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ESTIMATION OF DISTRIBUTION NETWORK STATE ON THE BASIS OF A MATHEMATICAL LOAD MODEL

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The target of estimating distribution network state is to refine the measurement data, but it is especially important to detect significant measurement errors or mistakes. Usually the redundancy of data required for estimation is not available in distribution networks – the number of measurements is not essentially bigger or is even smaller than the number of main state parameters. Resolving of this problem can be based on a mathematical load model, by means of which it is possible to calculate load values and characteristics for all loads at any time. Such a load model should consider all main regular changes of a load and take into account temperature dependency, stochastic nature of a load, and also frequency and voltage dependencies.

MV Distribution Network

It is possible to represent a MV distribution network with the help of feeders going out from a transmission substation and consisting of line parts and distribution substations that are connected to the ends of line parts (Figure). The configuration of a feeder can change by switching disconnectors at the switching substations. Also MV/MV stations and voltage regulators may be incorporated.

The load of a MV feeder is formed by active and reactive loads of distribution substations. The load of a distribution substation is formed as the sum of LV loads and losses of distribution transformers.

Dispatch system SCADA meters state parameters of a feeder periodically. Measured data may include values of active and reactive power, currents and voltages at various points of the feeder. Mainly these variables are measured at HV/MV substations. The metering frequency is usually one or more times per minute.

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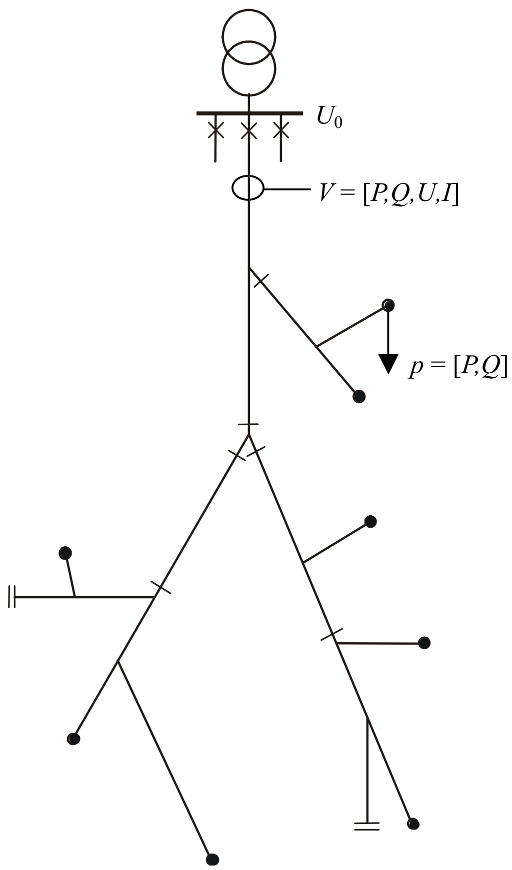

A distribution network

Changes of feeder configuration do not obstruct the estimation if the actual or expected circuit is known. Possible changes of the feeder circuit are recorded by SCADA, and these can be checked during the state estimation. However, SCADA does not observe changes of the circuit in LV networks, and that may result in abrupt changes of LV loads at distribution substations. Abrupt changes in a load may occur also because of other reasons, for example, due to the load irregularity of dominant consumers or switching on/off reactive power sources. In this paper such different states of the load are called load cases. On the basis of actual cases of several loads the load scenario is formed.

Mathematical Load Model

Mathematical model of a load (active power, reactive power or current) can be presented in the form of

$$P(t) = E(t) + R(t)\theta(t)$$

where $E(t)$ is mathematical expectation of the load;
$R(t)$ is rate of the load;
$\theta(t)$ is normalized deviation of the load.

Normalized deviation of the load consists of temperature dependency $\gamma(t)$ and stochastic component $\vartheta(t)$

$$\theta(t) = \gamma(t) + \vartheta(t)$$

Stochastic component in its turn is divided into three components:

$$\vartheta(t) = \zeta(t) + \xi(t) + \pi(t)$$

where $\zeta(t)$ is expected deviation of the load;

$\xi(t)$ is residual deviation of the load (white noise);

$\pi(t)$ is peak deviation of the load.

Finally

$$P(t) = E(t) + R(t)[\gamma(t) + \zeta(t) + \xi(t) + \pi(t)]$$

Mathematical expectation *E(t)* is the main component of the load model, which describes regular changes of the load and corresponds to normal temperature. Mathematical expectation is principally nonstochastic. In addition to mathematical expectation, standard deviation of the load is considered, which is the measure of load stochasticity.

Rate of the load *R(t)* is needed for comparison of different loads on the basis of model components. For example, it is practical to estimate temperature dependency $\gamma(t)$ simultaneously for a larger group of similar loads – for the whole load class. So the rate *R(t)* determines the level of temperature dependency for considered load. In addition, the rate of the load supports also the consideration of stochastic components.

Temperature dependency $\gamma(t)$ of the load describes the load deviation caused by deviation of outdoor temperature from its normal values. Normal temperature (mathematical expectation of temperature) is calculated as the average outdoor temperature of the last 30 years at certain time moments of the year. It is typical that temperature dependency of load is characterized by delay of about 24 hours. If the real temperature corresponds to normal temperature (considering delay), the temperature dependency does not exist $\gamma(t) = 0$.

Here expected deviation of the load $\zeta(t)$ describes the conditional mathematical expectation of the stochastic component, which is also needed for calculating load short-term (more accurate) forecast. Residual deviation of the load $\xi(t)$ describes a normally distributed noncorrelated stochastic process – white noise. Peak deviation of the load $\pi(t)$ corresponds to large positive or negative deviations of the load, which do not match with normal distribution. Such kinds of load deviations can occur from time to time.

Mathematical model describes the load, but it does not determine the values needed in practice directly (so-called load characteristics), for example it does not give forecasts directly. However, it is possible to calculate these load characteristics on the basis of the load model. Load characteristics can be divided into primary and derived characteristics. Primary characteristics are derived directly from the mathematical model. Derived characteristics are calculated by combining primary characteristics. Primary characteristics are load real values *P(t)*, mathematical expectation *E(t)*, standard deviation *S(t)*, temperature dependency $\gamma(t)$, etc.

On the basis of these components, it is possible to find the next characteristics, for example:

$P_F(t) = E(t) + R(t)\gamma'(t)$ – long-term forecast of the load

$P_{SF}(t) = E(t) + R(t)[\zeta_\tau(t) + \gamma_\tau(t)]$ – short-term forecast of the load

$P_N(t) = P(t) - R(t)\gamma(t)$ – normalized load

$P_S(t) = P(t) - R(t)[\gamma(t) - \gamma'(t)]$ – simulated load.

Here τ is forecast anticipation time, and $\gamma'(t)$ – influence of simulated temperature. It is possible to simulate temperature, for example, by adding specified deviation to normal temperature, or by using some other year temperature data (cold winter, warm autumn). The long-term forecast of the load can be based on normal temperature (temperature dependence is absent), or on simulated temperature.

Mathematical expectation, standard deviation and rate are complicated time functions. Depending on the needed accuracy, also other components of the model, especially temperature dependency, may be described in detail. However, sometimes the load changes are so much irregular that accurate describing of such loads is impossible. In that case mathematical expectation, standard deviation and rate of these loads are calculated as constants, and corresponding models are called trivial models. In the case of such a trivial model the difference between load value and mathematical expectation is a stochastic component, for which it is possible to calculate the expected deviation, residual deviation and peak deviation of the load. So, it is possible both to analyze and to forecast the load by means of the trivial model. It is important to note that the above-mentioned trivial model in principle corresponds to traditional load forecast models that are generally used for short-term forecasting.

In order to describe a certain load, it is necessary to estimate the parameters of mathematical model on the basis of its load data. At the point of estimation the mathematical model consists of components and factors.

Model components that include most parts of model parameters are relatively stable. The components can be estimated in the course of load research, and they will stay the same during several years, until the character of the load does not essentially change. If there is not enough initial data (hourly data at least of one year) about the considered load, it is possible to use components of some other load model with similar structure. In this case the estimation and application of type load models is necessary.

Level of the load, shape of the load curve, temperature dependency and other current changes are described by model factors. As the number of the factors is not large, it is possible to refine them on the basis of a relatively small amount of initial data. If there is still not enough initial data, it is possible to derive needed factors partially from type models. The refining of factors is called model editing. Model parameters considered below are factors.

State Estimation

The target of state estimation is to refine measurement data, but it is especially important to clarify significant measurement errors or mistakes. As the redundancy of data required for the estimation is usually not available in distribution networks, traditional estimation methods used under conditions of main grid are not directly applicable in distribution networks. For getting required additional data load models are used [1]. The present work is based on the same conception, but the load model described above is essentially more perfect. This model has been used also before in connection with the network steady-state monitoring [2], but only under conditions of main grids. The calculating of feeder state parameters is based on network equations through main operating parameters – supply voltage and node loads

$$\mathrm{V}_j(U_0, p_1, p_2, \ldots p_n)$$

Symbol p_i corresponds here to both active and reactive power. So the general number of loads n is equal to double number of nodes and so there are $n + 1$ main operating parameters $U_0, p_1, p_2 \ldots p_n$.

Supposing that for considered moment there are m measurements $\widetilde{V}_j$ (P, Q, U or I), it is possible to obtain the refined operating parameters (refined measurements) V_j from the criterion

$$\sum_j \left[\widetilde{V}_j - V_j\right]^2 = \min, \; j = 1 \ldots m$$

It must be taken into consideration that in distribution networks the needed data redundancy does not exist in most cases – the number of measurements m does not essentially exceed the number of main operating variables $n + 1$, or is even smaller. For resolving this problem the boundary conditions are applied, which demand that the increments of state parameters have to be as small as possible

$$\sum_i \Delta^2 p_i = \min, \; i = 1 \ldots n$$

In other words, it is needed that the state of the network would be as close as possible to that, which is forecasted by means of the load models.

Let us see the network equations being linearized over main operating parameters, when marking $p_0 = U_0$

$$\Delta V_j = \beta_{0j}\Delta p_0 + \beta_{1j}\Delta p_1 + \beta_{2j}\Delta p_2 + \ldots + \beta_{nj}\Delta p_n$$

where $\beta_{ij} = \dfrac{\partial V_j}{\partial p_i}$, $i = 0 \ldots n$

In the form of matrix

$$\mathbf{\Lambda} = \mathbf{B\Delta}$$

where $\mathbf{\Lambda} = [\Delta V_0, \Delta V_1 ... \Delta V_m]$ is the vector of increments of measurement data;

$\mathbf{\Delta} = [\Delta p_0, \Delta p_1 ... \Delta p_n]$ is the vector of increments of main state parameters;

$\mathbf{B} = \begin{vmatrix} \beta_{10} & \beta_{11} & \cdots & \beta_{1n} \\ \beta_{20} & \beta_{21} & \cdots & \beta_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \beta_{m0} & \beta_{m1} & \cdots & \beta_{mn} \end{vmatrix}$ is the sensitivity matrix (Jacobean)

Here the increments Δp_i are found on the basis of the values calculated by a load model (short-term forecasts). It is possible to compose the mathematical model also for the supply voltage U_0. For simplicity it is the trivial model, where mathematical expectation and standard deviation of voltage are constant and deviation is described by means of *Box-Jenkins* model.

Let us see the extended vector of increments of measurement data

$$\widetilde{\mathbf{\Lambda}}_0 = [\Delta \widetilde{V}_1, \Delta \widetilde{V}_2 ... \Delta \widetilde{V}_m, 0, 0 ... 0]$$

in which the n last components are zeros.

Let us compose matrix B_0 with $n + 1$ columns where on the first m rows there are elements of sensitivity matrix B, and on the next n there is a zero vector and an identity matrix.

$$\mathbf{B}_0 = \begin{vmatrix} \beta_{10} & \beta_{11} & \cdots & \beta_{1n} \\ \beta_{20} & \beta_{21} & \cdots & \beta_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \beta_{m0} & \beta_{m1} & \cdots & \beta_{mn} \\ 0 & 1 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 1 \end{vmatrix}$$

As mentioned above, conditions are simultaneously satisfied, if

$$\left(\widetilde{\mathbf{\Lambda}}_0 - \mathbf{B}_0\mathbf{\Delta}\right)^T \left(\widetilde{\mathbf{\Lambda}}_0 - \mathbf{B}_0\mathbf{\Delta}\right) = \min_{\mathbf{\Delta}}$$

From here a system of linear equations over the increment vector $\mathbf{\Delta}$ follows

$$\mathbf{B}_0^T \mathbf{B}_0 \mathbf{\Delta} = \mathbf{B}_0^T \widetilde{\mathbf{\Lambda}}_0$$

The increment vector $\mathbf{\Delta}$ allows to refine all state parameters – it is possible to calculate both the specified measured parameters (estimates) and whatever other operating parameters.

For finding the elements of the sensitivity matrix B it is necessary, at first, to assign the increments of the model parameters $\Delta p_0 ... \Delta p_n$ in turn, then to find the corresponding load values and, at last, to calculate the increments ΔV_j of considered state parameters on the basis of non-linearized network equations. As a result

$$\beta_{ij} = \frac{\Delta V_j}{\Delta p_i}, \quad i = 0...n, j = 1...m$$

The results are authentic, if there are no bad data and exclusive states are not observed. If bad data are obtained, they have to be removed and the estimation procedure must be repeated. It is also possible to attempt to define the exclusive states more closely. If the exclusive state is still unclear, the estimation of this state is not possible, and the measurements of this moment are not used. As a result, dispersion of the short-term operating forecasts increases to a certain degree, but in principle this does not obstruct the process of estimation.

Bad data and exclusive loads are tried to disclose one by one or as scenarios. Errors among measured data may be combined with one another, for example due to telecommunication line interference. The symptoms of exclusive situations are detected by rating the load deviations and the elements of the sensitivity matrix.

Editing the Load Models

For considering possible changes in the load character, it is necessary to edit (specify) the model parameters. In this connection the model components are fixed.

Let us see the node loads p_i with parameters of the model $a_{i1}, a_{i2}...a_{ir}$ at the moment t_k

$$p_i(t_k, a_{i1}, a_{i2} ... a_{ir}), \quad i = 1...n$$

where r is the number of parameters that have to be estimated.

Next the network equations are presented

$$V_j = \mathrm{V}_j(U_0, p_1, p_2, ... p_n)$$

describing the state parameters *via* the parameters of the load model

$$V_{jk} = \mathrm{F}_{jk}(a_1, a_2, ... a_l), \quad j = 1...m$$

where $l = nr$ is the overall number of the parameters of all load models. The function F_{jk} corresponds to the moment t_k on which the load values and supply voltage depend.

The parameters of the load model are calculated from criterion

$$\sum_j \sum_k \left[\widetilde{V}_{jk} - V_{jk}\right]^2 = \min$$

where $\widetilde{V}_{jk}$ is the measurement data at the moment k.

Network equations, linearized over the model parameters, are

$$\Delta V_{jk} = \alpha_{1jk}\Delta a_1 + \alpha_{2jk}\Delta a_2 + \ldots + \alpha_{ljk}\Delta a_l$$

where $\alpha_{sjk} = \dfrac{\partial V_{jk}}{\partial a_s}$, $s = 1 \ldots l$

In the form of matrix

$$\mathbf{\Lambda}_k = \mathbf{A}_k \mathbf{\Delta}$$

where $\mathbf{\Lambda}_k = \left[\Delta V_{1k}, \Delta V_{2k} \ldots \Delta V_{mk}\right]$ is the increment vector of measured parameters;

$\mathbf{\Delta} = \left[\Delta a_1, \Delta a_2 \ldots \Delta a_l\right]$ is the increment vector of model parameters;

$\mathbf{A}_k = \begin{vmatrix} \alpha_{11k} & \ldots & \alpha_{1lk} \\ \ldots & \ldots & \ldots \\ \alpha_{m1k} & \ldots & \alpha_{mlk} \end{vmatrix}$ is the sensitivity matrix (Jacobean).

If the vector of metering deviations is

$$\widetilde{\mathbf{\Lambda}}_k = \left[\Delta \widetilde{V}_{1k}, \Delta \widetilde{V}_{2k} \ldots \Delta \widetilde{V}_{mk}\right]$$

the increments for model parameters are calculated from the criterion

$$\sum_k \left(\widetilde{\mathbf{\Lambda}}_k - \mathbf{A}_k \mathbf{\Delta}\right)^T \left(\widetilde{\mathbf{\Lambda}}_k - \mathbf{A}_k \mathbf{\Delta}\right) = \min_{\mathbf{\Delta}}$$

that gives the system of linear equations over the vector $\mathbf{\Delta}$

$$\sum_k \left(\mathbf{A}_k^T \mathbf{A}_k\right)\mathbf{\Delta} = \sum_k \mathbf{A}_k^T \widetilde{\mathbf{\Lambda}}_k$$

The load values, calculated by load models (short-term forecasts), form the basis for linearizing the network equations. Metering deviations corresponding to the same values are found. Linearization is acceptable, if the metering deviations are not too large. Otherwise it is necessary to use the iterative process, according to which the increments are added to the model parameters, and new state parameters and metering deviations are calculated. In this connection repeated linearization (calculation of the new sensitivity matrix) is not needed, and it is possible to continue with the old matrix. The elements of sensitivity matrix are calculated as follows

$$\alpha_{sjk} = \frac{\Delta V_{jk}}{\Delta a_s}, \quad s = 1 \ldots l, j = 1 \ldots m.$$

The number of measurement data must be larger than the number of parameters being estimated. For example, if the number of the loads is 10 (5 distribution substations), parameters 10 and measurements 10, the number of the required values is l = 100, and so more than 10 metering hours are needed. Therefore the daily metering is enough to get the results. However, the authenticity of results is a separate question. It is clear that the model, estimated on the basis of the data of one or some days, is not usable for a longer period. For example, the data of winter loads is not representative for summer loads. Generally it is an adaptation problem, which can be resolved taking into account the essential meaning of the model parameters.

The degree of the equation system, obliged to be resolved, is high (for the above-mentioned example it is 100). It means that the volume of calculations is too large, and it may mean that the equation system is ill-conditioned. The way out is to use the results of state estimation, by means of which the node loads are always found independently from the structure of measured state parameters. Consequently, it is possible to receive all load values, state parameters and edit the parameters of all load models. If m = 1 and $l = r$ we can get

$$\mathbf{A}_k = \left|\alpha_{1k} \ldots \alpha_{rk}\right|$$

$$\mathbf{\Delta} = \left[\Delta a_1, \Delta a_2 \ldots \Delta a_r\right]$$

$$\widetilde{\mathbf{\Lambda}}_k = \left[\Delta \widetilde{V}_k\right] = \widetilde{p}_k$$

$$\sum_k \left(\mathbf{A}_k^T \mathbf{A}_k\right)\mathbf{\Delta} = \sum_k \widetilde{p}_k \mathbf{A}_k^T$$

where $\widetilde{p}_k$ is the estimated value of the observed load at the moment k. The degree of the equation system is now r, or for the above-mentioned example it is now 10, and problems about calculation volumes are settled.

Conclusions

The method of distribution network estimation described above is based on the physically well-grounded mathematical model that considers time-dependencies, stochasticity, temperature dependency, and also voltage and frequency dependencies of a load.

The efficiency of the estimation method described above depends on the accuracy of the load models, which in turn depends on the regularity of load changes. For irregular loads it is possible to use the trivial model. If the number of irregular loads is large, or the values of these loads are relatively large, editing of the models does not give essential refinements. Traditional load models are generally similar to the trivial model, and so they are not effective enough for estimation of the distribution network state.

By means of the method described above it is possible to learn how the network state is compatible with both the load and network models, and whether the network state is familiar or not. It is possible to present exceptional states like events and fix that the computation on the basis of available data is not adequate. Though the deficit of needed data does not allow refining the measurement data, it is still possible to clarify the faulty measurements. When the method is utilized, there are no problems about computer resources, because there are no large volumes of computations.

It is important that both the state estimation and the editing of load models will take place. So, the models always correspond to the real situation in the distribution network. On the basis of the load models it is possible to calculate, forecast and analyze the network states for both the short-term and long-term time intervals.

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