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STATICAL ANALYSIS OF GIRDER- OR CABLE-STIFFENED SUSPENDED STRUCTURES

Valdek KULBACH

Tallinna Tehnikaülikooli Ehitiste Projekteerimise Instituut (Department of Engineering Design, Tallinn Technical University), Ehitajate tee 5, EE-0026 Tallinn, Eesti (Estonia)

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Abstract. This paper presents a calculation method for the prestressed suspension structures stiffened by a girder or a stretching cable. For both structures, the relative deflection was found from a cubic equation where support displacements were taken into account. Under the action of the temporary half-span load, the summary load was distributed into the symmetrical and antisymmetrical parts. After calculation of deflections and inner forces under the action of the symmetrical load, for secondary loading, the changed geometrical and statical parameters were used. For both structures, numerical examples are given. This paper outlines our research and adds to our previous studies.

Key words: suspended structure, hanging roof, suspension bridge, prestressed cable system, cable structure, carrying cable, stretching cable.

1. INTRODUCTION

The behaviour of prestressed suspension structures stiffened by girders and stretching cables was analysed. The common assumptions about the linear elastic strain–stress dependence of materials and absence of elongations of hangers were taken into account. The cables were regarded as geometrically nonlinear rods without bending rigidity, and stiffening girders – as bended linear bars. The mutual action between the carrying cables and stiffening members was regarded as a continuous contact load. The action of the uniform whole-span and half-span vertical loads was investigated. Those loads are typical of both roof structures and pedestrian bridges.

2. INITIAL EQUATIONS

Under prestressing, the condition of equilibrium for a cable may be written as (Fig. 1*A*)

$$H_0 \frac{d^2 z}{dx^2} = p_0 \tag{1}$$

or

$$z = f\frac{x^2}{a^2}, \tag{2}$$

where

- x, z – initial coordinates of the cable;
- H_0 – initial value of the horizontal component of the inner force;
- p_0 – prestressing contact load;
- f – flexure of the cable.

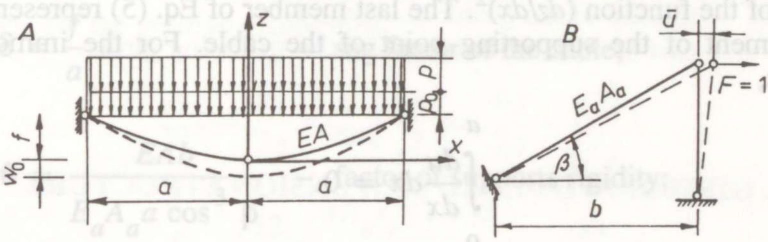

Fig. 1

Under the action of the permanent or temporary outer loads, we may write

$$H\left(\frac{d^2 z}{dx^2} + \frac{d^2 w}{dx^2}\right) = p_0 + p, \tag{3}$$

where

- w – vertical displacement of the cable;
- H – horizontal component of the total inner force;
- p – additional vertical load.

The compatibility condition of the relative elongation of the cable may be expressed as the equation of equality of geometrical and elastic deformations

$$\frac{1}{1+\left(\frac{dz}{dx}\right)^2}\left[\frac{du}{dx} + \frac{dw}{dx}\left(\frac{dz}{dx} + \frac{1}{2}\frac{dw}{dx}\right)\right] = \frac{H - H_0}{EA}\left[1 + \left(\frac{dz}{dx}\right)^2\right]^{1/2}, \tag{4}$$

where

u – horizontal displacement of the cable element;

EA – rigidity of the cable in tension.

To eliminate displacement u, we have to integrate Eq. (4) over the cable span. For symmetrical loading, we may write

$$\int_0^a \frac{dw}{dx}\left(\frac{dz}{dx}+\frac{1}{2}\frac{dw}{dx}\right)dx = \frac{H-H_0}{EA}\int_0^a \left[1+\left(\frac{dz}{dx}\right)^2\right]^{3/2} dx - \int_0^a \frac{du}{dx}dx. \quad (5)$$

For the integral

$$\int_0^a \left[1+\left(\frac{dz}{dx}\right)^2\right]^{3/2} dx,$$

it is useful to develop the expression in square brackets as a series in the powers of the function $(dz/dx)^2$. The last member of Eq. (5) represents the displacement of the supporting point of the cable. For the immovable supports,

$$\int_0^a \frac{du}{dx}dx = 0.$$

For the linearly elastic supports, the displacement concerned may be expressed as the product of the cable force and the translation of the support under the action of the unit force. For the scheme in Fig. 1*B*, we may write

$$\int_0^a \frac{du}{dx}dx = \frac{(H-H_0)\,b}{E_a A_a \cos^3\beta}, \quad (6)$$

where

b – horizontal projection of the anchor cable;

β – angle of inclination of the anchor cable;

$E_a A_a$ – rigidity of the anchor cable in tension.

With a single cable loaded by the uniformly distributed load p, we obtain a cubic equation for the relative deflection $\zeta_0 = w_0/f$

$$\zeta_0^3 + 3\zeta_0^2 + (2+p_0^*)\,\zeta_0 = p^*, \quad (7)$$

where

$p_0^* = \dfrac{H_0}{\Phi}$ – prestressing factor;

$p^* = \dfrac{P}{\Phi}$ – loading parameter;

$H_0 = \dfrac{p_0\, a^2}{2f}$ – horizontal component of the initial force of the cable;

$P = \dfrac{pa^2}{2f}$ – horizontal component of the loaded cable force;

$\Phi = \dfrac{2EA\delta^2}{3\,(1+\kappa)}$ – rigidity factor of the cable;

$\delta = \dfrac{f}{a}$ – sag factor of the cable;

$\vartheta = \dfrac{EAb}{E_a A_a a \cos^3 \beta}$ – factor of supports rigidity;

$\kappa = 2\delta^2 + 1{,}2\,\delta^4 + \vartheta$ – geometrical factor.

The horizontal component of the cable force is

$$H = H_0 + \Phi\zeta_0\,(2+\zeta_0)\,. \tag{8}$$

Equation (7) corresponds to the deflection function

$$w = w_0\left(\frac{x^2}{a^2} - 1\right). \tag{9}$$

If we approximate the cable deflection by a trigonometric function

$$w = -w_0 \cos\frac{\pi x}{2a} \tag{10}$$

and use Eqs. (2), (3), and (4), then as a result of the Galjorkin procedures, we obtain the cubic equation for the determination of relative displacement the coefficients of which are very close to the coefficients of Eq. (7):

$$\zeta_0^3+\frac{96}{\pi^3}\zeta_0^2+\left(\frac{2048}{\pi^6}+\frac{32}{3\pi^2}p_0^*\right)\zeta_0=\frac{1024}{3\pi^5}p^*. \tag{11}$$

For the inner forces of the cable, we obtain

$$H = H_0+\frac{3}{\pi}\Phi\zeta_0\left(2+\frac{\pi^3}{32}\zeta_0\right). \tag{12}$$

A comparison of deflection functions (9) and (10) is useful for the analysis of the behaviour of a girder-stiffened system because the fourth derivative of function (9) is lacking. For the stiffening girder, we have the condition of equilibrium defined by:

$$E_bI_b\frac{d^4w}{dx^4}+p'' = 0, \tag{13}$$

where

E_bI_b – rigidity of the girder in bending;
p'' – part of the load balanced by the stiffening girder.

3. GIRDER-STIFFENED SUSPENSION STRUCTURE

For the state of prestressing, we may apply Eq. (1), where the initial load p_0 is initiated by the weight of the girder and the beam system. Under the action of the outer load, we have Eq. (13) for the stiffening girder and for the cable (Fig. 2)

$$H\left(\frac{d^2z}{dx^2}+\frac{d^2w}{dx^2}\right) = p_0+p', \tag{14}$$

where

p' – part of the load balanced by the cable.

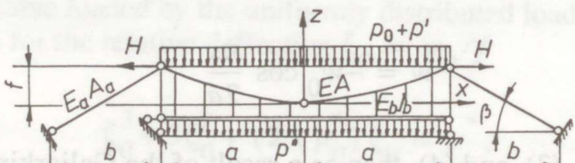

Fig. 2

After summing up Eqs. (13) and (14), we have the condition of equilibrium for the system as a whole:

$$EI\,\frac{d^4w}{dx^4}-H\left(\frac{d^2z}{dx^2}+\frac{d^2w}{dx^2}\right)+p=0, \tag{15}$$

where

$$p=p_0+p'+p'' \text{ – summary load of the structure.}$$

For the initial state, we have the parabolic cable form (2).

Using boundary conditions, after integrating Eqs. (14) and (15), we can obtain exact expression for the coefficient $c=(EI/H)^{1/2}$ in a complicated transcendental form [1]. On the other hand, after suitable approximation of the deflection function, we may obtain a proper result, very close to the exact solution. For the whole–span loading, the deflection function may be approximated by expression (10), which satisfies the boundary conditions. Then, we may write the condition of compatibility (5) as in (12). Based on the value (12) of H in the condition of equilibrium (15), we obtain a cubic equation for the relative displacement

$$\begin{aligned}&\zeta_0^3\cos\frac{\pi x}{2a}+\zeta_0^2\left(\frac{64}{\pi^2}\cos\frac{\pi x}{2a}+\frac{8}{\pi^2}\right)+\\&+\zeta_0\left[\frac{512}{\pi^5}+\frac{4E_bI_b(1+\kappa)}{EA_1f^2}\cos\frac{\pi x}{2a}+\right.\\&\left.+\frac{8p_0a^4(1+\kappa)}{\pi^2EAf^3}\cos\frac{\pi x}{2a}\right]=\frac{64pa^4(1+\kappa)}{\pi^4EAf^3}.\end{aligned} \tag{16}$$

Using the Galjorkin procedure and the designations of Eq. (7), we have finally

$$\zeta_0^3+\frac{96}{\pi^3}\zeta_0^2+\left[\frac{2048}{\pi^6}+\frac{4E_bI_b(1+\kappa)}{EAf^2}+\frac{32}{3\pi^2}p_0^*\right]\zeta_0=\frac{1024}{3\pi^5}p^*. \tag{17}$$

For the maximum bending moment of the stiffening girder, we obtain

$$\max M=-E_bI_b\frac{d^2w}{dx^2}=\frac{\pi^2E_bI_b}{4a^2}\zeta_0. \tag{18}$$

Applying the analogy of the equations for the single cable and using more exact values of the corresponding numerical factors, we obtain the equations for the relative deflection and inner forces

$$\zeta_0^3 + 3\zeta_0^2 + (2 + \rho + p_0^*)\,\zeta_0 = p^*, \tag{19}$$

$$H = H_0 + \Phi\zeta_0\,(2 + \zeta_0)\,, \tag{20}$$

$$\max M = \frac{16}{3}\rho\Phi f\,\zeta_0, \tag{21}$$

where

$$\rho = \frac{E_b I_b}{6\Phi a^2} \text{ – rigidity factor of the stiffening girder.}$$

Other designations are as in (7).

For the half-span loading by a live load, it is useful to distribute the summary load into symmetrical and antisymmetrical parts [2]. If we denote p_0 – the initial load, p_1 – the additional load over the whole span, and p_2 – the load on the right half of the span, then the symmetrical part of the load will be $p_s = p_1 + 0.5p_2$ and the antisymmetrical part $p_u = -0.5p_2$ sign x. For the load p_s, we may apply the formulae (19), (20), and (21). For the antisymmetrical load p_u, the deflection function may be approximated by the trigonometric expression

$$w = -w_1 \sin\frac{\pi x}{a}. \tag{22}$$

Then, we obtain the equation of deformation compatibility (5)

$$H = H_s + \frac{\pi^2 EAf^2}{4a^2(1+\kappa)}\zeta_1^2, \tag{23}$$

where

H_s – horizontal component of the cable force when load $p_0 + p_s$ is applied.

After using the Galjorkin procedures, the condition of equilibrium (3) for the load p_u gives the following cubic equation for the relative deflection $\zeta_1 = w_1/f$:

$$\zeta_1^3 + \left(\rho + \frac{32}{3\pi^2}p_s^*\right)\zeta_1 = \frac{1024}{3\pi^5}p_u^*, \tag{24}$$

where

$$p_s^* = \frac{H_s}{4\Phi} \text{ – prestressing factor},$$

$$p_u^* = \frac{3p_u a^4 (1+\kappa)}{64EAf^3} = \frac{P_u}{16\Phi},$$

$$P_u = \frac{p_u a^2}{2f}.$$

The bending moment of the stiffening girder is expressed by

$$M = \frac{\pi^2 E_b I_b f}{a^2} \zeta \sin \frac{\pi x}{a}. \tag{25}$$

More exact values of numerical factors give

$$\zeta_1^3 + (\rho + p_s^*)\zeta_1 = p_v^*, \tag{26}$$

$$H = H_s + 4\Phi\zeta_1^2, \tag{27}$$

$$\max M = 16\rho\Phi f\zeta_1. \tag{28}$$

As in the formulae (23), (24), (25) and in the formulae (26), (27), and (28), the geometrical and statical parameters

$$f, \delta, \kappa, \Phi, \rho$$

are to be taken as the corresponding values, changed under the action of the load p_s.

The summary displacement on the quarter-span of the cable has the value

$$\max w = -0.75\, w_0 - w_1. \tag{29}$$

The corresponding bending moment is

$$\max M = 0.75\rho\Phi w_0 + 4\rho\Phi w_1. \tag{30}$$

4. DOUBLE-CABLED SUSPENSION STRUCTURE

For a prestressed double-cabled suspension structure (Fig. 3), the conditions of equilibrium (1) and (3) and the equations of deformation compatibility (5) have to be written out for both the bearing and the stretching cables [3]. Let us denote the parameters of the bearing cable by index 1 and those of the stretching cable by index 2. The contact load between the cables, initiated by the prestressing forces, will change from the initial value p_0 to the value p_c for a loaded structure. To determine six unknowns (H_{01}, H_{02}, H_1, H_2, p_c, and w), we have six equations, which may be presented for parabolic cables and inclined anchor bars

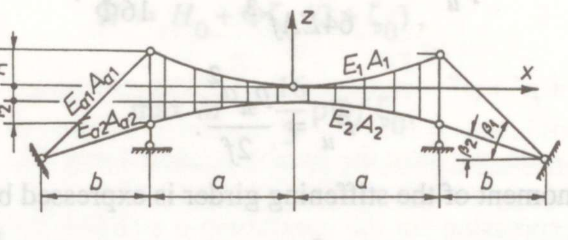

Fig. 3.

$$H_{01} = \frac{p_0 a^2}{2f_1}, \tag{31}$$

$$H_{02} = \frac{p_0 a^2}{2f_2}, \tag{32}$$

$$H_1\left(\frac{d^2w}{dx^2} + \frac{2f_1}{a^2}\right) = p_c + p, \tag{33}$$

$$2H_2\left(\frac{d^2w}{dx^2} - \frac{2f_2}{a^2}\right) = -p_c, \tag{34}$$

$$\int_{-a}^{a} \frac{dw}{dx}\left(\frac{2f_1 x}{a^2} + \frac{1}{2}\frac{dw}{dx}\right)dx = \frac{H_1 - H_{01}}{E_1 A_1}(1+\kappa_1), \tag{35}$$

$$\int_{-a}^{a} \frac{dw}{dx}\left(-\frac{2f_2 x}{a^2} + \frac{1}{2}\frac{dw}{dx}\right)dx = \frac{H_2 - H_{02}}{E_2 A_2}(1+\kappa_2), \tag{36}$$

where

$$\kappa_1 = 2\delta_1^2 + 1.2\delta_1^4 + \vartheta_1, \quad \delta_1 = \frac{f_1}{a},$$

$$\kappa_2 = 2\delta_2^2 + 1.2\delta_2^4 + \vartheta_2, \quad \delta_2 = \frac{f_2}{a},$$

$$\vartheta_1 = \frac{E_1 A_1 b_1}{E_{a1} A_{a1} a \cos^3 \beta_1}, \quad \vartheta_2 = \frac{E_2 A_2 b_2}{E_{a2} A_{a2} a \cos^3 \beta_2}.$$

First, we can eliminate the unknown contact load p_c, then, instead of Eqs. (33) and (34), we have only one equation

$$(H_1 + H_2) \frac{d^2 w}{dx^2} + \frac{2}{a^2} (H_1 f_1 - H_2 f_2) = p. \tag{37}$$

If the load is distributed over the whole span, we may approximate the deflection function as in (10) for a single cable or for a girder-stiffened structure. Then we obtain the following system of equations:

$$(H_1 + H_2) \frac{\pi^2 w_0}{4a^2} \cos \frac{\pi x}{2a} + \frac{2}{a^2} (H_1 f_1 - H_2 f_2) = p, \tag{38}$$

$$\frac{(H_1 - H_{01})(1 + \kappa_1)}{E_1 A_1} = \frac{4 f_1 w_0}{\pi a^2} + \frac{\pi^2 w_0^2}{16 a^2}, \tag{39}$$

$$\frac{(H_2 - H_{02})(1 + \kappa_2)}{E_2 A_2} = \frac{-4 f_2 w_0}{\pi a^2} + \frac{\pi^2 w_0^2}{16 a^2}. \tag{40}$$

To determine the relative deflection $\zeta_0 = w_0 / f$, after using the Galjorkin procedures we obtain the following cubic equation:

$$(1 + \psi) \zeta_0^3 + \frac{96}{\pi^3} (1 - \alpha\psi) \zeta_0^2 +$$

$$+ \left[\frac{2048}{\pi^6} (1 + \alpha^2 \psi) + \frac{32}{3\pi^2} p_0^* \right] \zeta_0 = p^*, \tag{41}$$

where

$$\alpha = \frac{f_2}{f_1}; \quad \psi = \frac{E_2 A_2 (1 + \kappa_1)}{E_1 A_1 (1 + \kappa_2)} \text{ – geometrical factors,}$$

$$p_0^* = \frac{H_{01} + H_{02}}{\Phi} \text{ – prestressing factor,}$$

$$p^* = \frac{P}{\Phi}; \quad P = \frac{p a^2}{2 f_1} \text{ – load parameters,}$$

$$\Phi = \frac{2E_1 A_1 \delta_1^2}{3(1+\kappa_1)} \text{ – cable rigidity parameter.}$$

For the horizontal components of the cable forces we have

$$H_1 = H_{01} + \frac{3}{\pi}\Phi\zeta_0\left(2 + \frac{\pi^3}{32}\zeta_0\right), \tag{42}$$

$$H_2 = H_{02} - \frac{3}{\pi}\psi\Phi\zeta_0\left(2\alpha - \frac{\pi^3}{32}\zeta_0\right). \tag{43}$$

Based on an approximate solution, we obtain Eqs. (41), (42), and (43). At the same time, the exact equations result in the following system:

$$(1+\psi)\zeta_0^3 + 3(1-\alpha\psi)\zeta_0^2 + \left[2(1+\alpha^2\psi) + p_0^*\right]\zeta_0 = p^*, \tag{44}$$

$$H_1 = H_{01} + \Phi\zeta_0(2+\zeta_0), \tag{45}$$

$$H_2 = H_{02} - \psi\Phi\zeta_0(2\alpha - \zeta_0). \tag{46}$$

For a live-load loading over the right half-span, the procedure was the same as for a girder-stiffened structure. For the symmetrical part of the load $p_s = p_1 + 0.5p_2$, we used Eqs. (44), (45), and (46). Next, the changed geometrical factors f_1, f_2, α, δ_1, δ_2, κ_1, κ_2, and ψ and also initial cable forces H_{s1} and H_{s2} were used for the antisymmetrical loading $p_u = -0.5p_2$ sign x. Approximating the deflection function as in (23) and applying it to Eqs. (3), (5), and (6) for the cable forces and displacements, we obtain

$$H_1 = H_{s1} + \frac{3\pi^2}{8}\Phi\zeta_1^2, \tag{47}$$

$$H_2 = H_{s2} + \frac{3\pi^2}{8}\psi\Phi\zeta_1^2, \tag{48}$$

$$(1+\psi)\zeta_1^3 + \frac{32}{3\pi^2}p_0^*\zeta_1 = \frac{1024}{\pi^5}p_v^*, \tag{49}$$

where

$$p_0^* = \frac{H_{s1}+H_{s2}}{4\Phi}, \qquad p_v^* = \frac{P_v}{16\Phi}, \qquad P_v = \frac{p_v a^2}{4f_1}.$$

For the parabolic approximation of the deflection function, we obtain correspondingly

$$(1+\psi)\,\zeta_1^3 + p_0^*\zeta_1 = p_v^*, \tag{50}$$

$$H_1 = H_{s1} + 4\Phi\zeta_1^2, \tag{51}$$

$$H_2 = H_{s2} + 4\psi\Phi\zeta_1^2. \tag{52}$$

5. NUMERICAL EXAMPLES

Let us have two types of bearing structures for a pedestrian bridge with the following parameters:

1) middle span $l = 2a = 64$ m
2) sag of the bearing cable $f = 6.4$ m
3) span of the anchor cable $b = 24$ m
4) inclination of the anchor cable tg $\beta_1 = 0.75$
5) rigidity parameters of the bearing cable ($\varnothing$ 73.5 mm)
   $E_1 = 0.16 \cdot 10^6$ N/mm^2, $A_1 = A_{a1} = 24.92$ cm^2
6) loads on the spanning member:
   the whole dead load 6.0 kN/m,
   the whole live load 12.0 kN/m.

5.1 Girder-stiffened structure

For the stiffening girder, a HEA 1000 section with the parameters $E_b = 0.21 \cdot 10^6$ N/mm^2, $I_x = 553{,}800$ cm^4, $W_x = 11{,}190$ cm^3 was chosen. The initial load, balanced by the cable, consisted of the weight of the erecting units of the girder system (part of the dead load) $p_0 = 2.0$ kN/m; the rest of the dead load $p_1 = 4.0$ kN/m, the whole additional load $p_1 + p_2 = 16$ kN/m.

Geometrical factors of the structure:

$$\delta = \frac{f}{a} = 0.2, \qquad \vartheta = \frac{EAb}{E_a A_a a \cos^3 \beta} = 1.465,$$

$$\kappa = 2\delta^2 + 1.2\delta^4 + \vartheta = 1.547 .$$

Rigidity parameters:

$$\Phi = \frac{2EA\delta^2}{3(1+\kappa)} = 4175 \cdot 10^3 \text{ N}, \qquad \rho = \frac{8E_b I_b (1+\kappa)}{EAf^2} = 0.725 .$$

Prestressing factors:

$$H_0 = \frac{p_0 a^2}{2f} = 160 \text{ kN}, \quad p_0^* = \frac{H_0}{\Phi} = 0.0383 .$$

Load parameters:

$$\text{dead load: } P_1 = \frac{4.0 \cdot 32^2}{2 \cdot 6.4} = 320 \text{ kN}, \quad p_1^* = \frac{P_1}{\Phi} = 0.07665 ,$$

$$\text{summary load: } P = \frac{16.0 \cdot 32^2}{2 \cdot 6.4} = 1280 \text{ kN}, \quad p^* = \frac{P}{\Phi} = 0.3066 .$$

From the cubic equation (19) for the dead load, we obtain $\zeta_0 = 0.02694$ and for the total load, $\zeta_0 = 0.0998$. The respective displacement $w_0 = \zeta_0 f$ has the values 0.172 and 0.639 m.

Deflection under the action of the live load was $w_0 = 0.639 - 0.172 = 0.467$ m.

The horizontal component of the cable force (20) had the value

$$H = H_0 + \Phi\zeta_0 (2 + \zeta_0) = 160 + 875 = 1035 \text{ kN}.$$

The maximum bending moment (21) of the stiffening girder was

$$\max M = \rho\Phi f\zeta_0 = 0.725 \cdot 4175 \cdot 6.4 \cdot 0.0998 = 1933 \text{ kN}\cdot\text{m} .$$

If a live-load is distributed over the right half of the span, we have to divide the load into symmetrical and antisymmetrical parts:

$$p_s = 4.0 + 0.5 \cdot 12.0 = 10.0 \text{ kN/m} ,$$

$$p_v = -0.5 \cdot 12.0 \operatorname{sgn} x \text{ kN/m} .$$

From Eqs. (19), (20), and (21) and the load p_s, we have

$$p_s = \frac{10.0 \cdot 32^2}{2 \cdot 6.4} = 800 \text{kN/m}, \quad p_s^* = \frac{800}{4175} = 0.1916 ,$$

$$\zeta_0 = 0.0647, \ w_0 = 0.414 \text{m}, \ H_s = 160 + 0.0647 \cdot 2.0647 \cdot 4175 = 718 \text{ kN},$$

$$M_{x=0.5a} = 0.75 \cdot 0.725 \cdot 4175 \cdot 6.4 \cdot 0.0647 = 940 \text{ kN/ m}.$$

For the load p_v, we took into account the changed geometrical and statical parameters

$$f = 6.4(1 + 0.0647) = 6.81 \text{ m}, \quad \delta = \frac{6.81}{32} = 0.2128 ,$$

$$\kappa = 1.558, \quad \Phi = 4885 \text{ kN}, \quad \rho = 0.620.$$

The prestressing and load parameters for Eq. (26) have the values

$$p^*_{0s} = \frac{H_s}{4\Phi} = \frac{718}{4 \cdot 4885} = 0.03675\,,$$

$$P_v = \frac{p_v a^2}{2f} = \frac{6.0 \cdot 32^2}{2 \cdot 6.81} = 451.1\ \text{kN}\,,$$

$$p^*_v = \frac{P_v}{16\Phi} = 0.00577\,.$$

From Eq. (26), we now obtain $\zeta_1 = 0.00878$, on the quarter of the span $w = 0.75\,w_0 \pm \zeta_1 f = 0.37$ m or 0.25 m, respectively.

The maximum bending moment (28) is

$$\max M = 940 + 4\rho\Phi f\zeta_1 = 940 + 681 = 1621\ \text{kN}\cdot\text{m}.$$

5.2. Double-cabled structure

For the stretching cable, the rope ∅ 63 mm with the rigidity parameters $E_2 = 0.16 \cdot 10^6\ \text{kN}\cdot\text{mm}^2$, $A_2 = A_{a2} = 18.34\ \text{cm}^2$ was chosen. The prestressing forces were induced by the initial contact load p_0, the minimum value of which was determined as $p_0 = 4.5$ kN/m to retain tension in the stretching cable; then, we took into account the whole dead and live load. So we applied $p_1 = 6.0$ kN/m and $p_1 + p_2 = 18.0$ kN/m.

The geometrical factors of the structure were:

$$f_1 = 6.4\ \text{m}, f_2 = 4.0\ \text{m}, \alpha = \frac{f_2}{f_1} = 0.625\,,$$

$$\delta_1 = 0.2, \delta_2 = 0.125, \kappa_1 = 1.547, \kappa_2 = 0.8850\,,$$

$$\psi = \frac{E_2 A_2 (1+\kappa_1)}{E_1 A_1 (1+\kappa_2)} = 0.9944, \quad \vartheta_1 = \frac{E_1 A_1 b_1}{E_{a1} A_{a1} a \cos^3 \beta_1} = 1.465\,,$$

$$\vartheta_2 = \frac{E_2 A_2 b_2}{E_{a2} A_{a2} a \cos^3 \beta_2} = 0.8535\,.$$

The value of the rigidity factor of the cable had the same value as that of the girder-stiffened structure:

$$\Phi = \frac{2E_1A_1\delta_1^2}{3(1+\kappa_1)} = 4175\ \mathrm{kN}.$$

The prestressing factors were:

$$H_{01} = \frac{p_0a^2}{2f_1} = 360\ \mathrm{kN}, \quad H_{02} = \frac{p_0a^2}{2f_2} = 576\ \mathrm{kN},$$

$$p_0^* = \frac{H_{01}+H_{02}}{\Phi} = 0.2242.$$

The load parameters for the dead load and the total load had the values:

$$P_1 = \frac{6.0\cdot 32^2}{2\cdot 6.4} = 480\ \mathrm{kN}, \quad p_1^* = \frac{P_1}{\Phi} = 0.1150,$$

$$P = \frac{18.0\cdot 32^2}{2\cdot 6.4} = 1440\ \mathrm{kN}, \quad p^* = \frac{P}{\Phi} = 0.3450.$$

From the cubic equation (44), we obtained the relative deflection $\zeta_0 = 0.0377$ and $\zeta_1 = 0.1095$. The corresponding displacements $w_0 = f\zeta_0$ had the values 0.701 m and 0.241 m. Maximum displacement under the action of live load was:

$$w_0 = 0.701 - 0.241 = 0.460\ \mathrm{m}.$$

The horizontal components of the cable forces were:

$$H_1 = H_{01} + \Phi\zeta_0(2+\zeta_0) = 360 + 964 = 1324\ \mathrm{kN},$$
$$H_2 = H_{02} - \psi\Phi\zeta_0(2\alpha-\zeta_0) = 576 - 541 = 35\ \mathrm{kN}.$$

To calculate the displacements and inner forces under the action of a live load on the right half of the span, we had the following symmetrical and antisymmetrical loads:

$$p_s = 12.0\ \mathrm{kN}, \quad p_v = -6.0\ \mathrm{sgn}\, x\ \mathrm{kN}.$$

For the symmetrical part of the load, we had:

$$\zeta_0 = 0.0743, \quad w_0 = 0.476\ \mathrm{m}.$$

The horizontal components of the cable forces were:

$$H_{s1} = H_{01} + \Phi\zeta_0(2+\zeta_0) = 1003\ \mathrm{kN},$$

$$H_{s2} = H_{02} - \psi\Phi\zeta_0(2\alpha - \zeta_0) = 198 \text{ kN}.$$

The changed geometrical and statical parameters for the load p_v were:

$$f_1 = 6.4 + 0.476 = 6.876 \text{ m}, f_2 = 4.0 - 0.476 = 3.524 \text{ m}, \alpha = 0.5125,$$

$$\delta_1 = 0.2149, \delta_2 = 0.1101, \kappa_1 = 1.560, \kappa_2 = 0.8779,$$

$$\psi = 1.003, \quad \Phi = \frac{2 \cdot E_1 A_1 \delta_1^2}{3}(1 + \kappa_1) = 4795 \text{ kN}.$$

The prestressing factor after loading by p_s was:

$$p_{0s}^* = \frac{H_{s1} + H_{s2}}{4\Phi} = \frac{1201}{4 \cdot 4795} = 0.06262.$$

The load parameters were:

$$P_v = \frac{6.0 \cdot 32^2}{2 \cdot 6.4} = 480 \text{ kN}, \quad p_v^* = \frac{P_v}{16\Phi} = 0.00626.$$

The relative displacement ζ_1 was found from Eq. (50):

$$\zeta_1 = 0.0822, \quad w_1 = 0.565 \text{ m}.$$

On the quarter-span we had:

$$w = 0.75w_0 + w_1 = 0.922 \text{ m}$$

$$\text{or} \quad w = 0.75w_0 - w_1 = -0.208 \text{ m}.$$

The horizontal components of the cable forces were:

$$H_1 = H_{s1} + 4\Phi\zeta_1^2 = 1003 + 130 = 1133 \text{ kN},$$

$$H_2 = H_{s2} + 4\psi\Phi\zeta_1^2 = 198 + 130 = 328 \text{ kN}.$$

6. COMPARATIVE ANALYSIS

The girder- and cable-stiffened suspension systems are similar in behavioural and calculational aspects. Both for a single cable and for the compound structures, the deflection parameter may be found from a cubic equation. The studies covered the whole– or half-span uniformly distributed loads for parabolic cable structures. It is interesting to mention that for both the single and the double cabled system, the parabolic distribution of displacements was in agreement with the exact solution of the problem [1]. For a girder-stiffened structure, the exact solution may be found from the complicated transcendental equation [3] and the parabolic distribution of the displacements can describe the behaviour of the structure only approximately. For a girder, it is related to the fourth derivative in the condition of equilibrium. Fortunately, the results of approximate solution were very close to the exact values.

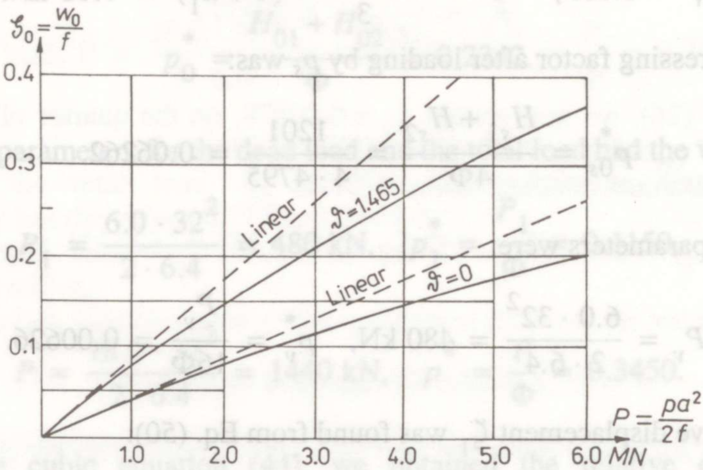

Fig. 4

Based on our analysis, the following conclusions can be drawn:

6.1. The relative displacements of different suspension structures may be found from similar equations (7), (19), and (44), which can be defined as slightly nonlinear (Fig. 4). The approximate value of ζ_0 may be found as the linear solution of the corresponding equation.

6.2. Displacements of supports influence the cable behaviour to a great degree. They are taken into account by the factors ϑ and κ; the latter is included into the parameters p_0, p_*, Φ, ρ, and ψ.

6.3. For part-span loaded structures, it is useful to divide the entire load into symmetrical and antisymmetrical parts. The principle of superposition as applied to geometrically nonlinear structures was discussed in [2]. To calculate the structure under the action of an antisymmetrical load, the changed geometrical and statical parameters of the system must be taken into account.

6.4. With whole–span loading, the girder-stiffened and double-cabled structures show similar quality of deformations. Because of parasitic influence of greater prestressing loads, the final cable forces have greater value for the double-cabled structures. The advantages of the girder-stiffened structures show in the case of part-span loading. The girder-stiffened structure is also characterized by somewhat smaller anchor forces. On the other hand, steel consumption for the girder is to an order greater than for the stretching cable.

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JÄIKURTALAGA VÕI PINGESTUSTROSSIGA JÄIGASTATUD RIPPKONSTRUKTSIOONI STAATILISE TÖÖ ANALÜÜS

Valdek KULBACH

Artiklis on esitatud ühtne meetod eelpingestatud rippkonstruktsiooni kandevõime arvutamiseks juhul, kui süsteemi jäigastamiseks on kasutatud kas jäikurtala või pingestustrossi. Mõlemal puhul saab konstruktsiooni suhtelise läbipainde leida kuupvõrrandist. Arvesse on võetud tugede rõhtsiirdeid. Ajutise koormuse mõjumisel poole silde ulatuses jaotatakse kogukoormus sümmeetriliseks ja antisümmeetriliseks osaks; viimase toime arvutamisel lähtutakse esimese koormusastme mõjul muutunud geomeetrilistest ja staatilistest parameetritest. Mõlema konstruktsiooni kohta on esitatud arvutusnäited.

АНАЛИЗ СТАТИЧЕСКОЙ РАБОТЫ ВИСЯЧИХ КОНСТРУКЦИЙ С БАЛКОЙ ЖЕСТКОСТИ ИЛИ С НАПРЯГАЮЩИМ ТРОСОМ

Валдек КУЛЬБАХ

Представлена общая методика расчета несущей способности предварительно напряженных висячих конструкций, жесткость которых обеспечивается или балкой жесткости, или напрягающим тросом. В обоих случаях относительный прогиб системы описывается кубическим уравнением. Приняты во внимание горизонтальные смещения опор. Для случая действия временной нагрузки на половине пролета суммарная нагрузка разделяется на симметричную и асимметричную части; при расчете системы на действие второй части нагрузки учитываются измененные геометрические и статические параметры системы. Для обоих видов конструкций приведены примеры расчета.