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OPTIMIZATION OF OPERATING RESERVES IN POWER SYSTEMS

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Three problems of planning and optimization of operational reserves in power system are considered in this paper: 1) planning of reserve size, 2) determination of costs related to the keeping of operating reserves, 3) optimal utilization of operating reserves. The size of control reserves has to be determined on the basis of probabilistic information about errors of load forecasting. Linear or non-linear models could be applied for the optimization of the costs of operating reserve keeping. For optimal utilization of reserves the application of dynamic characteristics of power units is recommended.

Introduction

There are various optimization problems in the power system operation and planning. Some of these problems are to be solved during real operation; however, many problems are planning problems for different time spans (from hours to several years).

Two tasks could be considered the main optimization problems:

1) optimization of expected operation of a power system for different time spans;
2) optimization of the distribution and utilization of reserves with reliability and security constraints.

In this paper the problem of optimizing the operating, control or spinning reserves in the power system is considered. These reserves are planned for the compensation of load deviations from the expected values and unexpected outages of generating capacities.

The operating reserves are usually divided into: 1) primary control reserve, 2) secondary control reserve, 3) tertiary control reserve (15-minute reserve); 4) slow scheduling reserve, 5) contingency reserve, including instant reserve, rapid reserve and slow reserve.

The problem of operating reserves is one of the problems most discussed by a number of researchers in different parts of the world ([1–9] and others).

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However, in spite of this many new aspects have not been considered sufficiently.

Planning of Operating Reserve Capacities

The reserve capacity required for the compensation of deviations in load demand from the expected values depends on the random character of deviations in load demand and on the models and methods of load forecasting. The spectrum of load deviations is very wide and comprises the changes of power demand with duration from some seconds to some days. The deviations in power demand increase with connecting wind-power units to the power systems.

The load demand of a power system $\widetilde{P}_D(t)$ is a non-stationary random process. Its density function is usually similar to the normal density function. We consider the power demand process consisting of two parts:

$$\widetilde{P}_D(t) = \overline{P}_D(t) + \Delta\widetilde{P}_D(t) \tag{1}$$

where $\overline{P}_D(t)$ – expected (predicted) values of power demand,

$\Delta\widetilde{P}_D(t)$ – random component of power demand or random errors of forecasting.

Forecast errors are positive and negative. Let $\Delta\widetilde{P}_D^{\max}(t)$ be the maximum error, then

$$-\Delta P_D^{\max}(t) \le \Delta\widetilde{P}_D(t) \le \Delta P_D^{\max}(t) \tag{2}$$

For the compensation of these errors, we must have power reserve

$$\overline{P}_R(t) = \pm\Delta\widetilde{P}_D^{\max}(t) \tag{3}$$

Assuming that load forecast errors are normally distributed random variables, the maximum error can be determined by standard deviation of forecast errors:

$$\Delta\widetilde{P}_D^{\max}(t) = 3\sigma_{\Delta P}(t) \tag{4}$$

where $\sigma_{\Delta P}(t)$ – standard deviation of $\Delta\widetilde{P}_D(t)$.

Information about the capacity of reserves, needed for compensation of load forecast errors, may be given by probability mass function of forecast errors, for instance, in the form of normal density (see below), where I_R is the interval of reserve and σ – standard deviation of load demand errors.

	1	2	3	4	5	6
I_R	$[-3\sigma;-2\sigma]$	$[-2\sigma;-\sigma]$	$[0;-\sigma]$	$[0;\sigma]$	$[\sigma;2\sigma]$	$[2\sigma;3\sigma]$
Probability	0.02	0.14	0.34	0.34	0.14	0.02

Reserve requirements for compensation of load forecast errors are:

$$\sum_{i=1}^{K} P_i^{\max} \geq \overline{P}_D(t) + \overline{P}_R^{+}(t) + \overline{P}_R^{out}(t) \tag{5}$$

$$\sum_{i=1}^{K} P_i^{\min} \leq \overline{P}_D(t) - \overline{P}_R^{-}(t) \tag{6}$$

where $P_i^{\min}$, $P_i^{\max}$ – minimum and maximum admissible loads of unit *I;*

$\overline{P}_R^{+}(t)$, $\overline{P}_R^{-}(t)$ – planned values of needed operating reserves for compensating forecast errors of load demand up and down (positive and negative errors);

$\overline{P}_R^{out}(t)$ – operating reserve needed for compensating unexpected outages of generating capacities;

K – number of units.

Reserve for unexpected outages of generating capacities could be taken equal to the capacity of the biggest unit in the power system or chosen by the desired reliability level of the system [4, 5].

Let us assume that the optimal unit commitment and optimization of load scheduling are made without reserve requirements. At that we have a certain natural reserve, but this may not satisfy the reserve requirements. For this reason the problem of planning reserves must be formulated separately as a sub-problem of optimal planning of power system operation. For the optimal planning of reserves, the linear or non-linear models could be used.

Cost of Operative Reserve Keeping

To keep the reserve capacities for regulating up (in the direction of load increase), the power system should operate with a set of running units that exceeds the optimal capacity, and to provide the reserve for regulating down the power system should operate with running units that are less than the optimal set. In case there does not exist an optimal set of operating units which complies with both conditions of regulating up and regulating down, it is necessary to plan the set of operating units separately for minimum load and maximum load periods.

Let us denote:

- *K* as the optimal combination of units for expected load demand for the time period *T* without reserve requirements,
- *L* as the optimal combination of units for expected load demand for the time period *T* with both reserve requirements (5) and (6),
- *M* as the optimal combination of units for expected load demand for the time period *T* with reserve requirements (5) to the plus (+) direction,
- *N* as the optimal combination of units for expected load for the time period *T* with reserve requirements (6) to the minus (–), direction.

In this case the cost of keeping of reserves for the time period *T*, when the power system is operating with commitment of given units, may be determined by equations:

$$\Delta F_R = F(L) - F(K) + G^{START}(K,L) \tag{7}$$

$$\Delta F_R^+ = F(M) - F(K) + G^{START}(K,M) \tag{8}$$

$$\Delta F_R^- = F(N) - F(K) - G^{START}(K,N) \tag{9}$$

where ΔF_R – cost of keeping of reserves for the up and down directions;

$F(K)$ – minimum total fuel cost for expected load demand if the power system is operating with combination of units *K*;

$G^{START}(K,L)$ – total start-up cost to change the unit commitment from the combination *K* to the combination *L*;

ΔF_R^+ – cost of keeping of reserves for the up direction;

ΔF_R^- – cost of keeping of reserves for the down direction.

The total costs $F(K)$, $F(L)$, $F(M)$ and $F(N)$ are obtained by economic load dispatch between all units.

Linear Model for Optimal Utilization of Reserves

Let $\overline{P}(t) = \langle \overline{P}_1(t),...,\overline{P}_K(t)\rangle$ be the vector of optimal planned loads of power units. Then the linear model for optimal utilization of reserves is as follows.

Find optimal unit commitment and optimal vector of reserve capacities $\overline{P}_R(t) = \langle P_{R1}(t),...,P_{RL}(t)\rangle$ that will minimize the cost of the utilization of reserves in the power system:

$$\min \sum_{t=1}^{S} \sum_{i=1}^{L} (c_{0i} + c_i \overline{P}_{Ri}(t))\Delta t \tag{10}$$

subject to:

$$\Delta\overline{P}_D(t) + \overline{P}_R^{outage} - \sum_{i=1}^{L} \overline{P}_{Ri}(t) = 0 \tag{11}$$

$$\sum_{i=1}^{L} P_i^{\max} \geq \overline{P}_D(t) + \Delta\overline{P}_D^+(t) + \overline{P}_R^{outage}(t) \tag{12}$$

$$\sum_{i=1}^{L} P_i^{\min} \leq \overline{P}_D(t) - \Delta\overline{P}_D^-(t) \tag{13}$$

$$P_i^{\min} - \overline{P}_i(t) \leq \overline{P}_{Ri}(t) \leq P_i^{\max} - \overline{P}_i(t), \ i = 1,\dots,L, \tag{14}$$

where $\Delta\overline{P}_D(t)$ – load deviation;

$\Delta\overline{P}_D^+(t)$ – maximum load deviation in the up direction;
$\Delta\overline{P}_D^-(t)$ – maximum load deviation in the down direction;
$\overline{P}_{Ri}(t)$ – optimal planned operating reserve of the *i*-th power plant;
$t = 1,\ldots, S$ – time intervals of the planning period;
L – number of units taking part in the reserve provision;
c_0, c_i – fixed and variable costs of the unit, respectively.

Number L includes the main units (K) and special units for reserves (gas turbines, diesel units, etc).

The problem (10)–(14) can be solved using methods of linear programming (LP).

Non-Linear Model for Optimal Utilization of Reserves

Find optimal unit commitment and optimal vector of reserves $\overline{P}_R(t) = \langle P_{R1}(t),\ldots,P_{RL}(t)\rangle$ that will minimize the cost of the utilization of reserves

$$\min \sum_{t=1}^{S}\sum_{i=1}^{L} C_{it}(\overline{P}_i(t) + \overline{P}_{Ri}(t))\Delta t \tag{15}$$

with the following constraints:

1) constraints (12)–(13) for the optimization of unit commitment,
2) constraints (11) and (14) for planning the distribution of reserve capacities between the units.

The Lagrange function for the reserve distribution problem can be written in the form:

$$\Phi = \sum_{t=1}^{S}\sum_{i=1}^{L} C_i(\overline{P}_i(t) + \overline{P}_{Ri}(t))\Delta t + \sum_{t=1}^{S} \mu(t)(\Delta\overline{P}_D(t) + \overline{P}_R^{outage} - \sum_{i=1}^{L}\overline{P}_{Ri}(t)) \tag{16}$$

with constraints (14).

Here $\mu(t)$ – Lagrange multiplier for *t*-th time interval.

The optimality conditions are as follows:

$$\frac{\partial\Phi}{\partial\overline{P}_{Ri}(t)} = \frac{\partial C_i(t)}{\partial\overline{P}_{Ri}(t)} - \mu(t)\begin{cases}\geq 0 & \overline{P}_{Ri} = P_i^{\min} - \overline{P}_i(t)\\ = 0, \text{ if } & P_i^{\min} - \overline{P}_i(t) < \overline{P}_{Ri} < P_i^{\max} - \overline{P}_i(t)\\ \leq 0 & \overline{P}_{Ri} = P_i^{\max} - \overline{P}_i(t)\end{cases} \tag{17}$$

with constraints (11) and (14).

For solution of the problem, the μ-iteration method, gradient methods and other non-linear programming methods may be used.

So, as the utilization of operative reserves is a random process, then it should be planned in the probabilistic form. For instance, the optimal utilization of reserve capacities could be determined in the form of distribution density of the optimal utilization of the reserve capacities

$f_{Ri}(\overline{P}_R,t)$ or distribution series, or as a diagrammed function of the optimal utilization of the reserve capacity of the system load deviation $\overline{P}_R(\Delta P_D)$. It is convenient to use the last form in the operation control.

Dynamic Characteristics of Regulating Units

At planning the operation of a power system, the static input-output characteristics of fuel costs are used conventionally. A static input-output characteristic of fuel cost is a function of unit load – $C(P)$.

Those characteristics suit well for the optimization of average load of the time intervals of one hour or longer. However, the units participating in the primary and secondary control react to rapid changes of the system load, occurring in minutes and seconds. As for the units participating in the power and frequency control, their characteristics and limitation on the capacity ramping should be considered. There is no need to consider the differential equation describing the whole dynamics, but the operational costs of such units could be determined sufficiently accurate by simplified dynamic characteristics. For the units participating in the power and frequency control (primary and secondary control) the authors recommend applying the dynamic cost characteristics, where the speed of output increase is considered, instead of the static input-output characteristics. In this case, we will have the following dynamic input-output characteristics of units:

1) dynamic cost characteristic:

$$C = C(P,\frac{\partial P}{\partial t}) = C(P)\times(1+a\frac{\partial P}{\partial t}) + b\frac{\partial P}{\partial t} + c(\frac{\partial P}{\partial t})^2 \qquad (18)$$

where *C(P)* is a static cost characteristic and *a, b, c* are coefficients for consideration of the speed of load changing;

2) dynamic incremental cost characteristic:

$$\beta(P,\frac{\partial P}{\partial t}) = \frac{\partial C(P,\frac{\partial P}{\partial t})}{\partial P} \qquad (19)$$

3) dynamic cost rate characteristic:

$$\delta(P,\frac{\partial P}{\partial t}) = \frac{C(P,\frac{\partial P}{\partial t})}{P} \qquad (20)$$

4) dynamic efficiency characteristic:

$$\eta(P,\frac{\partial P}{\partial t}) = \frac{P}{C(P,\frac{\partial P}{\partial t})} \qquad (21)$$

Additional costs due to the implementation of the control function by the participating units arise from

1) the non-linearity of the incremental cost characteristics of units, and
2) transient processes occurring in the units due to the dynamic nature of the control function, i.e. from the required higher speed of changing the output of units.

Conclusions

As a result of this study the following conclusions can be made:

3) The size of control reserves has to be determined on the basis of probabilistic information about load forecast errors.
4) The cost of reserve keeping could be determined as a difference between total costs estimated considering reserve constraints and total costs estimated without reserve constraints.
5) For optimal utilization of reserves, the linear and non-linear model presented in the paper could be used.
6) In the models of optimal utilization of reserves, application of simplified dynamic characteristics of power units is recommended.

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