energy equation

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}=\frac{k}{\rho c_p}\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}\right) \tag{2.1}$$

Here T is the temperature, u is the fluid velocity in axial direction, c_p, k, ρ are specific heat, conductivity and density of the fluid, respectively, x, r are axial and radial coordinates within the tube, and t is time.

We consider the pulsating flow when sufficient time has passed after the start of the pulsation, when the quasi-equilibrium has been obtained. Then the boundary conditions can be written as

$$T(0,r,t)=T_0, \tag{2.2}$$

$$\frac{\partial T}{\partial r}(x,0,t)=0, \tag{2.3}$$

$$k\frac{\partial T}{\partial r}(x,r_0,t)=q_w=\text{const.} \tag{2.4}$$

Here T_0 is the uniform temperature of the fluid when entering the region where the heat flux at the tube wall q_w is given, and r_0 is the radius of the tube.

Let us introduce following notations:

$$\theta=\frac{T-T_0}{q_w r_0/k},\quad v=\frac{u}{u_0}, \tag{2.5}$$

$$\xi = \frac{x}{r_0 \mathrm{Pr}\,\mathrm{Re}},\ \eta = \frac{r}{r_0},\ \tau = \frac{tk}{\rho c_p r_0^2}, \tag{2.6}$$

$$\mathrm{Pr} = \frac{\mu c_p}{k},\ \mathrm{Re} = \frac{\rho u_0 r_0}{\mu}, \tag{2.7}$$

where u_0 is constant reference velocity, μ is viscosity, and Pr, Re are Prandtl and Reynolds numbers, respectively.

Using Eqs. (2.5)–(2.7), the energy equation (2.1) and boundary conditions (2.2)–(2.4) take nondimensional form

$$\frac{\partial\theta}{\partial\tau} + v\frac{\partial\theta}{\partial\xi} = \frac{\partial^2\theta}{\partial\eta^2} + \frac{1}{\eta}\frac{\partial\theta}{\partial\eta}, \tag{2.8}$$

and

$$\frac{\partial\theta}{\partial\eta}(\xi, 1, \tau) = 1, \tag{2.9}$$

$$\theta(0, \eta, \tau) = 0, \tag{2.10}$$

$$\frac{\partial\theta}{\partial\eta}(\xi, 0, \tau) = 0. \tag{2.11}$$

Let us assume that the velocity of flow v at every moment of time is constant in any cross section of the tube, and equals to the average flow velocity. That is,

$$v = v_m(1 + \varepsilon \sin\omega\tau), \tag{2.12}$$

where v_m is the average flow velocity, ε is a pulsation parameter, and ω is the frequency.

Inserting Eq. (2.12) into Eq. (2.8), we obtain

$$\frac{\partial\theta}{\partial\tau} + v_m(1 + \varepsilon\sin\omega\tau)\frac{\partial\theta}{\partial\xi} = \frac{\partial^2\theta}{\partial\eta^2} + \frac{1}{\eta}\frac{\partial\theta}{\partial\eta}. \tag{2.13}$$

Multiplying Eq. (2.13) by η, integrating over interval (0, 1) and using the boundary condition (2.9), we obtain

$$\frac{\partial\bar{\theta}}{\partial\tau} + v_m(1 + \varepsilon \sin\omega\tau)\frac{\partial\bar{\theta}}{\partial\xi} = 2, \tag{2.14}$$

where

$$\bar{\theta} = 2\int_0^1 \theta\eta \mathrm{d}\eta. \tag{2.15}$$

Here $\bar{\theta}$ is the mean temperature which equals to the bulk temperature since the flow velocity in cross section of the tube is constant.

The general solution of the first order linear partial differential equation (2.14) is

$$f(\psi_1, \psi_2) = 0, \tag{2.16}$$

where f is an arbitrary function, and

$$\psi_1(\xi, \tau, \bar{\theta}) = \text{const}, \quad \psi_2(\xi, \tau, \bar{\theta}) = \text{const} \tag{2.17}$$

forms a solution of the ordinary differential equations

$$\frac{d\xi}{v_m(1 + \varepsilon \sin\omega\tau)} = d\tau = \frac{1}{2} d\bar{\theta}. \tag{2.18}$$

From Eqs. (2.16)–(2.18) follows

$$\psi_1 = \bar{\theta} - 2\tau, \tag{2.19}$$
$$\psi_2 = \xi - v_m \left(\tau - \frac{\varepsilon}{\omega} \cos \omega\tau \right),$$

and

$$f\left[\bar{\theta} - 2\tau, \xi - v_m \left(\tau - \frac{\varepsilon}{\omega} \cos\omega\tau \right) \right] = 0. \tag{2.20}$$

Using now the boundary condition (2.10), we obtain

$$\xi = \frac{1}{2} v_m \bar{\theta} + \frac{\varepsilon v_m}{\omega} \left[\cos\omega \left(\tau - \frac{\bar{\theta}}{2} \right) - \cos\omega\tau \right]. \tag{2.21}$$

From Eq. (2.21) the mean temperature $\bar{\theta}$ is determined implicitly as a function of the variables ξ and τ.

Next we use the method of perturbation to derive the explicit form of the function $\bar{\theta}$. Let us suppose that ε is a small parameter and the solution of Eq. (2.21) can be written as

$$\bar{\theta}(\xi, \tau, \varepsilon) = \bar{\theta}_0(\xi, \tau) + \varepsilon \bar{\theta}_1(\xi, \tau) + \varepsilon^2 \bar{\theta}_2(\xi, \tau) + \ldots \tag{2.22}$$

Substituting Eq. (2.22) into Eq. (2.21) and comparing the terms with equal powers of ε, we obtain

$$\xi - \frac{1}{2} v_m \overline{\theta}_0 = 0, \tag{2.23}$$

$$\frac{1}{2}\overline{\theta}_1 + \frac{1}{\omega}\left[\cos\omega\left(\tau - \frac{\overline{\theta}_0}{2}\right) - \cos\omega\tau\right] = 0, \tag{2.24}$$

$$\overline{\theta}_2 + \sin\omega\left(\tau - \frac{\overline{\theta}_0}{2}\right)\cdot\overline{\theta}_1 = 0, \tag{2.25}$$

$$\cdots$$

From Eqs. (2.22)–(2.25) follows

$$\overline{\theta} = 2\left\{\frac{\xi}{v_m} + \frac{1}{\omega}\left[-\varepsilon + \varepsilon^2 \sin\omega\left(\tau - \frac{\xi}{v_m}\right)\right]\cdot\left[\cos\omega\left(\tau - \frac{\xi}{v_m}\right) - \cos\omega\tau\right]\right\}. \tag{2.26}$$

Using the notation

$$\xi^* = \frac{\xi}{v_m}, \tag{2.27}$$

Eq. (2.26) can be written as

$$\overline{\theta} = \overline{\theta}_s + \overline{\theta}_p, \tag{2.28}$$

where

$$\overline{\theta}_s = 2\xi^*, \qquad \overline{\theta}_p = \frac{2\varepsilon}{\omega}\left[-1 + \varepsilon\sin\omega\left(\tau - \xi^*\right)\right]\cdot\left[\cos\omega\left(\tau - \xi^*\right) - \cos\omega\tau\right]. \tag{2.29}$$

Figure 1 shows the variation of the oscillatory mean temperature $\overline{\theta}_p$ with time τ for different co-ordinates ξ^*. Notice that there is a difference in the form of the curves for different values of the co-ordinate ξ^*.

To determine the effect of pulsation on the rate of heat transfer at the point ξ^*, we express the local Nusselt number as [5,6]

$$\mathrm{Nu} = \frac{2}{\left\langle \theta\left(\xi^*, 1\right)\right\rangle - \left\langle \overline{\theta}\left(\xi^*\right)\right\rangle}, \tag{2.30}$$

where the angle brackets “< >” indicate the temporal average over the complete cycle of pulsation:

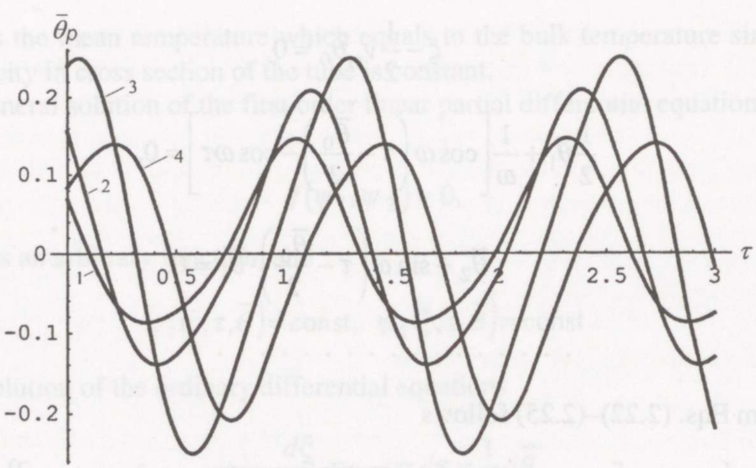

Fig. 1. Variation of the mean temperature $\overline{\theta}_p$ with time τ for different co-ordinates ξ^* by $\varepsilon = 0.2$, $\omega = 5$: $1 - \xi^* = 0.2\pi$; $2 - \xi^* = 0.3\pi$; $3 - \xi^* = 0.5\pi$; $4 - \xi^* = 0.6\pi$.

$$\left\langle \theta\left(\xi^*, 1\right) \right\rangle = \frac{1}{2\pi/\omega} \int_0^{2\pi/\omega} \theta\left(\xi^*, \tau, 1\right) \mathrm{d}\tau, \tag{2.31}$$

$$\left\langle \overline{\theta}\left(\xi^*\right) \right\rangle = \frac{1}{2\pi/\omega} \int_0^{2\pi/\omega} \overline{\theta}\left(\xi^*, \tau\right) \mathrm{d}\tau. \tag{2.32}$$

The corresponding local Nusselt number in a steady flow is given as

$$\mathrm{Nu}_0 = \frac{2}{\theta_s(\xi^*, 1) - \overline{\theta}_s(\xi^*)}. \tag{2.33}$$

The fractional change in the Nusselt number

$$\vartheta = \frac{\mathrm{Nu} - \mathrm{Nu}_0}{\mathrm{Nu}_0} \tag{2.34}$$

can be used as a measure of the effect of pulsation on the heat transfer at point ξ^* [5,6].

From Eqs. (2.30)–(2.34) we find

$$\vartheta = \frac{1}{\dfrac{2}{\mathrm{Nu}_0 \left\langle \overline{\theta}_p \right\rangle} - 1}. \tag{2.35}$$

Substituting Eq. (2.29) into Eq. (2.32), we obtain

$$\left\langle \overline{\theta}_p \right\rangle = \frac{\varepsilon^2 \sin\omega\xi^*}{\omega}, \tag{2.36}$$

therefore

$$\vartheta = \frac{\varepsilon^2 \mathrm{Nu}_0 \sin\omega\xi^*}{2\omega - \varepsilon^2 \mathrm{Nu}_0 \sin\omega\xi^*}. \tag{2.37}$$

The local fractional change in the Nusselt number ϑ has order of magnitude ε^2 and depends on the frequency ω. If the value of $\omega\xi^*$ is in the interval $2n\pi < \omega\xi^* < (2n+1)\pi$ $(n = 0, 1, 2, ...)$, then ϑ is positive and the Nusselt number increases, but if $(2n+1)\pi < \omega\xi^* < 2(n+1)\pi$, then ϑ is negative and the Nusselt number decreases. By increasing the frequency ω the absolute value of ϑ decreases.

Figure 2 shows the variation of fractional change in the Nusselt number with frequency ω at different co-ordinates ξ^*.

From Eq. (2.37) it follows that the fractional change in the Nusselt number ϑ is a periodic function, with period $2\pi/\omega$, of co-ordinate ξ^*. Let us consider the average value of ϑ over the period

$$\{\vartheta\} = \frac{1}{2\pi/\omega} \int_0^{2\pi/\omega} \vartheta\left(\omega, \xi^*\right) \mathrm{d}\xi^*. \tag{2.38}$$

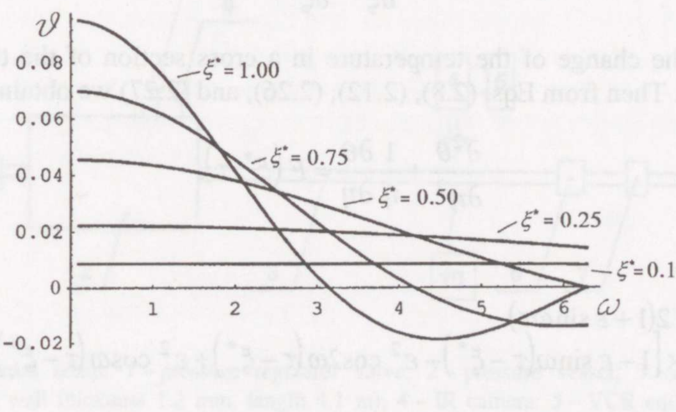

Fig. 2. Variation of the fractional change in the Nusselt number ϑ with frequency ω at different co-ordinates ξ^*.

Integrating Eq. (2.37) with $\frac{\varepsilon^2 \mathrm{Nu}_0}{2\omega} < 1$, we obtain

$$\{\vartheta\} = \frac{1}{\sqrt{1 - \frac{\varepsilon^4 \mathrm{Nu}_0{}^2}{4\omega^2}}} - 1. \tag{2.39}$$

If $\frac{\varepsilon^4 \mathrm{Nu}_0{}^2}{4\omega^2} << 1$, Eq. (2.39) can be approximately written as

$$\{\vartheta\} \approx \frac{1}{8} \frac{\varepsilon^4 \mathrm{Nu}_0{}^2}{\omega^2}. \tag{2.40}$$

From Eq. (2.40) follows that on average ϑ is positive, although for some values of ξ^* it has negative values. That is, the Nusselt number by pulsating flow increases on average. The average value $\{\vartheta\}$ increases by increasing the Nusselt number in steady flow Nu_0 and by decreasing the frequency ω.

3. EXPERIMENTAL ANALYSIS

The available experimental results about heat transfer in pulsating flow are inconclusive and contradictory. First of all, to compare the obtained results with the theoretical analysis, we find the equation for computing the wall temperature. Let us assume that the following equality holds

$$\frac{\partial \theta}{\partial \xi} = \frac{\partial \overline{\theta}}{\partial \xi}, \tag{3.1}$$

and that the change of the temperature in a cross section of the tube is quasi-stationary. Then from Eqs. (2.8), (2.12), (2.26), and (2.27) we obtain

$$\frac{\partial^2 \theta}{\partial \eta^2} + \frac{1}{\eta} \frac{\partial \theta}{\partial \eta} = F\left(\xi^*, \tau\right), \tag{3.2}$$

where

$$F\left(\xi^*, \tau\right) = 2(1 + \varepsilon \sin \omega \tau) \\ \times \left[1 - \varepsilon \sin \omega\left(\tau - \xi^*\right) - \varepsilon^2 \cos 2\omega\left(\tau - \xi^*\right) + \varepsilon^2 \cos \omega\left(\tau - \xi^*\right) \cos \omega \tau\right]. \tag{3.3}$$

Integration of the Eq. (3.2) with boundary condition (2.11) yields

$$\theta = \frac{\eta^2}{4} F\left(\xi^*, \tau\right) + \theta_c\left(\xi^*, \tau\right), \tag{3.4}$$

where θ_c is the temperature in the centre of the tube.

If we express θ_c through the mean temperature $\overline{\theta}$, we obtain

$$\theta = \overline{\theta} + \left(\frac{\eta^2}{4} - \frac{1}{8}\right) F\left(\xi^*, \tau\right). \tag{3.5}$$

Accordingly, the temperature at the wall of the tube ($\eta = 1$) can be written as

$$\theta_w = \overline{\theta} + \frac{1}{8} F\left(\xi^*, \tau\right). \tag{3.6}$$

Experiments were made at Satakunta Polytechnic, Pori. Measurements were conducted on an experimental setup which gave us the possibility to measure integral parameters of the flow. The principal scheme of the set-up is given in Fig. 3.

The setup was connected to the laboratory water supply system and was during the experiments under the network pressure, approximately equal to 4 bars. The pulsating flow was generated by a solenoid valve, which was controlled by PC. Flow rate and pressure measurements were recorded with the frequency 400 Hz per channel. The scanning rate of the IR camera was 50 Hz.

The pipe surface in the IR camera measurement area was heated by a 500 W heater. This makes convective heat transfer processes, taking place inside of the flow, well trackable from outside measurement of the pipe wall temperature by the IR camera.

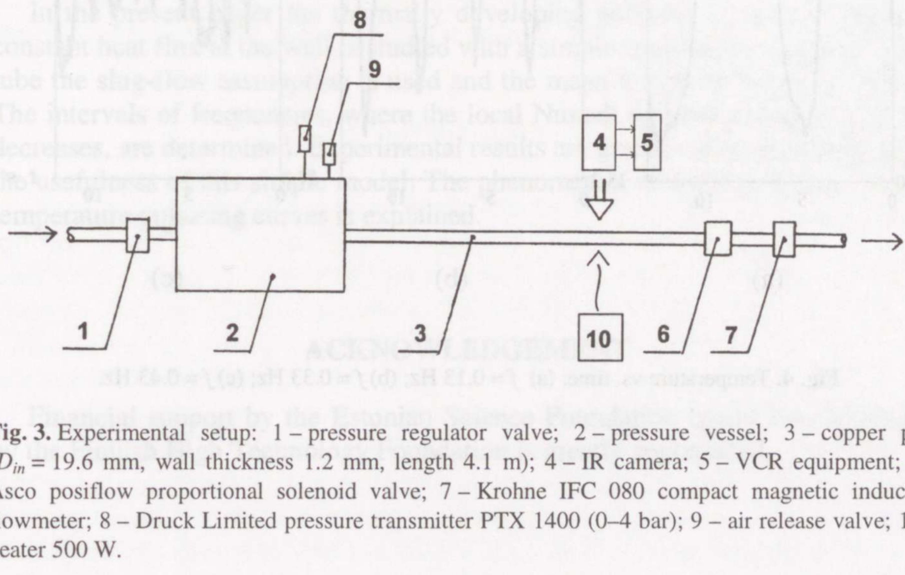

Fig. 3. Experimental setup: 1 – pressure regulator valve; 2 – pressure vessel; 3 – copper pipe (D_{in} = 19.6 mm, wall thickness 1.2 mm, length 4.1 m); 4 – IR camera; 5 – VCR equipment; 6 – Asco posiflow proportional solenoid valve; 7 – Krohne IFC 080 compact magnetic inductive flowmeter; 8 – Druck Limited pressure transmitter PTX 1400 (0–4 bar); 9 – air release valve; 10 – heater 500 W.

The IR camera used for the measurements was Inframetrics Model 740 IR imaging radiometer with wavelength 8–12 μm. Spatial resolution of the camera is 194 elements horizontally (50% SFR) and 240 vertically. Horizontal scanning rate is 7812 Hz and vertical 50 Hz. By using this kind of camera it is possible to detect frequencies up to 50 Hz. Line scans of the temperature were recorded with usual VCR equipment for further analysis.

The temperature changes of the pipe wall due to the cooling effect of the flowing water are relatively low. In order to bring out the convective heat transfer effects in the flow more precisely, the measurement range of the IR camera was set to 2 °C, the smallest possible measurement range of the camera.

Some results of these experiments are shown in Fig. 4. Figure 5 shows corresponding wall temperatures calculated from Eq. (3.6). Good qualitative agreement is observed between experimental and theoretical results. From these results we can also conclude that the phenomenon of cutting off the peaks in temperature pulsating curves is due to the developing character of the temperature in the downstream direction.

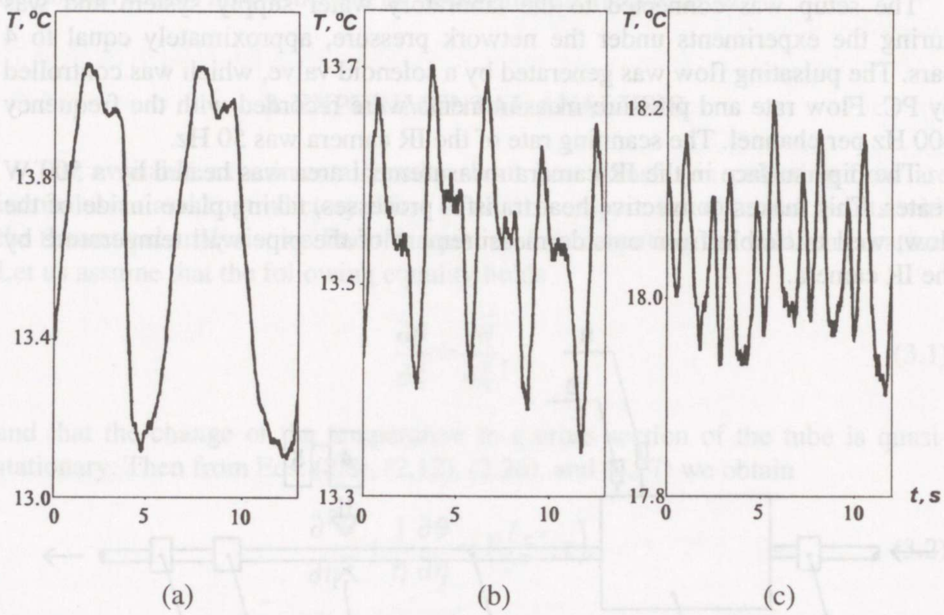

Fig. 4. Temperature vs. time: (a) $f = 0.13$ Hz; (b) $f = 0.33$ Hz; (c) $f = 0.43$ Hz.

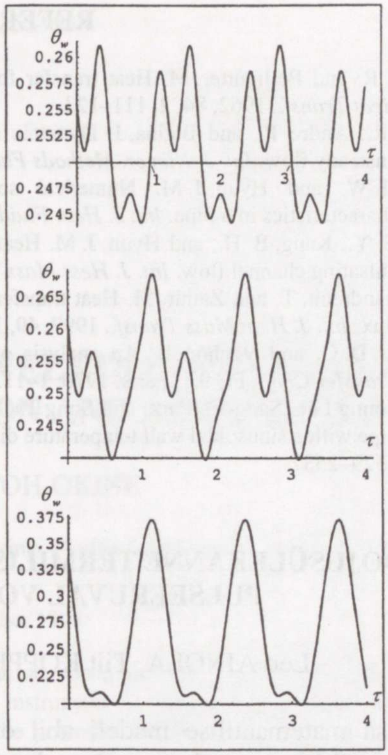

Fig. 5. Calculated temperature at the wall of the tube vs. time for $\omega = 5$, $\varepsilon = 0.4$: (a) $\xi^* = 0.010$; (b) $\xi^* = 0.019$; (c) $\xi^* = 0.100$.

4. CONCLUSIONS

In the present paper the thermally developing pulsatile flow in a tube with constant heat flux at the wall is studied with a simple mathematical model. In the tube the slug-flow assumption is used and the mean temperature is investigated. The intervals of frequencies, where the local Nusselt number either increases or decreases, are determined. Experimental results are given, which in general prove the usefulness of this simple model. The phenomenon of cutting off the peaks of temperature pulsating curves is explained.

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SOOJUSÜLEKANNE TERMILISELT ARENEVAS PIIRKONNAS PULSEERUVAL VOOLAMISEL TORUS

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Lihtsa matemaatilise mudeli abil on käsitletud soojusülekannet termiliselt arenevas piirkonnas pulseeruval voolamisel torus, kui toru välisseinale on rakendatud muutumatu soojuskoormus. Mudeli koostamisel on lähtutud eeldusest, et voolamine on ühtlase kiirusjaotusega, ning vaadeldud on keskmist temperatuuri. Arvutustega on määratud voolamise sageduse piirkonnad, kus lokaalne Nusselti arv kas suureneb või väheneb. Artiklis esitatud eksperimentaalsed tulemused kinnitavad kasutatud teoreetilise mudeli sobivust. Töös on selgitatud ka temperatuuri muutuste puhul esinevat temperatuuri pulseerumise kõvera maksimumi lõikamise efekti.