Proc. Estonian Acad. Sci. Engng., 1997, **3**, 1, 23–34
https://doi.org/10.3176/eng.1997.1.03

PILLAR STRENGTH AND FAILURE MECHANISMS

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Received 5 March 1996, revised 29 May 1996, accepted 11 December 1996

Abstract. This paper analyses the pillar failure mechanisms and strength in a mining block, using the two-dimensional theoretical and numerical modelling. Investigations were based on the Mohr–Coulomb failure criterion. Calculations were performed by the FLAC program, using the average strength and elasticity parameters. The results demonstrate that the type of failure mechanism (in vertical or inclined sections) depends on the stress field at the boundary of a pillar, which was found stronger at the inclined section.

Key words: pillar load, pillar failure mechanism, Mohr–Coulomb failure criterion, sliding surface, numerical modelling, stress field, pillar strength.

1. INTRODUCTION

Pillar design is concerned with matching pillar strength against pillar loading conditions to establish the dimensions that cause the pillar to behave as required, i.e., to stand forever or for a limited time, or to fail in a controlled manner. Despite structural simplicity and detailed knowledge of rock behaviour, obtained over the past few years, pillar design has changed very little during the present century.

As a result of the analysis of the pillar behaviour under overburden load, based on the two-dimensional theoretical and computational methods, we can estimate the pillar strength, depending on the type of failure. Here, the theory of plasticity is almost exclusively used to find upper bounds for ultimate loads. This is accomplished by the calculations based on the failure mechanism.

Our theoretical investigations were based on the Mohr–Coulomb failure criterion. Based on the tests demonstrating pillar failures, we can develop a reliable hypothesis for the failure mechanism. For instance, an excellent example of this is the failure of a pillar under an overburden loading in an Estonian oil-shale mine. The observations have shown that a pillar with vertical or subvertical and inclined cracks may fail.

To determine the loads and stability of the pillar, the computational method of stress analysis was employed. In this paper, the two-dimensional Fast Lagrangian Analysis of Continua (FLAC. Version 3.22) was used [1]. FLAC has several built-in material behaviour models and is particularly suitable for modelling non-linear, large strain and physically unstable continuous systems. All the calculations were performed in the Laboratory of Rock Engineering of the Helsinki University of Technology.

The analysis, based on both theoretical and computational methods, allows an estimation of the strength of the pillar, depending on the type of the failure.

2. THEORETICAL MODELS

Borissov [2] and Parker [3] have suggested a qualitative description of pillar loads within a mining block and a pillar failure mechanism. They claim that the pillar loads within a mining block depend upon the strength-stress relationship, a function of span and other variables not yet subject to precise calculations. Various theories of failure have been developed, but in practice, the problem of rock mass failure is a qualitative rather than a quantitative phenomenon.

2.1. Pillar loads

The pillar loads depend on the width of the mining block, leading to the concept of the critical width. The critical width is the greatest width that the rock above the mine can span before its failure or, if there are pillars, the width to which we must mine before the pillars accept the full weight of the overlying materials [2, 3]. Many theories available offer an excellent explanation for this process and help to calculate the critical width. It can be expressed by the following equation [2]:

$$\frac{L}{H} \geq 0.8-1.0, \qquad (1)$$

where L, width of the mining block; H, thickness of the overburden rocks.

In fact, the best indicator of critical width in a given situation will be provided by the records of failures and surface subsidence from measuring roof-to-floor convergence in the mine. Borissov's calculation scheme describes the behaviour of the overburden rocks in the conditions of Estonian oil-shale mines adequately. It is important to notice that pillar loads vary from place to place within a mining block, depending mostly on the extent of the roof sags at each place. Typically, the roof will sag closest to the centre of the mining block. Thus, higher loads are likely to occur there.

Two different assumptions regarding the pillar loads in a mining block are represented here:

a) A roof will sag closest to the centre of the mining block, where higher loads are likely to occur. Accordingly, the vertical stress on the pillar predominates close to the centre of the mining block.

b) On the pillars, both the normal and shear stresses predominate towards the margins of the mining block.

2.2. Pillar failure mechanisms

Of various theories of failure, for rock, the Mohr´s theory, based on the Coulomb´s law of friction, has been widely accepted. Mohr–Coulomb assumed that both constant internal cohesion and the internal cohesion, proportional to the normal pressure on the sliding surface, had to be overcome in the sliding surface. This assumption can be formulated in a *x*, *y*-coordinate system as follows [4]:

$$(\sigma_{xx} - \sigma_{yy})^2 + 4\tau_{xy}^2 = \sin^2\varphi(\sigma_{xx} + \sigma_{yy} + 2C\cot\varphi)^2, \quad (2)$$

where *C*, cohesion; φ, friction angle; σ_{xx}, σ_{yy}, τ_{xy}, components of the total stresses at the boundary.

According to the Mohr–Coulomb´s failure hypothesis, the middle principal stress has no effect on the failure process.

If a uni-axial compressive strength σ_c is introduced as the stress field defined by $\sigma_{xx} \neq 0$, $\sigma_{yy} = 0$ and $\tau_{xy} = 0$ (or $\sigma_{xx} = 0$, $\sigma_{yy} \neq 0$, $\tau_{xy} = 0$), then the pillar failure is of the type shown in Fig. 1 [4]:

$$\sigma_c = \frac{2C\cos\varphi}{1 - \sin\varphi}. \quad (3)$$

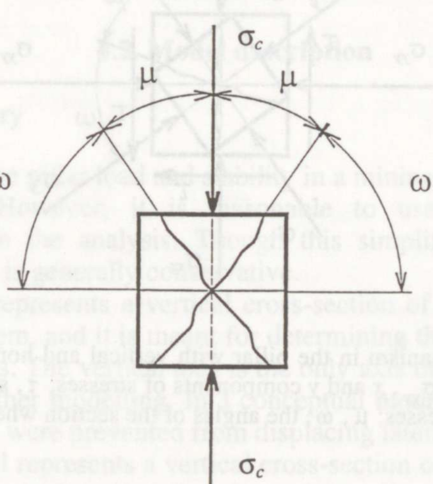


Fig. 1. The failure mechanism in the pillar with uni-axial compression: σ_c, uni-axial compression stress; μ, ω, the angles of the section where the failure criterion is satisfied.

In the case of uni- and bi-axial compression, the failure criterion is satisfied in the section, forming the angle μ between the major principal stress and the sliding surface, and the angle ω between the minor principal stress and the sliding surface (Fig. 1). Here [4]:

$$\mu = \frac{\pi}{4} - \frac{\varphi}{2}, \qquad \omega = \frac{\pi}{4} + \frac{\varphi}{2}. \tag{4}$$

This failure mechanism is often observed for the pillars in the centre of a mining block, where the major principal stress develops from the overburden loading.

Another type of pillar failure occurs when the failure criterion is satisfied in the section within the right angle from the minor principal stress (parallel to the major principal stress). This phenomenon is possible if the horizontal stress occurs on the top of the pillar. Figure 2 demonstrates the calculation method, showing that if the angle α between σ_{xx} and σ_1 equals μ, the aforementioned failure mechanism is possible. The angle α can be calculated by [2]:

$$\tan 2\alpha = \frac{2\tau_{xy}}{(\sigma_{xx} - \sigma_{yy})} \tag{5}$$

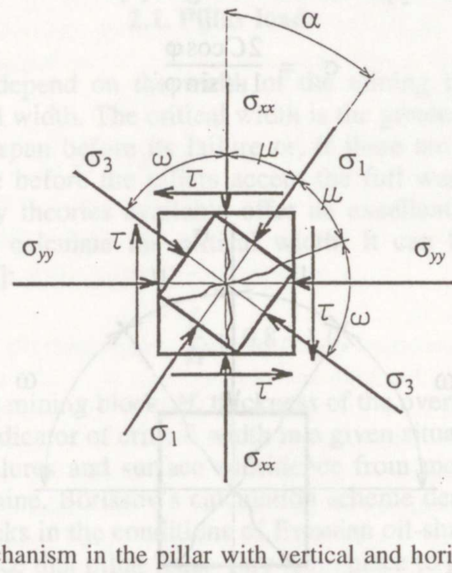


Fig. 2. The failure mechanism in the pillar with vertical and horizontal stresses: σ_1, σ_3, principal stresses; σ_{xx}, σ_{yy}, x and y components of stresses; τ, shear stress; α, the angle between σ_{xx} and σ_1 stresses; μ, ω, the angles of the section where the failure criterion is satisfied.

Through Eqs. (2), (4) and (5), we can forecast the conditions necessary for this failure mechanism. If the stress field is given by $\sigma_{xx} \neq 0$, $\sigma_{yy} = 0$ and $\tau_{xy} \neq 0$ (or $\sigma_{xx} = 0$ $\sigma_{yy} \neq 0$ $\tau_{xy} \neq 0$), then failure occurs:

$$\tau_{xy} = C\,, \qquad \sigma_{yy} = 2C\tan\varphi\,. \tag{6}$$

In this case, if the stress conditions are satisfied, vertical cracks appear in the pillars at the perimeter of a mining block. Accordingly, the uni-axial compressive strength equals the vertical stress:

$$\sigma_c = \sigma_{yy}\,. \tag{7}$$

Analyses show that the pillar is stronger if the failure criterion is satisfied in the inclined section.

3. NUMERICAL MODELLING

3.1. Estonia's oil-shale mining system

Estonia's oil-shale mines use the room-and-pillar mining system. The field of an oil-shale mine is divided into panels, which are subdivided into mining blocks, each approximately 300–350 m in width and from 600 to 800 m in length. The oil-shale bed is embedded at the depth of 40–75 m. Its height corresponds to the thickness of the commercial oil-shale bed, approximately 2.8 m. The width of the room is determined by the stability of the roof. It is very stable when it has a dimension of 6–10 m. In this case, the immediate roof must still be supported by bolting. Actual mining practice has shown that pillars with a square cross-section suit best. The cross-sectional area of the pillars is 30–40 m^2, depending on the depth of the oil-shale bed. Because Estonia's rock is heterogeneous, the stability analysis is complex.

3.2. Model description

3.2.1. Model geometry

The problem of the pillar load and stability in a mining block is fundamentally three-dimensional. However, it is reasonable to use the two-dimensional continuous model in the analysis. Though this simplification leads to some inaccuracy, the error is generally conservative.

The first model represents a vertical cross-section of a mining block by the room-and-pillar system, and it is meant for determining the load and deformation in the roof and pillars. The vertical axis is the only axis of symmetry that enables simplification of further modelling. In a conceptual model, both the vertical and the bottom boundary were prevented from displacing laterally.

The second model represents a vertical cross-section of a pillar, an immediate roof and a floor. It is intended for determining the character of the pillar destruction. For the pillar, the width and height ratio (W : H) was 2 : 1. The load of the pillar top was vertical or inclined. Table 1 shows the general parameters of the room-and-pillar mining system in an Estonian oil-shale mine, used for the modelling.

Table 1

Room-and-pillar mining system parameters in Estonian oil-shale mine used for the modelling

Parameter	Value
Depth of the mined oil-shale bed, m	40
Density of overburden rock, kg/m^3	2500
Pillar cross-sectional area, m^2	36
Width of the room, m	9
Width of the mining block, m	300

3.2.2. Model properties

In Estonian oil-shale mines, the pillars and the roof are non-homogeneous and structurally complicated. They consist of the oil-shale and limestone layers of different properties and thickness. In the calculations, average rock properties were used, determined by the following formulas [5]:

a) density:

$$\gamma_a = \frac{\sum \gamma_i m_i}{\sum m_i}, \tag{8}$$

where γ_i, density of layers; m_i, thickness of layers.

b) cohesion:

$$C_a = \frac{\sum C_i\, l_i}{\sum l_i}, \tag{9}$$

where C_i, cohesion of layers; l_i, length of the sliding surface.

c) friction angle:

$$\tan\varphi_a = \frac{\sum \sigma_i\, l_i \tan\varphi_i}{\sum \sigma_i\, l_i}, \tag{10}$$

where φ_i, friction angle of layers; σ_i, normal stress of the sliding surface.

The cohesion of the rock mass C_M can be calculated using the expression presented in [5]:

$$C_M = \frac{C_a}{1 - a \ln H/L}, \tag{11}$$

where a, coefficient, depending on the structure of the rock mass (a = 0.5–10); H, height of the construction; L, average dimension of the blocks.

The strength and elasticity parameters for oil-shale and limestone were derived from the tests conducted by Reinsalu [6]. In the calculation by the above-mentioned method, the average data of the rock and rock mass were used. The rock mass was assumed to behave as an elasto-plastic material with the Mohr–Coulomb yield criterion. Table 2 shows rock mass properties in the modelling described in this paper.

Table 2

Rock mass properties used for the modelling

Parameter	Value	
	Pillar	Roof and floor
Density, kg/m^3	1900	2400
Bulk modulus, GPa	9.0	30.0
Shear modulus, GPa	4.0	17.0
Friction angle, deg.	37.7	45.0
Cohesion, MPa	4.0	16.0
Tensile strength, MPa	0.8	2.5

3.2.3. Calculations and calculation sequence

The calculation sequence for the FLAC program is as follows:
step 1 – equilibrium under in-situ stresses;
step 2 – excavation of the rooms;
step 3 – equilibrium under stresses in novel conditions.

Between calculation steps 1 and 2, all displacements were reset. This did not affect the calculations, but enabled us to evaluate the incremental deformation response at each step. Because this study focuses on the rock mass and stability of the pillar response caused by the stress and displacement, only the results of step 3 are discussed here. The resulting rock-mass responses are presented in terms of induced deformation and stresses and plasticity indicators.

4. RESULTS

Figures 3 and 4 show the vertical and horizontal displacement contours in the pillars and in the vicinity of roof and floor under overburden loading within the mining block. Our analysis shows that the roof sags closest to the centre of the mining block and higher vertical loads are likely to occur there. In the pillars, both the vertical and horizontal displacements occur towards the margins of the mining block. In consequence, the vertical and horizontal stresses appear there.

Figure 5*a* illustrates the failure of a pillar with uni-axial compression. Localization of plastic strain occurred with the band angle of approximately 64° to the horizontal. This angle is consistent with the calculation method (Eq. (4)). Figure 5*b* illustrates the same results with the principal stresses indicated. The analysis showed that the pillar rock is most restrained by friction at the roof and floor and least restrained at the mid-height. So, a pillar can be expected to develop a waistline, and the zone of a damaged rock cannot bear much stress. Consequently, outer zones will fail, but the core will be much stronger.

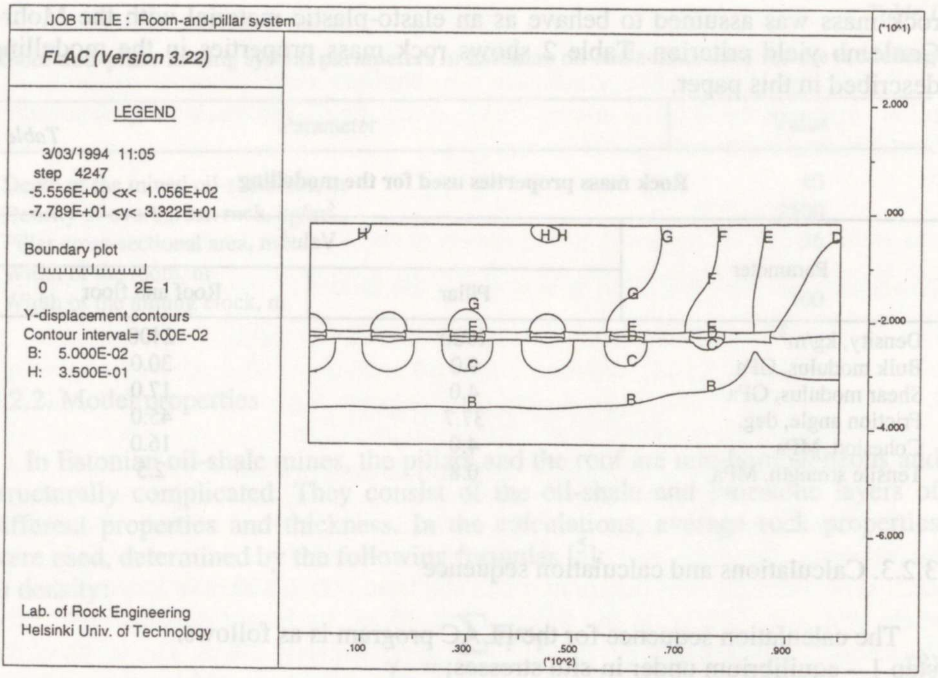


Fig. 3. Vertical displacement contours in a vertical cross-section of a mining block.

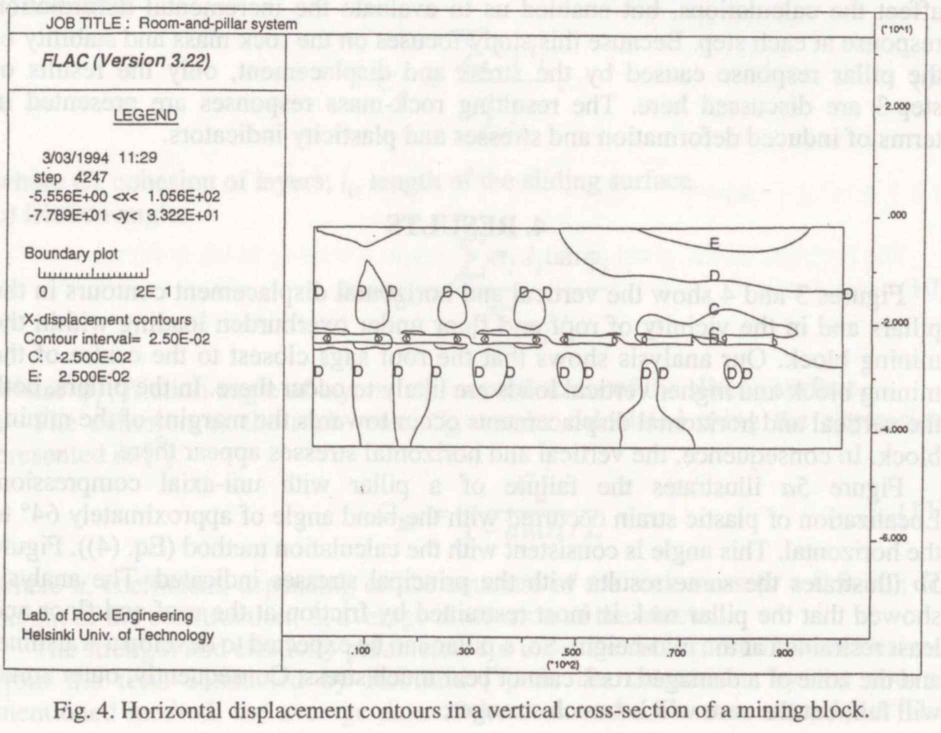


Fig. 4. Horizontal displacement contours in a vertical cross-section of a mining block.

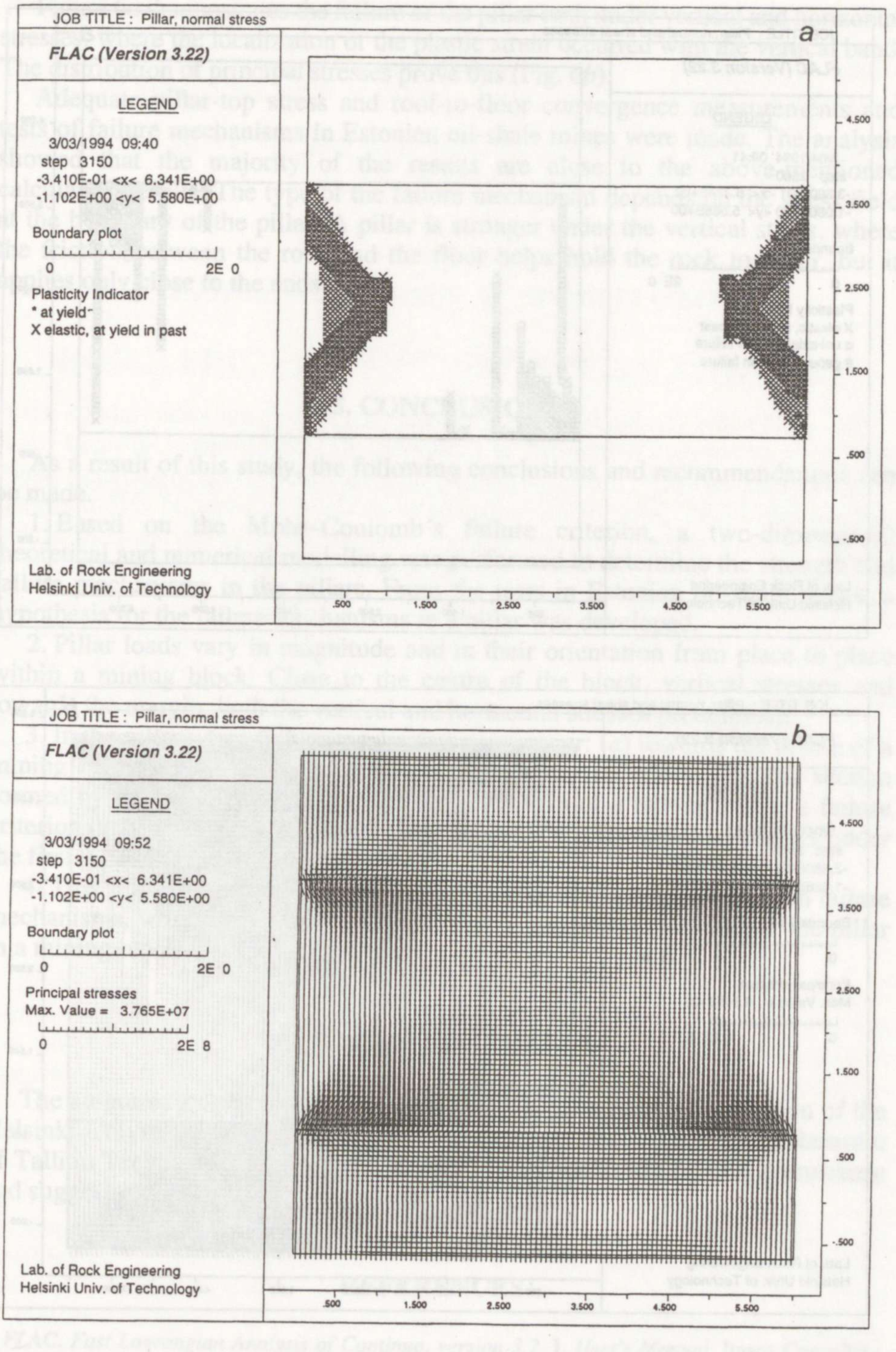
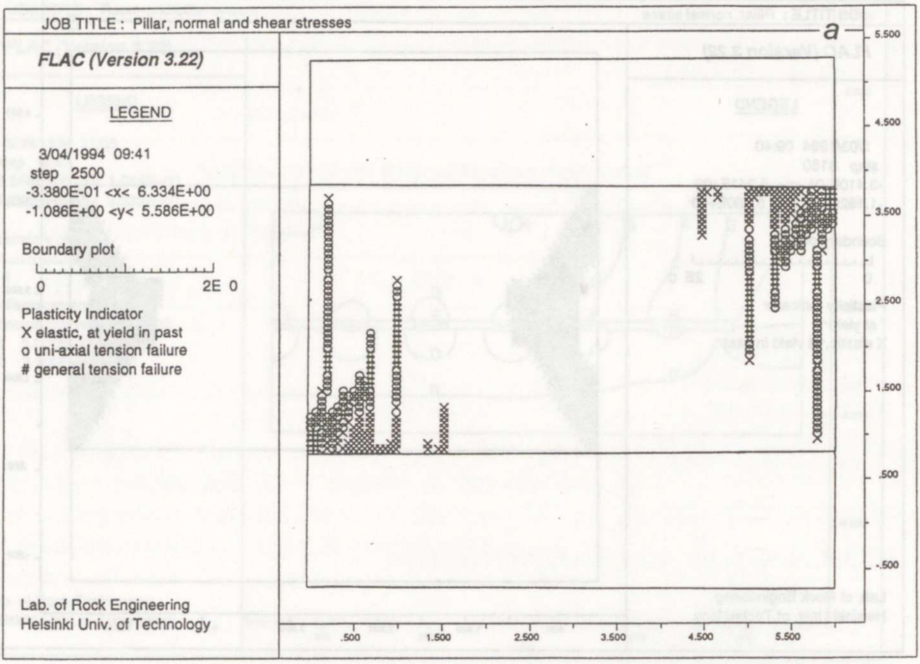


Fig. 5. The failure mechanism in the pillar with uni-axial compression. *a*, plasticity indicator; *b*, principal stresses.



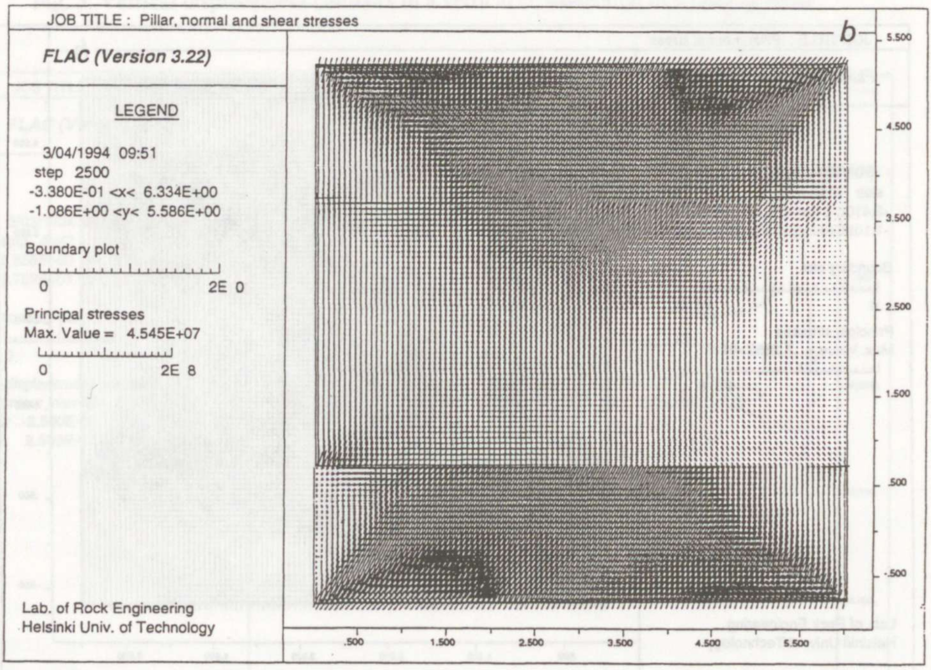


Fig. 6. The failure mechanism in the pillar with vertical and horizontal stresses. *a*, plasticity indicator; *b*, principal stresses.

Figure 6*a* demonstrates the failure of the pillar both under vertical and horizontal stresses, where the localization of the plastic strain occurred with the vertical band. The distribution of principal stresses prove this (Fig. 6*b*).

Adequate pillar-top stress and roof-to-floor convergence measurements and tests of failure mechanisms in Estonian oil-shale mines were made. The analysis showed that the majority of the results are close to the above-mentioned calculation results. The type of the failure mechanism depends on the stress field at the boundary of the pillar. A pillar is stronger under the vertical stress, where the friction between the roof and the floor helps hold the rock together, but it applies only close to the ends.

5. CONCLUSIONS

As a result of this study, the following conclusions and recommendations can be made.

1. Based on the Mohr–Coulomb´s failure criterion, a two-dimensional theoretical and numerical modelling was performed to determine the strength and failure mechanisms in the pillars. From the tests in Estonian oil-shale mines, a hypothesis for the failure mechanisms in a pillar was developed.

2. Pillar loads vary in magnitude and in their orientation from place to place within a mining block. Close to the centre of the block, vertical stresses and towards the margin, both the vertical and horizontal stresses predominate.

3. In the pillars, two failure mechanisms can occur: (*a*) towards the centre of a mining block, a failure criterion applies which can be met within the section formed by the angle ω and (*b*) towards the margin of a mining block, a failure criterion in the vertical section which may be satisfied. The pillar is stronger under the first type of the stress field.

4. This study showed how important it is to take into consideration both failure mechanisms, which determine the stability, strength and dimensions of the pillar in a mining block.

ACKNOWLEDGEMENTS

The author expresses his deep gratitude to professor Raimo Matikainen of the Helsinki University of Technology, professors Alo Adamson and Enno Reinsalu of Tallinn Technical University for their great assistance and helpful discussions and suggestions.

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TERVIKU TUGEVUS JA PURUNEMISMEHHANISM

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On analüüsitud terviku tugevust ja purunemismehhanismi kambriplokis kasutades kahemõõtmelist teoreetilist ja numbrilist modelleerimist. Uuringud põhinesid Mohri–Coulomb´i tugevusteoorial. Protsessi numbriliseks modelleerimiseks on kasutatud FLAC-programmi. Andmebaasi loomisel on lähtutud kivimite keskmistest tugevus- ja elastsusparameetritest. Analüüs näitas, et terviku purunemine kas vertikaalselt või kaldu olevate lõhedega sõltub rakendatava koormuse tüübist. Kaldlõhede korral on tervik tugevam.