# ON THE LOCAL AND GLOBAL OPTIMUMS IN DISTRIBUTION FEEDER RECONFIGURATION 

## O. TERN ${ }^{\circ}$

Department of Electrical Power Engineering, Tallinn University of Technology, 5, Ehitajate Rd., Tallinn 19086, Estonia


#### Abstract

A simple algorithm, based on iterative stepwise shifting of loads (switch exchange) between distribution network feeders, is presented. The results of a computer experiment in local distribution network are presented and compared with the results of other researches.


## Introduction

The scope of one feeder in the radial distribution network is determined by breakpoints (lines with switches open) separating this feeder from the rest of the network. Depending on cable lengths, their cross-sections and customers' loads, different feeder configurations give different total losses in the distribution network. Reduction of the losses might be significant, and therefore the problem of optimal feeder configuration has been one of the actual problems in the research of distribution networks for a long time.

The present development of SCADA systems and remote control facilities make the problem actual. Various methods are applied for optimizing feeder configuration: simple exchange of tie and sectionalizing of switches [5], more sophisticated sequential switch opening [10], network partitioning [11], application of generic algorithms [17], reduced-order binary decision diagrams [19]. Simple switch exchange methods are considered inefficient as they often lead to local optimality "traps", and for large networks, only methods based on heuristics and on combinatory methods are considered suitable.

Nevertheless, the simplest approach for reconfiguration of distribution feeder is used in the algorithm presented below. It is found to be usable, fast, and it has given a considerable reduction of losses. It was found that different switching order produces different optimal network configurations. Therefore, the question about local and global optimality remains open.

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Fig. 1. A breakpoint between feeders F1 and F2

## An Iterative Method of Shifting of Breakpoints

The method presented here is a modification of classical switch exchange method. The objective of the algorithm is to determine the optimal location of each breakpoint individually. Therefore, as each breakpoint is considered separately, the problems like the $2^{n}$ or similar problems of combinatory analysis do not occur. Nevertheless, network losses can be not the only criteria considered in such approach.

Suppose the initial configuration of a distribution network with radial structure is given, where no normal constraints are violated, all line currents and bus voltages are within normal limits. As a simplification, the loads are modeled as constant currents. The following algorithm will be applied to this network:

1. Load-flow calculation is made for the whole network in order to determine the voltages of the bus-bars of supply substations

$$
\dot{U}_{s}=U_{s}^{a}+j U_{s}^{r}, s=1,2, \ldots S
$$

where $S$ - the set of supply substations.
2. A list of all breakpoints will be composed. The way of its composition and the order of the breakpoints in the list are free. Suppose the number of breakpoints is $B$, breakpoints numbers are $N_{b}, b=1,2,3, \ldots . B$, respectively. Additional information on every breakpoint and every switch is needed whether it is permitted to close this breakpoint (switch) a. The same applies to every line switch - whether it is permitted to switch this line off (usual operational limitations should be taken into consideration, if those exist).
3. Beginning with the first one in this list the following is calculated:
3.1. Breakpoint will be checked for the possibility of its shifting. If the closure of this switch is not permitted, the next breakpoint in the list will be studied.
3.2. Two vectors of lines will be composed: one from one side of the breakpoint to the supply bus $s^{\prime}$ (feeder $F 1$ ), and second from the other side of the breakpoint to the supply bus $s$ " (feeder F2) (i.e. circuits from the breakpoint to the supply buses (Fig.1).
Suppose the first vector (circuit) having $N$ elements ( $n=1,2, \ldots N$ ) and the second one $M$ elements ( $m=1,2, \ldots M$ ). The elements of those vectors (lines) are numbered starting from supply buses.
3.3. Calculate voltages or voltage drops on both sides of the breakpoint

$$
\begin{align*}
& U_{1}=\dot{U}_{s^{\prime}-} \sum_{n=1}^{N}\left(\dot{I}_{n} * \dot{Z}_{n}\right), \forall(n, n=1, \ldots N), \\
& U_{2}=\dot{U}_{s^{\prime \prime}}-\sum_{m=1}^{M}\left(\dot{I}_{m} * \dot{Z}_{m}\right), \forall(m, m=1 \ldots M) \tag{1}
\end{align*}
$$

3.4. Compare voltages $U_{1}$ and $U_{2}$. Relation $\left|\dot{U}_{1}\right|>\left|\dot{U}_{2}\right|$ indicates that the voltage drop in feeder $F 1$ is less than in feeder $F 2$. It could indicate that feeder $F 2$ is carrying more load than feeder $F 1$, and there could be some reason to switch some load from feeder $F 2$ to feeder $F 1$, closing the breakpoints switch and opening the switch of line number $M$, thus transferring load $I_{M}$ to feeder $F 1$.
3.5. Check if the switching off of line number $M$ is permitted. If it is, then proceed with the algorithm, if not - go to the next breakpoint.
3.6. Calculate the losses in the same circuits where the voltage drop was calculated

$$
\begin{align*}
& \Delta P_{1}^{\prime}=\sum_{n=1}^{N}\left\{\left(I_{a, n}^{2}+I_{r, n}^{2}\right) * r_{n}\right\}, \\
& \Delta P_{2}^{\prime}=\sum_{m=1}^{M}\left\{\left(I_{a, m}^{2}+I_{r, m}^{2}\right) * r_{m}\right\}  \tag{2}\\
& \Delta P_{\Sigma}^{\prime}=\Delta P_{1}^{\prime}+\Delta P_{2}^{\prime} .
\end{align*}
$$

3.7. Calculate losses in the same circuits with load $I_{M}$ switched from feeder F1 to feeder $F 2$

$$
\begin{align*}
& \Delta P_{1}^{\prime \prime}=\sum_{n=1}^{N}\left[\left(I_{a, n}+I_{a, M}\right)^{2}+\left(I_{r, n}+I_{r, M}\right)^{2}\right]^{*} r_{n}+\left(I_{a, M}^{2}+I_{r, M}^{2}\right) * r_{0} \\
& \Delta P_{2}^{\prime \prime}=\sum_{m=1}\left[\left(I_{a, m}-I_{a, M}\right)^{2}+\left(I_{r, m}-I_{r, M}\right)^{2}\right] * r_{m}  \tag{3}\\
& \Delta P_{\Sigma}^{\prime \prime}=\Delta P_{1}^{\prime \prime}+\Delta P_{2}^{\prime \prime}
\end{align*}
$$

3.8. Calculate possible change of losses by shifting of the breakpoint

$$
\begin{equation*}
\Delta \Delta P=\Delta P_{\Sigma}^{\prime}-\Delta P_{\Sigma}^{\prime \prime} \tag{4}
\end{equation*}
$$

3.9. If $\Delta \Delta P>0$, the shifting of the breakpoint or the load $I_{M}$ from feeder F1 to feeder $F 2$ results in loss reduction.
If the shapes of the load curves of different buses are sharply different, the energy losses might be considered instead of power losses.

The levels of line currents and bus voltages are checked. If they are within admissible limits, the new breakpoint will be fixed, topology of the network will be changed, the old breakpoint in the list of breakpoints will be replaced with the new one.
3.10. Go to the next breakpoint in the list.
4. After first inspection of all breakpoints, the next iteration begins. All breakpoints are tested in the same order. The iterations are repeated until no breakpoint could be shifted during whole iteration.
5. Change the order of testing the breakpoints in order to find a possible more optimal configuration, and start the whole procedure from the beginning.

## Computer Experiments in Local Distribution Network

For testing the applicability and efficiency of the developed algorithm, calculation was made with actual data of a part of the local distribution network. The results of this computer experiment are given in Table 1.

Different paths shown here have the following meaning:
Path 1 - the breakpoints are handled in the order of their listing.
Path 2 - the testing of breakpoints begins from the end of their list.
Path 3 - breakpoints are taken alternatively from the beginning and from the end of the breakpoints list.
Path 6 - the breakpoints are sorted by the possible loss reduction that could be obtained by their shifting.

Then, beginning from the breakpoint with the maximal loss reduction down to the end of the list the shifting process is executed. The difference of the loss reductions for different order of breakpoints is $0.6 \%$ for the first network and $2.1 \%$ for the second network. Bearing in mind that all bus loads are given with an accuracy of about $5 \%$, these differences could be considered acceptable. Reduction of losses by around $10 \%$ could also be considered essential.

Table 1. Search for Optimal Feeder Configuration in Local City Network

|  | North-east dis | tricts (10 kV) | South-west districts (6kV) |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Number of supply substations |  | 3 |  | 7 |  |
| Number of feeders |  | 42 |  | 83 |  |
| Number of load buses | 52 | 22 |  | 55 |  |
| Number of breakpoints | 23 | 7 |  | 72 |  |
| Number of fixed breakpoints |  | 22 |  | 83 |  |
| Total load in the network, MW |  | 63.33 |  | 79.64 |  |
| Total line losses, initial network, kW |  | 8.0 |  | 79.8 |  |
| Number of the path (test order) | 1 : 2 | $3: 6$ | 1 : 2 | 3 | 6 |
| Iterations calculated | 5 : 5 | $5: 10$ | 4 : 4 |  |  |
| Shifted breakpoints | 41 : 39 | $36: 42$ | $41: 42$ |  |  |
| Line losses after optimisation, kW | 434.7: 435.8 | 438.0: 436.4 | 518.7: 508.7 | 508.3 | 520.7 |
| Reduction of losses, kW | 83.4: 82.1 | 80.1: 81.6 | 61.1: 71.0 | 71.4 | 59.1 |
| Reduction of losses, \% | 16.1: 15.9 | 15.5: 15.8 | 10.5: 12.3 | 12.3 | 10.2 |

Nevertheless, the question remains - how far is the best solution found here from the global optimum. To answer this question a new algorithm using GA or similar heuristics had to be developed. Another idea for testing the algorithm developed here is to use this algorithm against the work of other authors; using the data of their test networks to compare our results with the results of algorithms developed by other authors. In all papers listed in the reference [1-16], the test networks were too small, or the data given in the
paper was incomplete. A comparison possibility appeared in 2002, when J. Z. Zhu published his paper in the journal "Electric Power System Research" [17], giving complete data of the test network he had used.

## A Comparison with the Results of Zhu [17]

In the fifth chapter of his paper, J. Z Zhu tests his approach to the feeder configuration optimization on 16-bus and 33-bus distribution systems. We consider here the 33-bus network. The configuration of this system is given in Fig. 2, the data and parameters are brought in Table 2.

Table 2. Parameters and Loads of the Network in Fig. 2

| Line No. | Node $I$ | Node $J$ | $\mathrm{R}[\Omega]$ | $\mathrm{Xl}[\Omega]$ | $\mathrm{P}_{\mathrm{j}}[\mathrm{kW}]$ | $\mathrm{Q}_{\mathrm{j}}[\mathrm{kVar}]$ | $\mathrm{V}_{\mathrm{j}}$ Mod p.u. | $\mathrm{V}_{\mathrm{j}} \Delta[\mathrm{rad}]$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | 1 | 2 | 0.0922 | 0.047 | 100.00 | 60.00 | 0.997 | 0.0145 |
| 2 | 2 | 3 | 0.493 | 0.2512 | 90.00 | 40.00 | 0.9829 | 0.096 |
| 3 | 3 | 4 | 0.3661 | 0.1864 | 120.00 | 80.00 | 0.975 | 0.0617 |
| 4 | 4 | 5 | 0.3811 | 0.1941 | 60.00 | 30.00 | 0.9681 | 0.2283 |
| 5 | 5 | 6 | 0.819 | 0.707 | 60.00 | 20.00 | 0.9497 | 0.1339 |
| 6 | 6 | 7 | 0.1872 | 0.6188 | 200.00 | 100.00 | 0.9462 | -0.0964 |
| 7 | 7 | 8 | 0.7115 | 0.2351 | 200.00 | 100.00 | 0.9413 | -0.0603 |
| 8 | 8 | 9 | 1.0299 | 0.74 | 60.00 | 20.00 | 0.9351 | -0.1334 |
| 9 | 9 | 10 | 1.044 | 0.74 | 60.00 | 20.00 | 0.9292 | -0.1959 |
| 10 | 10 | 11 | 0.1967 | 0.0651 | 45.00 | 30.00 | 0.9284 | -0.1887 |
| 11 | 11 | 12 | 0.3744 | 0.1298 | 60.00 | 35.00 | 0.9269 | -0.1785 |
| 12 | 12 | 13 | 1.468 | 1.1549 | 60.00 | 35.00 | 0.9208 | -0.2698 |
| 13 | 13 | 14 | 0.5416 | 0.7129 | 120.00 | 80.00 | 0.9185 | -0.3485 |
| 14 | 14 | 15 | 0.5909 | 0.526 | 60.00 | 10.00 | 0.9171 | -0.3862 |
| 15 | 15 | 16 | 0.7462 | 0.5449 | 60.00 | 20.00 | 0.9157 | -0.4094 |
| 16 | 16 | 17 | 1.2889 | 1.721 | 60.00 | 20.00 | 0.9137 | -0.4868 |
| 17 | 17 | 18 | 0.732 | 0.5739 | 90.00 | 40.00 | 0.9131 | -0.4963 |
| 18 | 2 | 19 | 0.164 | 0.1565 | 90.00 | 40.00 | 0.9965 | -0.0037 |
| 19 | 19 | 20 | 1.5042 | 1.3555 | 90.00 | 40.00 | 0.9929 | -0.0633 |
| 20 | 20 | 21 | 0.4095 | 0.4684 | 90.00 | 40.00 | 0.9922 | -0.0827 |
| 21 | 21 | 22 | 0.7089 | 0.9373 | 90.00 | 40.00 | 0.9916 | -0.103 |
| 22 | 3 | 23 | 0.4512 | 0.3084 | 90.00 | 50.00 | 0.9794 | -0.065 |
| 23 | 23 | 24 | 0.898 | 0.7091 | 420.00 | 200.00 | 0.9727 | -0.0237 |
| 24 | 24 | 25 | 0.8959 | 9.7071 | 420.00 | 200.00 | 0.9694 | -0.0674 |
| 25 | 6 | 26 | 0.2031 | 0.1034 | 60.00 | 25.00 | 0.9477 | 0.1734 |
| 26 | 26 | 27 | 0.2842 | 0.1447 | 60.00 | 25.00 | 0.9452 | 0.2295 |
| 27 | 27 | 28 | 1.0589 | 0.9338 | 60.00 | 20.00 | 0.9337 | 0.3124 |
| 28 | 28 | 29 | 0.8043 | 0.7006 | 120.00 | 70.00 | 0.9255 | 0.3904 |
| 29 | 29 | 30 | 0.5074 | 0.2585 | 200.00 | 100.00 | 0.9219 | 0.4956 |
| 30 | 30 | 31 | 0.9745 | 0.9629 | 150.00 | 70.00 | 0.9178 | 0.4112 |
| 31 | 31 | 32 | 0.3105 | 0.3619 | 210.00 | 100.00 | 0.9169 | 0.3882 |
| 32 | 32 | 33 | 0.3411 | 0.5302 | 60.00 | 40.00 | 0.9166 | 0.3805 |
| 33 | 25 | 29 | 0.5 | 0.5 |  |  |  |  |
| 34 | 8 | 21 | 2 | 2 |  |  |  |  |
| 35 | 12 | 22 | 2 | 2 |  |  |  |  |
| 36 | 9 | 15 | 2 | 2 |  |  |  |  |
| 37 | 18 | 33 | 0.5 | 0.5 |  |  |  |  |



Fig. 2. 33-bus test system in [17]
Table 3. Optimization of the 33-Bus Test Network by the Method Presented in This Paper

| Break-point No. | Existing open switch No | Possible <br> loss reduction, W | New open switch No. | Real <br> loss reduction, W |
| :---: | :---: | :---: | :---: | :---: |
| Iteration No. 1 |  |  |  |  |
| 1 | 33 | 16290 | 28 | 16290 |
| 2 | 34 | 14850 | 7 | 14850 |
| 3 | 35 | 13211 | 11 | 13211 |
| 4 | 36 | 1140 | 14 | 1102 |
| 5 | 37 | 140 | 32 | 199 |
| Iteration No. 2 |  |  |  |  |
| 1 | 28 | 2410 | 33 | 2435 |
| 2 | 7 | -19270 | 7 | 0 |
| 3 | 11 | 910 | 10 | 885 |
| 4 | 14 | -3170 | 14 | 0 |
| 5 | 32 | -620 | 32 | 0 |
| Iteration No. 3 |  |  |  |  |
| 1 | 33 | -24640 | 33 | 0 |
| 2 | 7 | -17090 | 7 | 0 |
| 3 | 10 | 680 | 9 | 699 |
| 4 | 14 | -5040 | 14 | 0 |
| 5 | 32 | -820 | 32 | 0 |
| Iteration No. 4 |  |  |  |  |
| 1 | 33 | -24640 | 33 | 0 |
| 2 | 7 | -17680 | 7 | 0 |
| 3 | 9 | -6370 | 9 | 0 |
| 4 | 14 | -5040 | 14 | 0 |
| 5 | 32 | -820 | 32 | 0 |
| Total loss reduction 49,670 W |  |  |  |  |

The 33-bus test system consists of one supply transformer and 32 load points (distribution transformers). There are five initially open switches in the test system - "33", "34", " 35 ", " $36 "$ and " 37 ". The total system load is 3.715 MW, the basis of the system is $V=12.66 \mathrm{kV}$ and $S=10 \mathrm{MVA}$.

Applying his method to the network presented in Fig. 2 and Table 2, J. Z. Zhu finds optimal configuration of this network by given loads with open switches No. 7, 9, 14, 32 and 33. Applying the method proposed in [6] to the same network, J. Z. Zhu finds optimal configuration to be with open switches No. 7, 10, 14, 33 and 37 and total losses by this configuration being about $10 \%$
more than by the configuration found by his method. So, J. Z. Zhu concludes that global optimum is found in his paper and not in [6].

Two calculations were made for the network presented in Fig. 2 and Table 2. In order to get some comparison with the efficiency of the algorithm developed in this paper, in the first case the initial state of the network was taken with open switches No. 33, 34, 35, 36 and $37-$ the initial state of the network. The path and results of this calculation are presented in Table 2.

As one can see, after the fourth iteration the configuration of networks has converged to the configuration obtained by the method published in [17].

In the second case the optimization result by the method published in [6] was taken for the initial state - switches No. 7, 10, 14, 33 and 37 are open. The results are given in Table 4.

The applied algorithm has found by the first step, two possible ways for loss reduction in the network configuration, which was considered as optimal in [6].

Table 4. Optimization of the "Optimized" by the Method of [6] Network

| Break-point No. | Existing open switch No | Possible <br> loss reduction, W | New open switch No. | Real <br> loss reduction, W |
| :---: | :---: | :---: | :---: | :---: |
| Iteration No. 1 |  |  |  |  |
| 1 | 7 | -17110 | 7 | 0 |
| 2 | 10 | 480 | 9 | 503 |
| 3 | 14 | -4010 | 14 | 0 |
| 4 | 33 | -13540 | 33 | 0 |
| 5 | 37 | 800 | 32 | 851 |
| Iteration No. 2 |  |  |  |  |
| , | 7 | -14680 | 7 | 0 |
| 2 | 9 | -6370 | 9 | 0 |
| 3 | 14 | -5040 | 14 | 0 |
| 4 | 33 | -11710 | 33 | 0 |
| 5 | 32 | -820 | 32 | 0 |
| Total loss reduction 1,354 W |  |  |  |  |

## Conclusions

A variation of the classical switch exchange method is presented in this paper. The developed algorithm and software are fast and enable online control of large networks with considerable reduction of network losses. A comparison with the method using GA [17] is given showing on the test network similarity. However, it will be inconsiderate to declare that the algorithm presented here gives always the same results as methods based on GA. It is obvious that simple networks are not suitable for comparing different optimization methods. Decision on the suitability of control and optimization methods should be made based on the experiments in actual networks. More comparisons of different methods for optimization of configuration of distribution networks are needed. In addition, one more question needs to be
answered - how does the accuracy of the load data used by optimization influence the accuracy of optimum determination. Does the search for global optimum have a meaning at all, or are the local optimums found here sufficient for practical applications?

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[^0]:    * E-mail olaf.terno@ttu.ee

