

*John Tagore TEVET*

## ON A DISCRETE MODEL OF ECOLOGICAL PROCESSES

**Abstract.** In the present paper one of the subtypes of discrete models — graph theoretical models are presented and illustrated in the form of ontogeny, coenogenesis, and bioindicative models. To present the graph theoretical model concept in the case of an ecological system it is necessary to proceed from two principles: (1) any observation of an ecological system must be dealt with as a certain state of this system, and (2) an ecological system itself is treated as probable succession series of the states of this system. The use of discrete graph theoretical models opens new aspects for research into the structure and dynamics of ecological systems. The effectiveness of the model depends very much on the intuition of the investigator at formulating a suitable research aspect.

**Key words:** bioindicative model, coenogenesis model, discrete model, ecological niche, ecological state, graph theoretical model, ontogeny model, succession series, synusia.

In the present paper an attempt is made to show that many ecological and biological problems can be interesting not only from the practical and theoretical points of view, but they can also serve as an incentive for the creation of new discrete mathematical methods (those methods, in their turn, may turn out to be interesting from the theoretical aspect).

It has been stated that discrete mathematical models are exactly suitable for describing and studying such “large” and “complex” systems as social, organizational, ecological, and biological ones (Roberts, 1976). The use of discrete methods is not necessarily the criterion for estimation, but these methods have been useful nevertheless.

The present paper presents one of the subtypes of discrete models — graph theoretical models, and illustrates their use giving some examples. Graph theory originates from the well-known problem of Königsberg bridges; afterwards the development of this theory has been stimulated by the research of chemical compounds structures, game analyses, problems of genetics and communications systems. In the present case it has been stimulated by the problems of ecological states structures and their dynamics.

Ecological phenomena often turn out to be extremely complicated and can be described only by a large number of difficult-to-determine variables and their interrelations. To solve these problems, it is often necessary to make substantial simplifications, presumptions, and abstractions, which makes simulation (i. e. modelling) of ecological phenomena to a certain extent subjective. However, this does not mean that the adequacy of the model is lost, provided that the investigator knows exactly from which aspect the ecological object is studied. As it is known, every system and phenomenon is many-sided.

If the necessary presumptions and abstractions are exactly formulated, or correspond fully to certain mathematical (e. g. graph theoretical) terms, ambiguity will be avoided and it will be possible to use the advantages of mathematical treatment.

The procedure which describes an ecological problem as mathematical or graph theoretical manifestations and later on analyses this problem with the help of mathematical or graph theoretical means, is called here mathematical or graph theoretical simulation.

\* Tallinn Botanical Garden, Kloostrimetsa Rd. 44, 200019 Tallinn, Estonia.



Before going on to the concepts of graph theoretical simulation of ecological objects, it seems to be necessary to explain this model from the graph theoretical aspect. It can be done as follows:

Simulation (modelling) is based on the semantic-syntactical interpretation of some graph theoretical problems ensuing from a more specific description of binary relations (i. e. actual and potential edges of the graph) by the so-called binary words. The initial task here is to determine the graph's node and edge orbits, from which the following well-known graph theoretical problems arise: complete system of graph invariants, graphs isomorphism, graphs reconstructions, and random graphs. Special attention has been paid to strongly regular graphs. This group of problems represents the static aspect of the model.

From the group of static problems new problems crop up: elementary variations of the graph and their genetics, algebraic problem of elementary variations, and the most important one — elementary variations as a dynamic system. These problems have not been treated much in graph theory.

There exists a direct relation between the randomness (existence probability), the information amount contained, and symmetry of the graph. Problems of elementary variations can be developed when the attributes of these variations are combined (or brought in conformity) with system-theoretical axiomatics.

These graph theoretical problems are presented in earlier papers by the present author (Tevet, 1987; 1990a).

The problem of discrete variations dynamics also exists in the structurality principle of theoretical biology and can be applied in various ecological models. In constructing these models, the main mathematical and system-theoretical postulates were taken into consideration.

To apply the above-formulated graph theoretical model concept in the case of an ecological system, it is necessary to proceed from two main principles:

**I. Observations of an ecological phenomenon (system) must be treated as states of this phenomenon (system).** It is essential to present the states structure in the form of a graph, whose nodes correspond to the list of the observed structural elements (e. g., species, individuals, factors, etc.) with necessary additional data, and edges to the relations between structural elements (e. g., contacts, neighbourhood, dependence, equivalence, sympathy, etc.). The structure represents a way of the elements organization or composition on the basis of their interrelations and corresponds to the isomorphic graphs class.

**II. Ecological system itself is treated as probable succession series of the states of this system.** This ensues from the fact that in an ecological system, which is a dynamic system, the states change continuously due to internal impacts (environmental impacts), i. e. in the case of the given discrete model it is its structure that changes. Therefore, the model of states succession series is a set of chains of structure variations.

Let us consider three applications of the graph theoretical model as examples. One is based on the relatively abstract ontogenesis process, the other two on observation data and are therefore stochastic.

**1. ONTOGENY MODEL.** The ontogeny hypothesis is based on the postulates on living system dynamics formulated by L. von Bertalanffy and N. Wiener, well-known in system theory, such as: a) organization (pro: discreteness) of living systems and its change in time; b) monotonous increase of the living system complexity and information amount until ripeness and their decrease after passing through ripeness; c) probability of reaching ripeness is much smaller than the probability of cessation; d) monotonous decrease of entropy; e) concentration of living system is maximum at its origin and zero at cessation.

All kinds of elementary variations of graphs, or, to be more exact, their structures, form a certain diagram of structure variations *DSV* in which all system-theoretical postulates are valid. In diagram *DSV* it is possible to determine structures which correspond to the extreme states of life systems — ORIGIN, RIPENESS, CESSATION. The characteristics men-



tioned above in ontogeny process postulates can be measured in structures and they are treated as output values  $Y$ . System inputs  $X$  are external impacts which annul the relations between structural elements, thus continuously changing the structure of living systems.

The problem of modelling lies in the determination of structural variations monotonous chains  $MCSV$  corresponding to the ontogeny hypothesis in diagram  $DSV$ . The number of such chains among various others is very small and they represent the ontogeny process model. In the case of this model the living system's structural elements and their interrelations are not fettered. The meaning of the model lies in the research and interpretation of the structural changes of states simulating an ontogeny process.

The ontogeny model has been presented in Tevet (1983; 1990b).

2. COENOGENESIS MODEL. The problem of the dynamics of synusia structures is based on the use of observation data. Each observation has been presented in the form of species and a graph depicting the relations between them, on their basis the structure of the coenogenetical state of the given synusia component will be determined. In the case of relatively ideal environmental conditions the following assumption could be used as a hypothesis: in the course of the coenogenesis process the total coverage increases. This could serve as basis for sequencing coenogenetic states.

Possible neighbour states can be determined on the basis of a corresponding succession function — maximum coincidence of structures. Such dynamic series of neighbour states chains form a coenogenetic aspect model of the studied synusiae. The output values  $Y$  of this model are structural complexity  $\eta$  of the synusia and evolutionary (i. e. life or growth form) complexity  $\mu$ . Argument characteristics  $X$  are represented by coenogenetic states structures and their corresponding coenogenesis discrete stages  $\{t\}$ .

Such a discrete model makes it possible to study changes in the synusia structure in the course of coenogenesis in the phase space  $R(\eta, \mu, t)$  and to determine the corresponding ecological niches. For a description of such a coenogenetic model see Martin, Tevet (1988); Tevet, Martin (1986; 1987; 1988).

3. BIOINDICATIVE MODELS. Bioindicative problems based on observation data may be formulated in different ways. The starting point, similarly to the previous model, is the determination of states structures and characteristics corresponding to them. By formulating various investigation aspects and setting different hypotheses, it is possible, with the help of graph theoretical models, to analyse the relations between environmental pollution (e. g. by the content of chemicals in substrate), association structure, pH content, and coverage and to find the corresponding ecological niches.

Such an approach has been used in Martin (1981); Мартин (1984; 1985a, 1985b); Nilson (1988). In these works the problem has not yet been studied from the structural-analytical aspect, but the experience of using the coenogenesis model suggests that discrete models are justified also in the case of bioindicative problems.

As can be seen, the use of discrete graph theoretical models in the case of ecological objects opens new aspects for investigating the structure and dynamics of an ecological system.

From the theoretical point of view the ontogenesis process model is the most correct and general, but its interpretation for a concrete living system type remains obscure. The coenogenesis model is the most "practical" as it interprets empirical data; however, it is not easy to find theoretically correct succession functions between random states. This is evidently also true about bioindicative models.



Graph theoretical models have so far been implemented in ecology only proceeding from the structural-static aspect. For example, oriented and interval graphs have been used to study species competition, nutrition schemes and chains, and trophical status (Wilson, Bossert, 1971) — thus, traditionally from the aspect of communication networks analysis. The present graph theoretical modelling (simulation) concept which is based on structurality principle and structural dynamics is evidently different from the earlier ones. This concept has not yet been widely accepted, but it has not got any principal resistance either.

It could be said for conclusion that the structural (pro: graph theoretical) characteristics of ecological phenomena make it possible to distinguish various stages of ecological processes and ecological niches, while the changes of characteristics and the relations between them enable to determine ecological regularity. The effectiveness of the model depends very much on the intuition of the investigator at formulating a suitable research aspect.

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Presented by J. Martin, D. Sc.,  
Member of the Estonian Academy of Sciences

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