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ISOQUANTA AND THE STRUCTURE OF INTERRELATIONS BETWEEN LAKE CHARACTERISTICS OBTAINED BY THEIR MEANS

Up to the present time, the criteria characteristic of lakes have been based on grouping and classification. Good results have been obtained by the classification of 132 Estonian lakes using the method of principal coordinates and principal components (Mäemets, Raitviir, 1977).

The purpose of the present work is a profound analysis of interrelations between the main criteria typical of lakes, i. e. the complex interrelations of some criteria groups to the behaviour of other ones.

The reflection of a priori tendencies in intercriterion dependences by correlation coefficients, regression equations and dispersion estimation are widely used. Such patterns of dependences may prove inadequate because they do not correspond to the real condition (or situation), i. e. to the real combination of criteria in the given phenomenon.

The general principles of the method are given by J. T. Teevet (Тээвет, Кяби, 1982). The term "isoquantum" has been taken over from modern economic investigations (Vensel, 1979). In case of analogous patterns some authors use the term "isoline" (Вознесенский, 1981).

This method has been successfully used in several fields of investigation, such as the process of solubilization of cancerogenic substances in cells (Teevet, Krasnoštšokova, 1983), the influence of medicines and the postoperative treatment of patients (Planken, Teevet, 1984), the classification of the marine islets (Rebassoo, 1984), the analysis of the influence of microclimate (Бударин et al., 1981), the rheology of materials (Кяби, Тээвет, 1982).

The method of isoquanta is compiled on the basis of principles of regression analysis, dispersion analysis and some principles of cluster analysis. The key point of the method is the decomposing of factor space into situation clusters, which permits to get additional information about the dependences.

In that way, the given method differs from

a) the traditional regression analysis because we do not try to form a comprehensive smooth regression equation but aim at forming an assemblage of autonomous isoquanta;

b) the traditional dispersion analysis, because we do not try to estimate the general influence but to determine the concrete values of regression functions and to estimate them;

c) the traditional cluster analysis, because we do not try to find continuous homogeneity domains but to decompose space into situation clusters.

To proceed from the real situation of lake characteristics, we have made an attempt to reveal statistically the structure of interrelations between the criteria.

We have chosen the following ten criteria typical of lakes: (1) average depth (m); (2) transparency of water in summer (m); (3) the HCO'_3

content (mg/l) in the surface layer; (4) oxygen saturation (%) of the bottom water in summer; (5) number of macrophyte species; (6) degree of water-bloom intensity (0, 1, 2, 3); (7) number of fish species; (8) amount of *Daphnia cucullata* (0, 1, 2, 3, 4, 5); (9) amount of *Bosmina longispina* (0, 1, 2, 3, 4, 5); (10) amount of *Chydorus sphaericus* (0, 1, 2, 3, 4, 5).

To analyse the structure of complex interrelations these criteria have been divided into aspects of investigation. Each of these aspects represents two sets of criteria, one of which is called the input or the X-criterion, while the other one is called the output or the Y-criterion.

Within the bounds of each investigation aspect, the interrelations between the X- and the Y-criteria (or the influence upon the Y-criteria) $F(X_1, .., X_n, .., X_N) = Y_1, .., Y_m, .., Y_M$ have been analysed. No restrictions have been placed upon such aspects of investigation, but a proper interpretation of the facts is needed.

We have formulated the following aspects of investigation:

1. The aspect of the main criteria. The most significant criteria identified by the method of main components serving as X-criteria: the HCO'_3 content in the surface layer $X^{(3)}$ and the average depth $X^{(1)}$. Here the interrelations between this complex of X-criteria and the following Y-criteria have been revealed: the number of macrophyte species $Y^{(5)}$; the degree of water-bloom intensity $Y^{(6)}$; the number of fish species $Y^{(7)}$; the amount of *Daphnia cucullata* $Y^{(8)}$; the amount of *Bosmina longispina* $Y^{(9)}$; the amount of *Chydorus sphaericus* $Y^{(10)}$

$$F[X^{(3)}, X^{(1)}] = Y^{(5)}, Y^{(6)}, Y^{(7)}, Y^{(8)}, Y^{(9)}, Y^{(10)}.$$

2. The aspect of the zooplankton. The input criteria are the amount of *Daphnia cucullata* $X^{(8)}$, the amount of *Bosmina longispina* $X^{(9)}$ and the amount of *Chydorus sphaericus* $X^{(10)}$. Here the interrelations between the set of zooplankton and the following Y-criteria have been analysed: average depth $Y^{(1)}$; transparency of water in summer $Y^{(2)}$; the HCO'_3 content in surface layer $Y^{(3)}$; number of macrophyte species $Y^{(5)}$; number of fish species $Y^{(7)}$; the degree of water-bloom intensity $Y^{(6)}$

$$F[X^{(8)}, X^{(9)}, X^{(10)}] = Y^{(1)}, Y^{(2)}, Y^{(3)}, Y^{(5)}, Y^{(6)}, Y^{(7)}.$$

3. The aspect of selective criteria. The input criteria consist of the HCO'_3 content in the surface layer $X^{(3)}$, the degree of water-bloom intensity $X^{(6)}$ and the amount of *Daphnia cucullata* $X^{(8)}$. Hereby we have revealed the interrelations between the X-criteria and the following Y-criteria: the average depth $Y^{(1)}$; transparency of water in summer $Y^{(2)}$; oxygen saturation of the bottom water in summer $Y^{(4)}$; number of macrophyte species $Y^{(5)}$; the amount of *Chydorus sphaericus* $Y^{(10)}$

$$F[X^{(3)}, X^{(8)}, X^{(6)}] = Y^{(1)}, Y^{(2)}, Y^{(4)}, Y^{(5)}, Y^{(10)}.$$

The general structure of the dependences between the criteria is given as an assemblage of isoquanta. An isoquantum $R^{C(X)}W$ is called a regressive curve with one independent X-variable where the other values of its X-criteria have been fixed at certain levels. The formation of isoquanta is basically the levelling of the values of X-criteria, while the optimal number of the levels K_ξ within the bounds of the investigation aspect can be found by the Stergess formula $K_\xi = 3.322 \log I + 1 = 3.322 \log 132 + 1 \approx 8$, where $I = 132$ is the number of objects (lakes).

Proceeding from the aspects formulated above, we can determine the levels of X-criteria as shown in Table.

For every investigation aspect, by levelling the input criteria, the N-dimensioned factor space $R(X)$ will be decomposed into N-dimensional subspaces $C(X)$, $C(X) \subset R(X)$.

Level	Criteria					
	HCO′$_3$ content ($X^{(3)}$)	Average depth ($X^{(1)}$)	Amount of *D. cucullata* ($X^{(8)}$)	Amount of *B. longispina* ($X^{(9)}$)	Amount of *Ch. sphaericus* ($X^{(10)}$)	Degree of water-bloom intensity ($X^{(6)}$)
$k=1$	low content up to 60 mg/l	low level up to 2 m	low level 0	low level 0	low level 0—1	low intensity 0—1
$k=2$	below average over 60 up to 120 mg/l	average level over 2 up to 4 m	average level 1—3	average level 1—2	average level 2—3	high intensity 2—3
$k=3$	above average above 120 up to 180 mg/l	high level above 4 m	high level 4—5	high level 3—5	high level 4—5	
$k=4$	high content over 180 mg/l					

These subspaces, limited by the level surfaces containing real objects, are called situation clusters $C(X)$.

Thus the situation clusters $C(X)$ are formed within the bounds of X-criteria of lakes with identical levels or, in other words, the lakes in one and the same subspace $C(X)$ are considered to be in an identical situation or condition.

The identification attribute of a situation cluster $C(X)$ is a situation vector $<XK> = <k_1, \dots k_n, \dots k_N>$, where k_n is the number of the level of the nth X-criterion. Thus the situation vector represents the coordinates of the corresponding subspace. For example, the situation vector of the aspect of the influence of zooplankton $<XK> = <2.1.3>$, characterizing the situation cluster which is composed of the average number of *Daphnia cucullata*, a small number of *Bosmina longispina* and a large number of *Chydorus sphaericus*.

To create a basis for forming isoquanta, the conditional expectation of output values (or regression functions) $M^{C(X)}Y$ of the situation cluster $C(X)$ is created with the set $\{w\}$:

$$w_j = \begin{cases} M^{C(X)}Y_1 = F_1[C_j(X)]; \\ \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \\ M^{C(X)}Y_m = F_m[C_j(X)]; \\ \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \\ M^{C(X)}Y_M = F_M[C_j(X)], \end{cases}$$

where $j \in [1, J]$ is the number of the situation cluster. Thus for example: the set of the regression functions of the clusters of the aspect of main criteria is

$$w_j = \begin{cases} M^{C(X)}Y^{(5)} = F^{(5)}[C_j(X^{(3)}, X^{(1)})]; \\ \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \\ M^{C(X)}Y^{(10)} = F^{(10)}[C_j(X^{(3)}, X^{(1)})]. \end{cases}$$

The set $\{w\}$ belongs to the behaviour or phase space $R(Y)$ of the output criteria $\{w\} \subset R(Y)$. Thus, every situation cluster $C(X)$ of the factor space $R(X)$ has only one corresponding point w in the phase space $R(Y)$.

The optimization of the system clusters takes place by means of minimizing the innercluster dispersions $D^{C(X)}X$ and the conditional dispersion of the outputs $D^{C(X)}Y$.

To estimate the reliability of the situation cluster $C(X)$ and its conditional expectation $M^{C(X)}Y$ we shall use the following criteria:

1) The statistical convergence of the output of the cluster $C(X)$

$$Q^{C(X)}Y = \sqrt{\left|1 - \frac{D^{C(X)}Y}{DY}\right|}.$$

where $D^{C(X)}Y > DY$ in case $Q = -Q$, and this shows that the output is not converging. Here DY is the general dispersion of the given criteria.

2) As the situation clusters $C(X)$ are obtained by means of decomposing the factor space $R(X)$, they are obviously conventional subsamples of a more general sample, and their dispersions $D^{C(X)}X$ do not depend on the dispersion of a more general sample DX. The fact that every $C(X)$ has a corresponding subsample of output values $\{Y_j\}$ of a more general sample $\{Y_j\} \subset \{Y\}$ implies the independence between the output dispersions $D^{C(X)}Y$ and DY. Thus, it can be concluded that the variations of the output also follow Fisher's F-distribution: $F = DY : D^{C(X)}Y$. By means of these critical values the confidence limits of the output values Y_j can be found. If the output values of clusters are reliable, the values of the corresponding regression functions can also be considered true.

The curve connecting the corresponding regression functions w, w' of the chain of the neighbouring clusters is thus the isoquantum $R^{C(X)}W$. We call neighbouring clusters such a pair of clusters as $C(X)$, $C'(X)$, the situation vectors $<XK>$, $<XK>'$ of which within the limits of one X-criterion (i. e. co-ordinate) differ from each other by just one level. The interval of an isoquantum e, $e \in R^{C(X)}W$ between the points w, w' is directed from the lower level to the higher one. In other words: an isoquantum is formed by such intervals $e_1 \cup e_2 \cup \ldots = R^{C(X)}W$ connecting the points w where the level of one X-variable changes. The reliability of an isoquantum is determined by the reliability coefficient and the statistical convergence Q of the output values $Y_{j,m}$ of the situation clusters $C(X)$ forming it.

Isoquanta and the cluster system for their formation make up a model of the investigated phenomenon presented in the form of the graph $\Gamma(w, e)$ within the behaviour or phase space $R(Y)$ where the set of nodes $\{w\}$ in its turn is formed of the reliable regression functions $w = \{M^{C(X)}Y_m\}$ where

$$\forall C(X) = \varnothing, \quad C(X) \subset R(X) \rightarrow \exists! w, \quad w \in R(Y);$$

the set of arcs $\{e\}$ is a set of intervals connecting the reliable pairs of nodes w and w'.

Thus we can conclude:

a) In each situation cluster various data are presented, such as the list of objects (lakes), X-criterion levels of a cluster, the value of the regression function of its output value $M^{C(X)}Y$, and their reliability.

b) Each isoquantum $R^{C(X)}W$ contains information on variable and invariable X-criteria, on the reliability of the cluster which forms an isoquantum and the expected values of Y-criteria at every point, and the variation region of Y-criteria.

Further we shall discuss the structure of statistic regularities resulting from the isoquantum of the investigation aspects.

In the figures, numbers in the circles represent situation vectors, i. e. X-criterion levels. The continuous line marks the change of the first X-criteria, the broken line — the second X-criteria and the point-dash line — the change of the third X-criteria.

1. The aspect of the main criteria

The aspect of the main criteria represents an aspect of analysis considering the relation between the main criteria and the number of macrophyte and fish species, the water-bloom intensity, and the zooplankton.

1.1. Relation to the number of macrophyte species (Fig. 1)

$$Y^{(5)} = F^{(5)}[C(X^{(3)}, X^{(1)})]$$

A monotonous relation between the number of macrophyte species and the rise in the HCO$'_3$ content, and a certain nonlinear relation with respect to the depth is noticeable. A positive relation between the HCO$'_3$ content (or the total number of ions) and the number of macrophyte species has been determined earlier (Mäemets, Lokk, 1982; Мяэметс, 1981). The minimum number of macrophyte species exists in cases when the low HCO'_3 content is accompanied by the medium or large depth of the lake (usually the littoral zone is then narrow and there is little room for plants). The maximum number of species occurs with the medium depth if the HCO'_3 content is medium or high. In the case of a small or medium depth the increase in the number of macrophyte species is connected with the rise in the HCO'_3 content, while its maximum occurs in lakes of medium depth. In the case of deep lakes the increase in the number of macrophyte species is connected with the rise in the HCO'_3 content up to the level below average. At a further rise in the HCO'_3 content the number of macrophyte species does not increase. In shallow lakes the number of macrophyte species is also connected with the HCO'_3 content but it remains smaller than in lakes of medium or large depth.

1.2. Relation to the water-bloom intensity (Fig. 2)

$$Y^{(6)} = F^{(6)}[C(X^{(3)}, X^{(1)})]$$

There is a general tendency that with the rising HCO'_3 content also the water-bloom intensity grows, being far from linear or monotonous but in accordance with the depth of the lake. The minimum water-bloom occurs in conditions where the HCO'_3 content is low and the depth of the lake is large (i. e. in oligo- and mesotrophic lakes); the maximum water-bloom takes place in cases when the HCO'_3 content in deep lakes is above the average level, or also in shallow lakes if the HCO'_3 content is high. If the depth of the lake is small or medium, the rise in the HCO'_3 content is accompanied by an increase of the water-bloom intensity. In deep lakes the rise in the HCO'_3 content above the average level is accompanied by a peak in the water-bloom intensity, while a further rise in the HCO'_3 content causes a fall in the water-bloom intensity. If the HCO'_3 content is above average, the water-bloom is the more intensive, the deeper the lake. If the HCO'_3 content is high, the water-bloom is the more intensive, the shallower the lake.

It is not sufficiently known how the water-bloom intensity is directly influenced by the HCO'_3 content and the lake depth, since the water-bloom intensity is known to depend above all on the amount of nutrients in the water and less likely on the HCO'_3 content and lake depth. At the same time it is known that in our conditions water-bloom does not occur in very HCO'_3-rich (HCO'_3 content above 200) lakes of medium depth (e. g. in alkalitrophic Äntu lakes). It is also known that Ca ions, as a rule, demobilize P and Fe in water (Pejler, 1965). On the other hand, it is evident that in the case of slow eutrophication of the Estonian lakes during earlier periods the rise in the HCO'_3 content and the increase in the concentration of nutrients were, in fact, proportional quantities. At the present time strong anthropogenic eutrophication has considerably spoilt this balance, and the accumulation of nutrients has become more rapid, giving rise to the so-called soft-water eutrophic lakes (Mäemets, Lokk, 1982), and thus changing the above-mentioned regularity between the rise in the HCO'_3 concentration and the water-bloom intensity. The depth of the lake influences the cyclic nature of water-bloom and its intensity evidently through the stratification level which determines the character of nutrient circulation and especially its cyclic nature. In highly stratified lakes large amounts of nutrients rise from the bottom layers to the upper ones only during the spring and autumn overturn, while in shallow lakes the cyclic nature is much less developed.

1.3. Relation to the number of fish species (Fig. 3)

$$Y^{(7)} = F^{(7)}[C(X^{(3)}, X^{(1)})]$$

The number of fish species increases quite monotonously mainly with the rise in the HCO'_3 content, as has been stated earlier (Мяэметс, 1981). The minimum number of fish species occurs with a low HCO'_3 content (such lakes usually have acid water as well), while the lake depth does not play a great role here. The maximum number of fish species is observed in circumstances where the HCO'_3 content is high and the lake is at least of a medium depth. Shallow and medium-depth lakes, whose HCO'_3 content is below the average, contain usually more fish species than the corresponding large-depth lakes. On the contrary, the shallow lakes whose HCO'_3 content is above the average, usually contain fewer fish species than the corresponding medium or large-depth lakes.

Of course, it is not probable that the HCO'_3 content itself might directly influence the number of fish species in the lake. The limiting factor could be the pH of water which depends largely on the HCO'_3 content. The role of pH in determining the number of fish species in the lake has been exaggerated in literature (Rahel, Magnusson, 1983); several other factors should also be considered (Tonn, Magnusson, 1982; Tonn et al., 1983).

1.4. Relation to the amount of *Daphnia cucullata* (Fig. 4)

$$Y^{(8)}=F^{(8)}[C(X^{(3)}, X^{(1)})]$$

It is evident that the amount of *D. cucullata* as the indicator of mesotrophic and eutrophic lakes depends quite monotonously on the HCO'_3 content of water and the lake depth. The minimum amount of *D. cucullata* occurs in shallow lakes with a low or below-average HCO'_3 content (i. e. in non-stratified soft-water lakes). The maximum amount of *D. cucullata* occurs in cases when the high HCO'_3 content is accompanied by the medium or large depth of the lake. In medium and deep lakes the rise in the HCO'_3 content brings about a direct increase in the amount of *D. cucullata*; in shallow lakes the amount of *D. cucullata* starts rising only when the HCO'_3 content is at least at the below-average level. The sharpest rise in the amount of *D. cucullata* in deep lakes takes place when the HCO'_3 content rises from the low level up to the below-average, while with the further rise the abundance falls. In cases when the HCO'_3 content is at the below- or above-medium level, the amount of *D. cucullata* rises together with the increasing depth of the lake. In the case of the low and high HCO'_3 content the amount of *D. cucullata* increases only with the increase of the depth up to the average degree. The sharpest rise in the amount of *D. cucullata* due to the lake depth occurs in circumstances when shallow lakes are transformed into lakes of the average depth. A general positive correlation between the abundance of *D. cucullata* and the HCO'_3 content has been also noted earlier (Мяэметс, 1981).

1.5. Relation to the amount of *Chydorus sphaericus* (Fig. 5)

$$Y^{(10)}=F^{(10)}[C(X^{(3)}, X^{(1)})]$$

The structure of the dependence of *Ch. sphaericus,* a well-known indicator of strong eutrophication, on the HCO'_3 content and lake depth is rather vague. In places the amount of *Ch. sphaericus* is well and reliably correlated both with HCO'_3 and the lake depth. The minimum number of *Ch. sphaericus* occurs with a low HCO'_3 content and great depth, which is quite logical. With extreme HCO'_3 values (low and high levels) the amount of *Ch. sphaericus* is related to the lake depth: the deeper the lake, the smaller the number of *Ch. sphaericus*. A reliable linear relation between HCO'_3 and the amount of relation between HCO'_3 and the amount of *Ch. sphaericus* is found in two cases:

(a) in shallow lakes where HCO'_3 rises from the below-average up to the

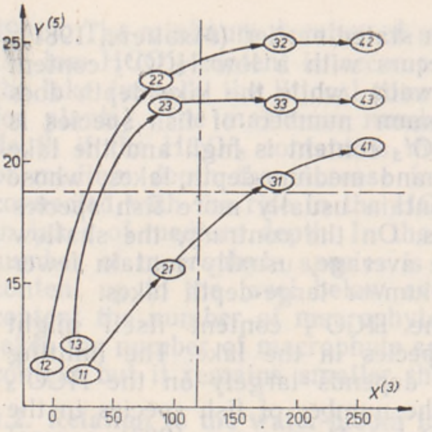


Fig. 1. Relation of the main criteria to the number of species macrophyte.

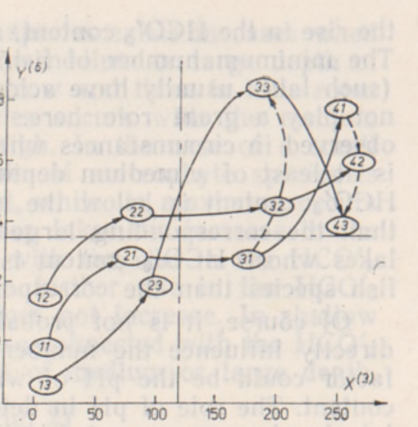


Fig. 2. Relation of the main criteria to the water-bloom intensitv.

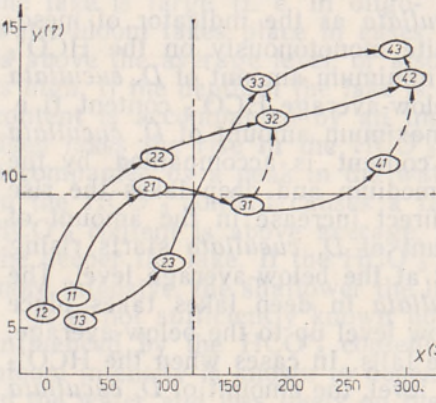


Fig. 3. Relation of the main criteria to the number of fish species.

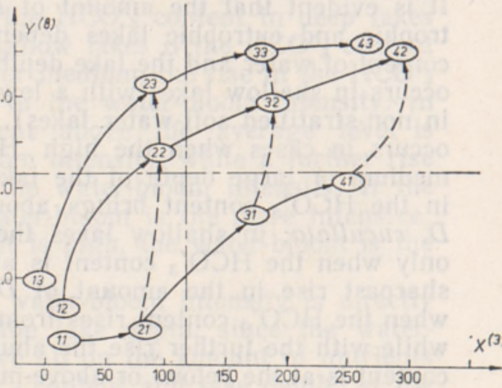


Fig. 4. Relation of the main criteria to the amount of *Daphnia cucullata*.

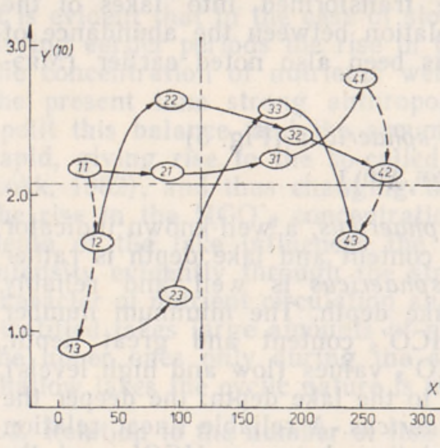


Fig. 5. Relation of the main criteria to the amount of *Chydorus sphaericus*.

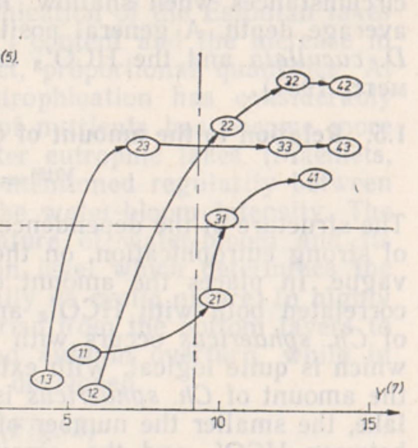


Fig. 6. Compatible relation of the main criteria to the number of macrophyte and fish species.

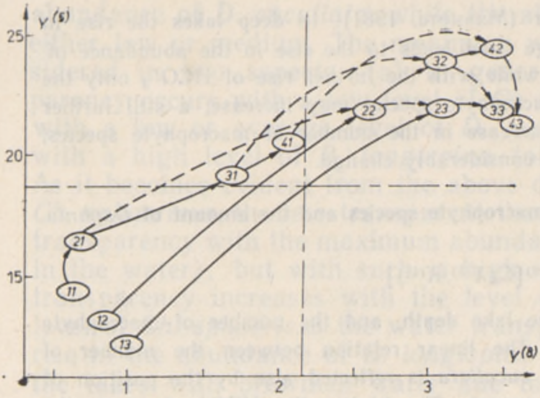
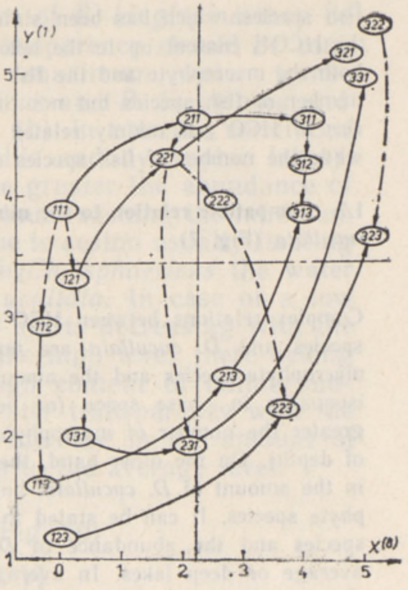


Fig. 7. Compatible relation of the main criteria to the number of macrophyte species and the amount of *Daphnia cucullata*.

Fig. 8. Relation of the amount of zooplankton to the lake depth. ▶



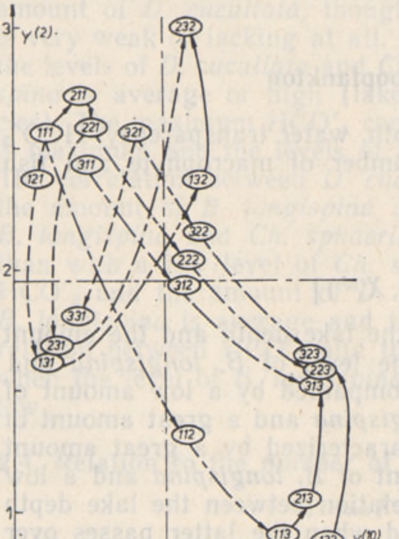


Fig. 9. Relation of the amount of zooplankton to the water transparency.

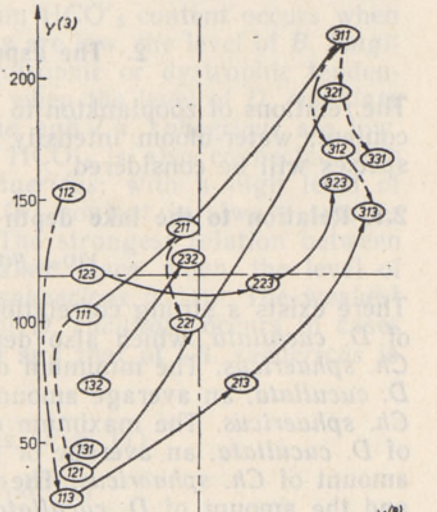


Fig. 10. Relation of the amount of zooplankton to the HCO'_3 content of water.

high level; (b) in deep lakes where HCO'_3 rises from the low up to the above-average level.

1.6. Compatible relation to the number of macrophyte and fish species (Fig. 6)

$$Y^{(5;\,7)} = F^{(5;\,7)}[C(X^{(3)}, X^{(1)})]$$

A strong linear relation between the macrophyte and the fish species is especially well revealed in lakes of the average depth. As a general logical rule, the rise in the HCO'_3 content is accompanied by the increase in the number of both the macrophyte and the fish species, which has been stated earlier (Мяэметс, 1981). In deep lakes the rise in the HCO'_3 content up to the below-average level leads to the rise in the abundance of both the macrophyte and the fish species, while with the further rise of HCO'_3 only the number of fish species but not that of macrophyte species may increase; a still further rise of HCO'_3 is mainly related to the increase in the number of macrophyte species, while the number of fish species does not considerably change.

1.7. Compatible relation to the number of macrophyte species and the amount of ***Daphnia cucullata*** (Fig. 7)

$$Y^{(5;\,8)}=F^{(5;\,8)}[C(X^{(3)},X^{(1)})]$$

Complex relations between HCO'_3 and the lake depth, and the number of macrophyte species and *D. cucullata* are noticeable. The linear relation between the number of macrophyte species and the amount of *D. cucullata* is reflected also by the position of isoquanta in phase space (or behaviour space). The higher the HCO'_3 content, the greater the number of macrophyte species and the amount of *D. cucullata* (irrespective of depth). On the other hand, the increase in the depth is somewhat related to the rise in the amount of *D. cucullata*, but the latter does not depend on the number of macrophyte species. It can be stated that in case of shallow lakes the number of macrophyte species and the abundance of *D. cucullata* are always smaller than in the case of average or deep lakes. In average and deep lakes the number of macrophyte species remains at the same level or even decreases while the abundance of *D. cucullata* rises still more with the increase in HCO'_3.

2. The aspect of the zooplankton

The relations of zooplankton to the lake depth, water transparency, HCO'_3 content, water-bloom intensity and the number of macrophyte and fish species will be considered.

2.1. Relation to the lake depth (Fig. 8)

$$Y^{(1)}=F^{(1)}[C(X^{(8)},X^{(9)},X^{(10)})]$$

There exists a strong correlation between the lake depth and the amount of *D. cucullata*, which also depends on the level of *B. longispina* and *Ch. sphaericus*. The minimum depth is accompanied by a low amount of *D. cucullata*, an average amount of *B. longispina* and a great amount of *Ch. sphaericus*. The maximum depth is characterized by a great amount of *D. cucullata*, an average or great amount of *B. longispina* and a low amount of *Ch. sphaericus*. The strongest relation between the lake depth and the amount of *D. cucullata* is revealed when the latter passes over from the average to the high level, if the level of *B. longispina* is high and that of *Ch. sphaericus* low. The weakest relation between the lake depth and the amount of *D. cucullata* occurs in cases when the levels of both *B. longispina* and *Ch. sphaericus* are low. At every fixed level of *D. cucullata* the increase in depth is generally related with the fall in the abundance of *B. longispina* or *Ch. sphaericus*. It is, however, possible that in the case of the average level of *D. cucullata* and high level of *B. longispina* the increase in depth brings about also a rise in the amount of *Ch. sphaericus*.

2.2. Relation to the water transparency (Fig. 9)

$$Y^{(2)}=F^{(2)}[C(X^{(8)},X^{(9)},X^{(10)})]$$

There is a dominating relation between the rise in water transparency and the fall in the amount of *Ch. sphaericus*. The water transparency is minimum with a great abundance of *Ch. sphaericus* and a low abundance of *D. cucullata*, while the abundance of *B. longispina* may be either low or medium. The maximum water transparency should be considered in two aspects: (a) in general, the maximum water transparency occurs with a low level of *Ch. sphaericus* and *B. longispina* and with a low or average level of *D. cucullata*; (b) in extreme conditions with a high level of *B. longispina* (oligotrophic and dystrophic lakes). As it becomes evident from the above data, the greater the abundance of *Ch. sphaericus*, the less transparent the water, thus reaching the minimum transparency with the maximum abundance (due to seston usually floating in the water), but with such a high level of *Ch. sphaericus* the water transparency increases with the level of *D. cucullata*. In case of a low level of *Ch. sphaericus* the water transparency starts decreasing with the rise in the abundance of *B. longispina* up to the high level (here belong the lakes with brownish water due to the high content of humic substances). An exceptional case is the rise in water transparency with the high level of *B. longispina* together with an increase in the amount of *D. cucullata* or *Ch. sphaericus* from the low up to the average level.

2.3. Relation to the HCO'_3 content of water (Fig. 10)

$$Y^{(3)} = F^{(3)}[C(X^{(8)}, X^{(9)}, X^{(10)})]$$

There exists a dominating relation between the HCO'_3 content and the amount of *D. cucullata*, though there are also cases when this relation is very weak or lacking at all. The minimum HCO'_3 content occurs when the levels of *D. cucullata* and *Ch. sphaericus* are low, the level of *B. longispina* is average or high (lakes with oligotrophic or dystrophic tendencies). The maximum HCO'_3 content occurs when the level of *D. cucullata* is maximum and the levels of *B. longispina* and *Ch. sphaericus* are low. The correlation between *D. cucullata* and HCO'_3 is also connected with the amount of *B. longispina* and *Ch. sphaericus*: with a high level of *B. longispina* and *Ch. sphaericus* the HCO'_3 content is always smaller than with a low level of *Ch. sphaericus*. The strongest relation between HCO'_3 and the amount of *D. cucullata* takes place when the level of *B. longispina* is average and that of *Ch. sphaericus* is low. The weakest relation between HCO'_3 and the amount of *D. cucullata* occurs in cases when the level of *B. longispina* is average and that of *Ch. sphaericus* is low.

2.4. Relation to the number of macrophytes (Fig. 11)

$$Y^{(5)} = F^{(5)}[C(X^{(8)}, X^{(9)}, X^{(10)})]$$

There is a dominating relation between the number of macrophyte species and the rise in the amount of *D. cucullata* (in exceptional cases this relation does not hold). The minimum number of macrophytes has been fixed in cases when the levels of *D. cucullata* and *Ch. sphaericus* are low, the level of *B. longispina* is at least average or high (in dyseutrophic lakes). The maximum number of macrophyte species occurs with a high level of *D. cucullata*, an average level of *B. longispina* and extreme (low or high) level of *Ch. sphaericus*. With a low level of *D. cucullata* the increase in the abundance of *B. longispina* is accompanied by a decrease in the number of macrophyte species. With higher levels of *D. cucullata* there appears a fall in the number of macrophyte species when the level of *B. longispina* is average or high. In both cases mainly dyseutrophic lakes are concerned. In circumstances where the level of *B. longispina* is low, the number of macrophyte species does not increase or increases very slightly with a rise in the amount of *D. cucullata* (regardless of the level of *Ch. sphaericus*). In case of an average or high level of *B. longi-*

spina the number of macrophyte species is directly related to the rise in the amount of *D. cucullata.*

2.5. Relation to the water-bloom intensity (Fig. 12)

$$Y^{(6)}=F^{(6)}[C(X^{(8)}, X^{(9)}, X^{(10)})]$$

The relations between the rise in the amount of *D. cucullata* and *Ch. sphaericus* and the growth in the water-bloom intensity can be considered as dominating. The minimum water-bloom intensity has been stated in cases where the level of *D. cucullata* is low, the level of *B. longispina* being high, and that of *Ch. sphaericus* the average (mainly in lakes with mesotrophic tendencies). The maximum water-bloom intensity occurs again in two aspects: (a) as a general tendency it is stated in cases when the levels of *D. cucullata* and *Ch. sphaericus* are high while that of *B. longispina* is low or average; (b) an extremely strong water-bloom intensity is stated at low levels of *D. cucullata* and *B. longispina* and at a high level of *Ch. sphaericus* (in strongly eutrophic lakes rich in colonies of blue-green algae). The strongest relation between the water-bloom intensity and the amount of *D. cucullata* occurs when the level of *B. longispina* is average or high. The relation between the water-bloom intensity and the level of *Ch. sphaericus* depends in turn on the abundance of *B. longispina*: (a) at a low level of *B. longispina* the relation between the amount of *Ch. sphaericus* and the water-bloom intensity is strongest; (b) at the average and high level of *B. longispina* this relation is either weaker or does not exist at all. At a low level of *B. longispina* the amount of *D. cucullata* is related with the water-bloom intensity through the number of *Ch. sphaericus*: (a) at a low level of *Ch. sphaericus* the rise in the amount of *D. cucullata* may bring about some increase in the water-bloom intensity; (b) at the average level of *Ch. sphaericus* the mentioned relation is not revealed; (c) at a high level of *Ch. sphaericus* the rise in the amount of *D. cucullata* is even accompanied by a fall in the water-bloom intensity.

2.6. Relation to the number of fish species (Fig. 13)

$$Y^{(7)}=F^{(7)}[C(X^{(8)}, X^{(9)}, X^{(10)})]$$

In cases when the level of *B. longispina* is low or average, there exists a dominating relation between the rise in the amount of *D. cucullata* and the number of fish species. The minimum number of fish species occurs in circumstances when the amount of *D. cucullata* and *Ch. sphaericus* is small, while that of *B. longispina* is average or great. The maximum number of fish species occurs when the level of *D. cucullata* is high and the level of *B. longispina* is average, while the amount of *Ch. sphaericus* does not play an essential role (mesotrophic lakes). At a high level of *B. longispina* accompanied by an increase in the amount of *D. cucullata* up to the medium level, also the number of fish species increases; with a further rise in the amount of *D. cucullata* the number of fish species may fall sharply. At a low level of *D. cucullata* some increase in the number of fish species can be observed also in cases when the level of *Ch. sphaericus* is high and the level of *B. longispina* rises from a low up to the average level (at a high level of *D. cucullata* the level of *Ch. sphaericus* does not play any role). The relation between the number of fish species and the amount of *D. cucullata* is strongest when the level of *B. longispina* is average and the level of *Ch. sphaericus* is low. The relation between the number of fish species and the level of *D. cucullata* is weakest in circumstances when the levels of both *B. longispina* and *Ch. sphaericus* are low.

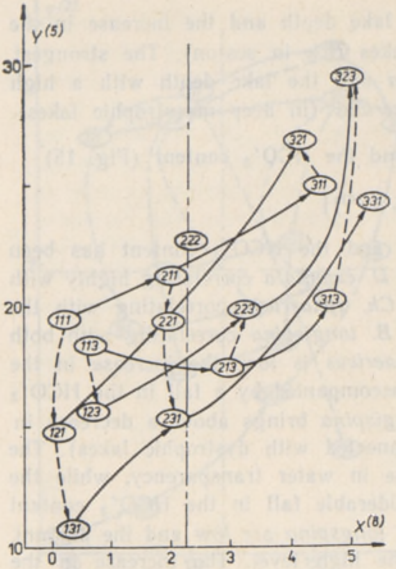


Fig. 11. Relation of the amount of zooplankton to the number of macrophytes.

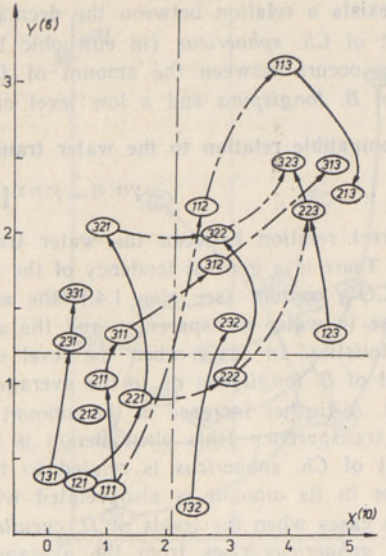


Fig. 12. Relation of the amount of zooplankton to the water-bloom intensity.

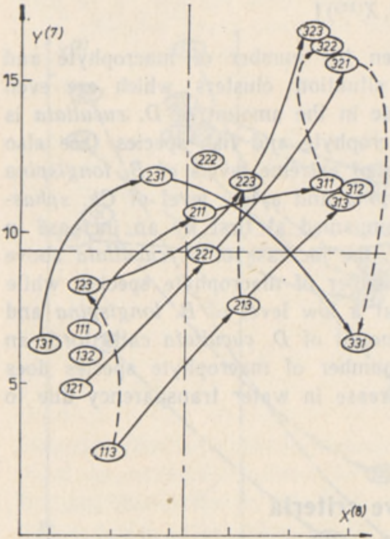


Fig. 13. Relation of the amount of zooplankton to the number of fish species.

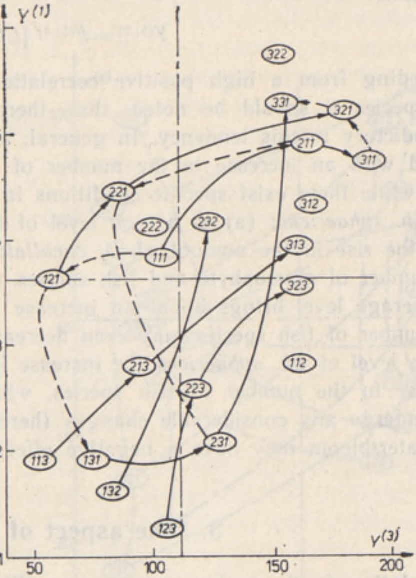


Fig. 14. Compatible relation of the amount of zooplankton to the main criteria.

2.7. Compatible relation to the main criteria (Fig. 14)

$$Y^{(1;\,3)} = F^{(1;\,3)}\left[C\left(X^{(8)}, X^{(9)}, X^{(10)}\right)\right]$$

The position of isoquanta gives evidence about the fact that there exists a certain relation between the lake depth and the HCO′$_3$ content. Almost in all circumstances there is a considerable relation between the increase in the amount of *D. cucullata* on the one hand, and the lake depth and the HCO′$_3$ level on the other. Nearly always there exists a relation between the decrease in the lake depth and the increase in the amount of *Ch. sphaericus* (in eutrophic lakes or lakes rich in seston). The strongest relation occurs between the amount of *D. cucullata* and the lake depth with a high level of *B. longispina* and a low level of *Ch. sphaericus* (in deep mesotrophic lakes).

2.8. Compatible relation to the water transparency and the HCO′₃ content (Fig. 15)

$$Y^{(2;3)} = F^{(2;3)}[C(X^{(8)}, X^{(9)}, X^{(10)})]$$

No direct relation between the water transparency and the HCO′₃ content has been found. There is a general tendency of the amount of *D. cucullata* correlating highly with the HCO′₃ content (see also 1.4.), the amount of *Ch. sphaericus* correlating with the decrease in water transparency, and the amount of *B. longispina* correlating with both characteristics. In cases when the level of *Ch. sphaericus* is low, the increase in the amount of *B. longispina* up to the average level is accompanied by a fall in the HCO′₃ content. A further increase in the amount of *B. longispina* brings about a decrease in water transparency (this phenomenon is mainly connected with dystrophic lakes). The amount of *Ch. sphaericus* is related to the decrease in water transparency, while the increase in its amount is also related with a considerable fall in the HCO′₃ content only in cases when the levels of *D. cucullata* and *B. longspina* are low and the amount of *Ch. sphaericus* rises from the average up to the high level. The increase in the amount of *D. cucullata* is related to the increase in the HCO′₃ content, while it is not directly related to water transparency (with some exceptions at a high level of *Ch. sphaericus*).

2.9. Compatible relation to the number of macrophyte and fish species (Fig. 16)

$$Y^{(5;7)} = F^{(5;7)}[C(X^{(8)}, X^{(9)}, X^{(10)})]$$

Proceeding from a high positive correlation between the number of macrophyte and fish species it should be noted that there exist situation clusters which are even contradictory to this tendency. In general, an increase in the amount of *D. cucullata* is related with an increase in the number of both macrophyte and fish species (see also 1.4.), while there exist specific conditions in the case of extreme levels of *B. longispina* and *Ch. sphaericus*: (a) at a high level of *B. longispina* and a low level of *Ch. sphaericus* the rise in the amount of *D. cucullata* is accompanied at first by an increase in the number of macrophyte and fish species. However, the increase of *D. cucullata* above the average level brings about an increase in the number of macrophyte species, while the number of fish species may even decrease; (b) at a low level of *B. longispina* and a high level of *Ch. sphaericus* the increase in the amount of *D. cucullata* calls forth an increase in the number of fish species, while the number of macrophyte species does not undergo any considerable changes (here the decrease in water transparency due to the water-bloom may have a negative effect).

3. The aspect of selective criteria

3.1. Relation to the water transparency (Fig. 17)

$$Y^{(2)} = F^{(2)}[C(X^{(3)}, X^{(8)}, X^{(6)})]$$

The relation between the water-bloom intensity and the water transparency is dominating. The minimum water transparency occurs in conditions where the HCO′₃ content is below the average, the level of *D. cucullata* low, and the water-bloom intensity high (soft-water eutrophic lakes). The maximum water transparency is observed when the level of HCO′₃ and the water-bloom intensity are low and the amount of *D. cucullata* is average. With a low level of water-bloom intensity the minimum water transparency occurs in cases when the HCO′₃ content is small or large and the amount of *D. cucullata* is small. With a high water-bloom intensity the maximum water transparency occurs when the HCO′₃ content is at the below- or above-average level and the amount of *D. cucullata* is average.

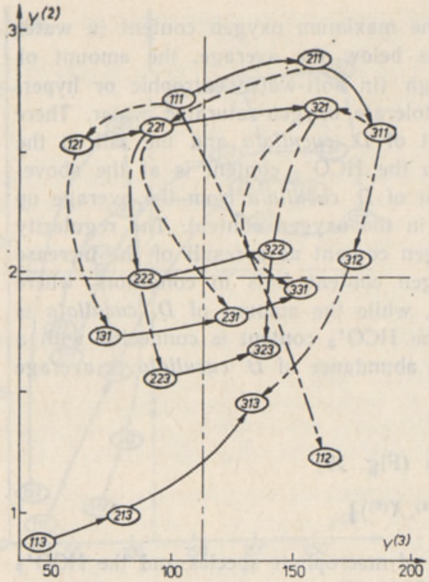


Fig. 15. Compatible relation of the amount of zooplankton to the water transparency and HCO′₃.

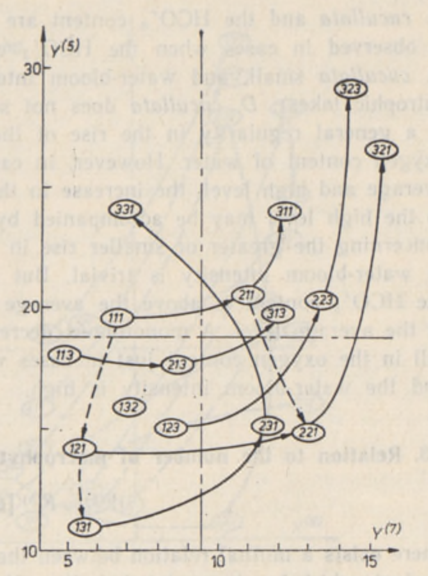


Fig. 16. Compatible relation of the amount of zooplankton to the number of macrophyte and fish species.

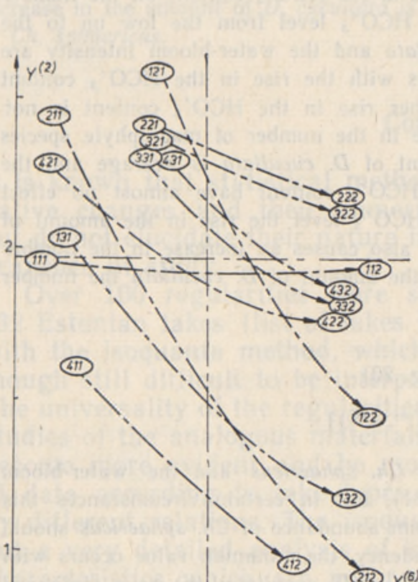


Fig. 17. Relation of selective criteria to the transparency of water.

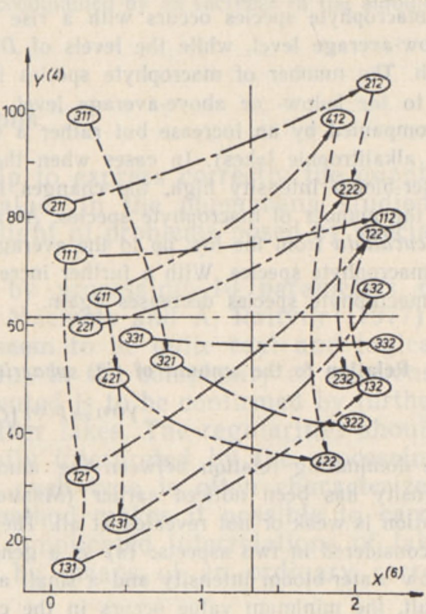


Fig. 18. Relation of selective criteria to the oxygen content.

3.2. Relation to the oxygen content (Fig. 18)

$$Y^{(4)} = F^{(4)}\,[C(X^{(3)}, X^{(8)}, X^{(6)})]$$

There exist quite distinct relations between *D. cucullata*, the HCO′₃ content, water-bloom intensity and the oxygen content of water. A reliable minimum oxygen content is observed when the water-bloom intensity is at a low level while the amount of

D. cucullata and the HCO'_3 content are high. The maximum oxygen content in water is observed in cases when the HCO'_3 content is below the average, the amount of *D. cucullata* small, and water-bloom intensity high (in soft-water eutrophic or hypereutrophic lakes). *D. cucullata* does not seem to tolerate oxygen-saturated water. There is a general regularity in the rise of the amount of *D. cucullata* and the fall in the oxygen content of water. However, in cases when the HCO'_3 content is at the above-average and high level, the increase in the amount of *D. cucullata* from the average up to the high level may be accompanied by a rise in the oxygen content. The regularity concerning the greater or smaller rise in the oxygen content as a result of the increase in water-bloom intensity is trivial. But the oxygen content falls in conditions where the HCO'_3 content is above the average or high, while the amount of *D. cucullata* is at the average level. A monotonous decrease in the HCO'_3 content is connected with a fall in the oxygen content just in cases when the abundance of *D. cucullata* is average and the water-bloom intensity is high.

3.3. Relation to the number of macrophyte species (Fig. 19)

$$Y^{(5)}=F^{(5)}\left[C\left(X^{(3)}, X^{(8)}, X^{(6)}\right)\right]$$

There exists a mutual relation between the number of macrophyte species and the HCO'_3 content, which has been stated earlier (Мяэметс, 1981). The minimum number of macrophyte species occurs when the levels of the HCO'_3 content, the amount of *D. cucullata* and the water-bloom intensity are low. The number of macrophyte species is maximum when the levels of HCO'_3, the abundance of *D. cucullata* and the water-bloom intensity are high. The strongest dependence of the increase in the HCO'_3 content on the number of macrophyte species occurs with a rise of the HCO'_3 level from the low up to the below-average level, while the levels of *D. cucullata* and the water-bloom intensity are high. The number of macrophyte species increases with the rise in the HCO'_3 content up to the below- or above-average level. A further rise in the HCO'_3 content is not accompanied by an increase but rather a decrease in the number of macrophyte species (in alkalitrophic lakes). In cases when the amount of *D. cucullata* is average and the water-bloom intensity high, the changes in the HCO'_3 content have almost no effect on the number of macrophyte species. At a low HCO'_3 level the rise in the amount of *D. cucullata* from the low up to the average level also causes an increase in the number of macrophyte species. With a further increase in the amount of *D. cucullata* the number of macrophyte species decreases again.

3.4. Relation to the amount of *Ch. sphaericus* (Fig. 20)

$$Y^{(10)}=F^{(10)}\left[C\left(X^{(3)}, X^{(8)}, X^{(6)}\right)\right]$$

The dominating relation between the amount of *Ch. sphaericus* and the water-bloom intensity has been noticed earlier (Мяэметс, 1981), but in certain circumstances this relation is weak or not revealed at all. The minimum abundance of *Ch. sphaericus* should be considered in two aspects: (a) as a general tendency, the minimum value occurs with a low water-bloom intensity and a small abundance of *D. cucullata*; (b) as a statistical result, the minimum value occurs in the case when the HCO'_3 content and the water-bloom intensity are high and the amount of *D. cucullata* is on the average level; the statistically maximum abundance of *Ch. sphaericus* occurs in the case when the HCO'_3 content is small, while the amount of *D. cucullata* is great and the water-bloom intensity is high. With a weak water-bloom intensity the amount of *Ch. sphaericus* is greatest in circumstances when the low HCO'_3 level is accompanied by an average or high level of *D. cucullata*. With a high water-bloom intensity the amount of *Ch. sphaericus* is related to the amount of *D. cucullata*. In the case of a high level of the latter, the amount of *Ch. sphaericus* is greater than in the case of the average or low level of abundance. In conditions of a high level of the water-bloom intensity and an average level of *D. cucullata*, the HCO'_3 content is accompanied by a decrease in the amount of *Ch. sphaericus*. When the water-bloom intensity is high and the HCO'_3 level low, the

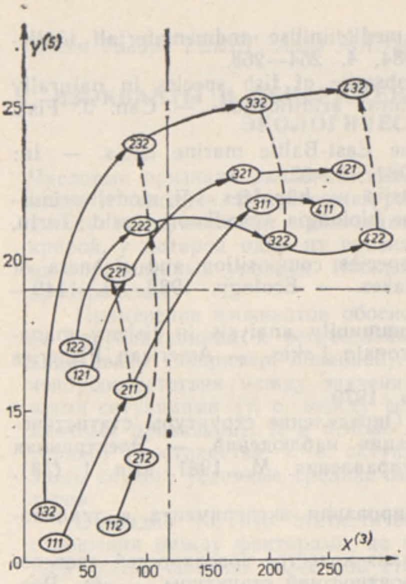


Fig. 19. Relation of selective criteria to the number of macrophyte species.

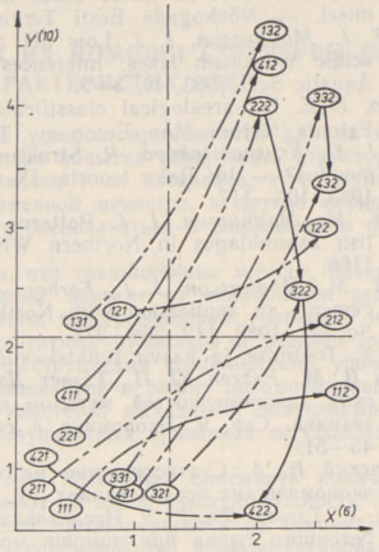


Fig. 20. Relation of selective criteria to the amount of *Chydorus sphaericus.*

increase in the amount of *D. cucullata* is also accompanied by an increase in the amount of *Ch. sphaericus.*

Conclusions

It is known that statistical methods help to express correctly the quantitative changes and their numerical values in the phenomena studied, but do not elucidate their nature in the light of problems posed by special branches of study.

Over 100 regularities were stated by processing 10 parameters of 132 Estonian lakes (list of lakes by A. Mäemets and A. Raitviir (1977)) with the isoquanta method, which all seem to be quite real and logical though still difficult to be interpreted due to the complexity of relations. The universality of the regularities presented is to be confirmed by further studies of the analogous materials of other lakes. The regularities should become more evident and be more easily interpreted by the processing of data according to lake types, since each type is often characterized by different relations. The isoquanta method makes it possible to carry out a very detailed analysis of highly complicated interrelations of lake characteristics which are not revealed by means of an ordinary correlation or regression analysis.

The method of isoquanta also makes it possible to express the stated statistical law-governed processes in a verbal way, as has been done above.

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ISOKVANDID JA NENDE ABIL ESITATUD SEOSTE STRUKTUUR JÄRVEDE ARVKARAKTERISTIKUTE VAHEL

Ökoloogilisi objekte iseloomustavad arvtunnused on laialdaselt kasutusel nende objektide takseerimisel ja klassifitseerimisel. Käesolevas töös on püütud isokvantide abil analüüsida seoste struktuuri seesuguste ökoloogiliste tunnuste vahel. Isokvant kujutab endast ühe sõltumatu tunnusmuutujaga regressioonikõverat, mille puhul teatud argumenttunnuste väärtused on fikseeritud mingitel kindlatel nivoodel. Isokvandid ise asuvad faasiruumis, mis võib olla mitmemõõtmeline.

Isokvantide kasutamine ökoloogiliste seaduspärasuste kindlakstegemiseks johtub asjaolust, et traditsioonilised, korrelatsioon- ja regressioonanalüüsi meetodid rajanevad aprioorselt etteantud seoseviisidele. Millegagi pole aga tagatud sellise etteantud seoseviisi (näiteks lineaarse) vastavus tegelikult eksisteerivale seoseviisile. Samuti pole regressioonivõrrandite puhul tagatud kooskõla resultaattunnuste väärtuste ja reaalselt eksisteerivate situatsioonide (s. t. argumenttunnuste nivoode tegelike kombinatsioonide) vahel. Isokvantide meetod baseerubki argumenttunnuste ruumi (s. t. faktorruumi) dekomponeerimisel alamruumideks, mida nimetatakse situatsiooniklastriteks. Regressioonifunktsioonide väärtusteks on siinjuures situatsiooniklastrite resultaattunnuste tinglikud keskväärtused.

On ilmne, et statistilised meetodid iseloomustavad vaid karakteristikutevaheliste seoste kvantitatiivset külge ega seleta nende ökoloogilist sisu.

Käesolevas artiklis on isokvantide abil kirjeldatud üle 100 statistilise seaduspärasuse, mis on saadud 132 Eesti järve vaatlusandmetest kümne tunnuse raames. Suurt osa saadud seaduspärasustest on kommenteeritud ja analüüsitud ökoloogia aspektist. Isokvantide kasutamise eeliseks on seegi, et saadud seaduspärasused on sõnastatavad.

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ИЗОКВАНТЫ И ОБНАРУЖЕНИЕ ПРИ ИХ ПОМОЩИ СТРУКТУРЫ СВЯЗЕЙ ЭКОЛОГИЧЕСКИХ ХАРАКТЕРИСТИК ОЗЕР

Числовые признаки экологических объектов успешно применяются для их классификации и таксации. В настоящей работе сделана попытка обнаружить структуру связей между экологическими признаками при помощи т. н. изокванта, т. е. регрессионной кривой, у которой один из признаков переменной меняется, а другие зафиксированы на определенных уровнях. Изоквант может располагаться в многомерном факторном пространстве.

Применение изоквантов обосновано тем, что традиционные методы, базирующиеся на корреляционном и регрессионном анализах, зависят от априорности задаваемого вида связи (например, линейной). Кроме того, в регрессионном уравнении не обеспечено соответствия между значениями результатных признаков и реально существующими ситуациями (т. е. между реально существующими комбинациями уровней аргументных признаков). А в основе метода изоквантов и лежит декомпонирование факторного пространства, т. е. ситуационные кластеры, выходными значениями которых здесь служат условные средние значения результатных признаков ситуационных кластеров.

Очевидно то, что статистические методы позволяют описывать количественные отношения между факторами, но не объясняют их экологическую сущность. В настоящей статье описано более 100 статистических закономерностей на основе 10 признаков 132 озер Эстонии. Большинство этих закономерностей прокомментировано с экологической точки зрения. Преимущество применения изоквантов состоит и в том, что полученные статистические закономерности можно изложить семантически, что и является основным содержанием этой статьи.