Oil Shale, 2005, Vol. 22, No. 2 Special
pp. 143-151
ISSN 0208-189X

https://doi.org/10.3176/oil.2005.2S.07

STOCHASTIC MODELS OF GENERATING UNITS

M. VALDMA,* J. SHUVALOVA

Department of Electrical Power Engineering,
Tallinn University of Technology
5, Ehitajate Rd., 19086 Tallinn, Estonia

A general approach to modeling of thermal and hydro generating units for optimal operation of power plants and systems is presented. Usually the characteristics of generating units are expressed as deterministic functions. This paper describes the models of input-output characteristics on the deterministic, probabilistic, uncertainty and fuzzy level. On the probabilistic level the input-output curves may be described by the functions of probabilistic characteristics of input. On the uncertainty level it is necessary to determine the intervals of curves or the intervals of probabilistic characteristics of curves. On the fuzzy level the input-output curves are determined by membership functions as the fuzzy zones of curves. The models presented in the paper can be used for the determination of unit's characteristics on the probabilistic, uncertainty and fuzzy form.

Introduction

For optimal planning and operation of power plants and systems the information about power units and plants (boilers, thermal turbines, boiler-turbine-generator units, water turbines-generators, thermal and hydroelectric power plants, and so on) is required.

Traditional models of generating units are their input-output characteristics. Input-output characteristics of power units are determined by the special tests or custom measures and presented in the table form, by curves or by mathematical functions (traditionally by polynomials) [1–4].

The input-output characteristics of generating units have a random character, and their changing in operation process and after repairs is random, too, because of the information about input-output characteristics is never exact.

This paper introduces a general approach to modeling of power units for optimal operation calculations, where input-output characteristics may be presented on the deterministic, probabilistic, uncertainty and fuzzy level [5, 6].

* Corresponding author: e-mail *mati.valdma@ttu.ee*

Deterministic Form of Characteristics

Generating unit as a dynamic system (Fig. 1) can be modeled by output functions F and state transition equations H:

$$Y(t) = F(X(t),V(t),t) \tag{1}$$

$$\frac{\partial X}{\partial t} = H(X(t),V(t),t) \tag{2}$$

where X , V , Y are the vectors of input, state and output variables;
t is time.

However, in normal conditions the generating units are described by static input-output characteristics where state variables are constant or they are the functions of output, too.

Fig. 1. Generating units are the dynamic systems

The following characteristics are used for modeling generating units in optimal operation and planning of power system:

1) Input-output characteristic

$$X = G(Y), \text{ where } V = const. \text{ or } V = V(Y) \tag{3}$$

2) Incremental input rate characteristic

$$\beta(Y) = \frac{\partial X}{\partial Y} = \frac{\partial G(Y)}{\partial Y} \text{ and } X_0 = G(Y^{min}) \tag{4}$$

3) Input rate characteristic:

$$\delta(Y) = \frac{X}{Y} = \frac{G(Y)}{Y} \tag{5}$$

4) Efficiency characteristic:

$$\eta(Y) = \frac{Y}{X} = \frac{Y}{G(Y)} \tag{6}$$

These are the static models of power units, where a concrete value of output determines the concrete value of input, incremental input rate, input rate or efficiency of unit.

Some typical curves of these characteristics are shown in Figs 2–4.

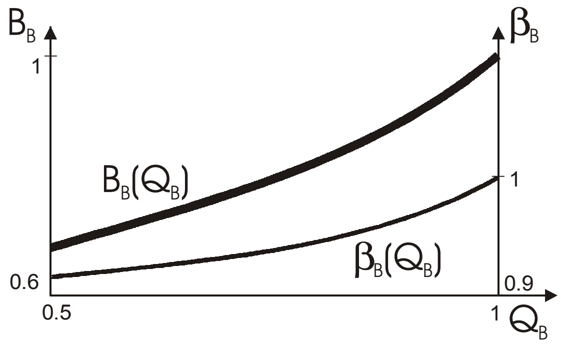

Fig. 2. Typical curves of boilers: $B_B(Q_B)$ – fuel curve, $\beta_B(Q_B)$ – incremental fuel curve

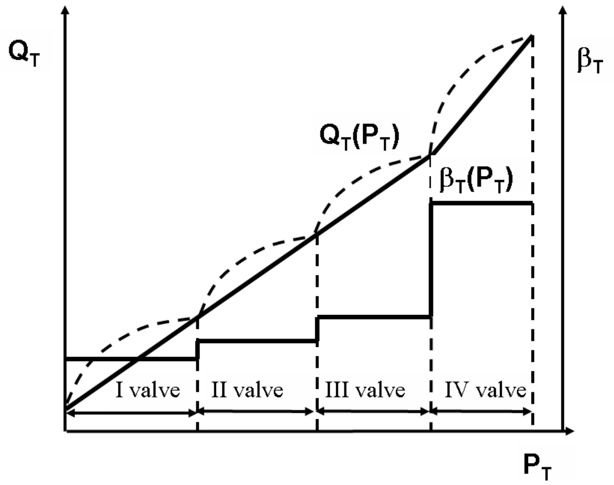

Fig. 3. Typical curves of condensing turbine + generator unit: $Q_T(P_T)$ – thermal input curve, $\beta_T(P_T)$ – incremental thermal rate curve

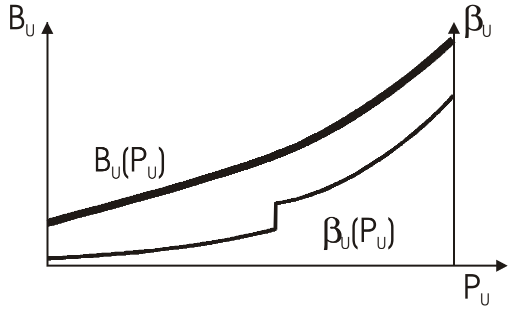

Fig. 4. Typical curves of thermal generating unit – boiler + turbine + generator: $B_U(P_U)$ – fuel curve of unit, $\beta_U(P_U)$ – incremental fuel rate curve of unit

Input-Output Characteristics in Probabilistic Form

In reality the input-output characteristics of units have a random character caused by errors of determination of initial characteristics and by random changing of characteristics in operation. In principle the characteristics of units are functions of random parameters, because the deterministic information about input-output characteristics is always inaccurate and incomplete. Information about random character of units may be presented in a probabilistic form by different ways.

We have studied the following stochastic models of characteristics.

Theoretical Model

Theoretically the input-output characteristic of a unit can be considered as a function of formal random parameter:

$$\tilde{X} = G(Y, \tilde{\omega}) \tag{7}$$

where $\tilde{\omega}$ – a formal random parameter with density function $f(\omega)$, $-\infty < \omega < \infty$.

Then the expected input-output characteristic of the unit is:

$$\overline{X} = EG(Y, \tilde{\omega}) = \int_{-\infty}^{\infty} G(Y, \omega) f(\omega) d\omega = \overline{G}(Y) \tag{8}$$

and the dispersion of input is the function:

$$D_X = E\left[X - \overline{X}\right]^2 = \int_{-\infty}^{\infty} \left[G(Y, \omega) - \overline{G}(Y)\right]^2 f(\omega) d\omega = D_G(Y) \tag{9}$$

Here E – operator of mathematical expectation.

In practical models we must describe a random character of characteristics by random variables.

Model P1

The model P1 contains the two random variables – multiplicative variable $\tilde{w}$ and additive variable $\tilde{\varepsilon}$:

$$\tilde{X} = \overline{G}(Y) \times (1 + \tilde{w}) + \tilde{\varepsilon} \tag{10}$$

where $E\tilde{w} = 0$ and $E\tilde{\varepsilon} = 0$.

We can describe the input-output characteristics approximately in a probabilistic form by input expectation characteristics and characteristics of dispersion or standard deviations:

$$\overline{X} = \overline{G}(Y) \tag{11}$$

and

$$D_X = \overline{G}^2(Y) \times D_w + D_\varepsilon \tag{12}$$

or

$$\sigma_X = \sigma_X(Y) \tag{13}$$

where

$$\sigma_X = \sqrt{D_X} = \sqrt{\overline{G}^2(Y) \times D_w + D_\varepsilon} \tag{14}$$

The model (10), which enables to take into account that random realizations of characteristics may be turned and dislodged relatively to expected input-output characteristics. Here the random variables describe the random errors of characteristics.

The complete probabilistic information about input-output characteristic of a unit may be given by expected characteristic $\overline{X} = \overline{G}(Y)$ and two-dimensional distribution function $F(w,\varepsilon)$ or density function $f(w,\varepsilon)$.

Figure 5 shows the functions of mathematical expectation $\overline{G}(Y)$ and standard deviation $\sigma_X(Y)$ of input-output characteristic of thermal generating unit.

The probabilistic information is more perfect than the deterministic information, because the probabilistic form enables to consider the probabilistic information about a random character of a characteristic. The model P1 enables to describe the random realization of characteristics and to consider a cumulative changing of input-output curves in operation [3–4].

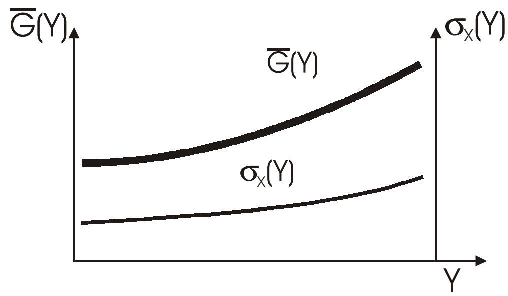

Fig. 5. Probabilistic information about input-output characteristic of thermal power unit:
$\overline{G}(Y)$ – function of mathematical expectation,
$\sigma_X(Y)$ – function of standard deviation of input

Input-Output Characteristic in Uncertainty Form

If the probabilistic information about input-output characteristic is lacking or is very approximate, we have to present the information in the uncertainty form.

There are two types of uncertainties [5, 6]:

1) **deterministic uncertainty** – only the intervals of uncertainty for values of input-output characteristics are given
2) **probabilistic uncertainty** – only the intervals of uncertainty for the probabilistic information of input-output characteristics are given.

The uncertainty form of information means that the limits of intervals for values or for probabilistic parameters are exactly given. However, inside of the intervals the values of parameters are not determined. In order to model characteristics in the uncertainty form the model P1 can be used.

Information about input-output characteristics in a uncertainty form may be presented in the following forms:

1) the deterministic uncertain information:

$$G^{min}(Y)=\overline{G}(Y)(1+w^{min})+\varepsilon^{min}\le G^{Actual}(Y)\le G^{max}(Y)=\overline{G}(Y)(1+w^{max})+\varepsilon^{max} \quad (15)$$

where the functions $G^{min}(Y)$ and $G^{max}(Y)$ or the function $\overline{G}(Y)$ and the limits of variables w^{min}, w^{max} and $\varepsilon^{min}, \varepsilon^{max}$ are given.

2) The probabilistic uncertain information:

a) uncertain information about the mathematical expectation of input

$$\overline{G}^{min}(Y)=\overline{G}(Y)(1+\overline{w}^{min})+\overline{\varepsilon}^{min}\le \overline{X}\le \overline{G}^{max}(Y)=\overline{G}(Y)(1+\overline{w}^{max})+\overline{\varepsilon}^{max} \quad (16)$$

where the functions $\overline{G}^{min}(Y)$, $\overline{G}^{max}(Y)$ or the function $\overline{G}(Y)$ and the values of expectations $\overline{w}^{min}, \overline{w}^{max}$ and $\overline{\varepsilon}^{min}, \overline{\varepsilon}^{max}$ must be given;

b) uncertain information about the standard deviations of input

$$\sigma_X^{min}(Y)\le \sigma_X\le \sigma_X^{max}(Y) \quad (17)$$

where the functions $\sigma_X^{min}(Y)$, $\sigma_X^{max}(Y)$ or the function $\overline{G}(Y)$ and the limits of dispersion D_w^{min}, D_w^{max}, D_ε^{min}, D_ε^{max} must be given.

Here

$$\sigma_X^{min}=\sqrt{\overline{G}^2(Y)\times D_w^{min}+D_\varepsilon^{min}} \quad (18)$$

$$\sigma_X^{max}=\sqrt{\overline{G}^2(Y)\times D_w^{max}+D_\varepsilon^{max}} \quad (19)$$

The examples of the input-output characteristic in the form of uncertainty are shown in Fig. 6.

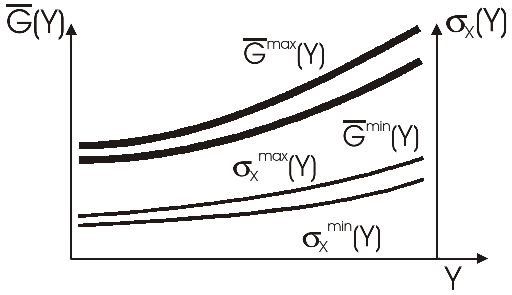

Fig. 6. Input-output characteristics in a form of uncertainty

Input-Output Characteristics in a Fuzzy Form

If the limits of intervals of uncertainty (15)–(17) are not assigned exactly, we have only the fuzzy information about input-output characteristics. The application of fuzzy set theory to power systems is a relatively new area of research. The interest towards using fuzzy information in power system operation and planning is increasing [7].

Information in a fuzzy form is presented by fuzzy sets. A fuzzy set is defined as a set of ordered pairs [8]:

$$Z'' = \{x, \mu_Z(x) / x \in Z\} \tag{20}$$

where $\mu_Z(x)$ is called the membership function of Z'', which indicates the degree that x belongs to Z'' [5].

Here Z is a crisp set, and Z'' is a fuzzy set, $0 \le \mu_Z(x) \le 1$. At that there are two different forms of fuzzy information [5, 6]:

1) deterministic fuzzy information – the zones of actual values of input-output characteristics are given in the fuzzy form;
2) probabilistic fuzzy information – the zones of probabilistic characteristics are given in the fuzzy form.

For demonstration the fuzzy forms of information we will use model P2 that is simplified model from P1:

$$\tilde{X}(Y, \tilde{w}) = \overline{G}(Y) \times (1 + \tilde{w}) \tag{21}$$

where $\tilde{w}$ is a fuzzy variable given by the membership function $\mu(w)$:

- if $w < w_1$ then $\mu(w) = 0$
- if $w_1 \le w \le w_2$ then $0 \le \mu(w) \le 1$
- if $w_2 \le w \le w_3$ then $\mu(w) = 1$
- if $w_3 \le w \le w_4$ then $0 \le \mu(w) \le 1$
- if $w > w_4$ then $\mu(w) = 0$.

The main curves for presentation of input-output characteristics in the fuzzy form are: $X(Y, w_1)$, $X(Y, w_2)$, $X(Y, w_3)$ and $X(Y, w_4)$. The input-output characteristic in a deterministic fuzzy form is presented in Fig. 7.

By analogy the fuzzy zones of expected input-output characteristics, characteristics of standard deviation and others probabilistic characteristics can be written. Using the fuzzy zones it is possible to include subjective information and intuition of specialists in the models of power units.

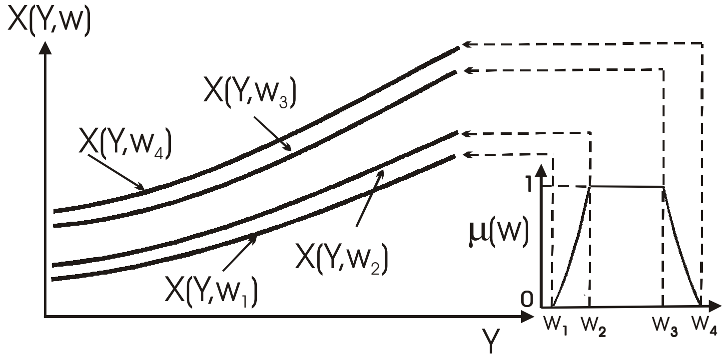

Fig. 7. Input-output characteristics in a deterministic fuzzy form

Conclusions

The input-output characteristics of power generating units may be presented in the deterministic, probabilistic, interval uncertainty or fuzzy form. The models described in the paper enable a much more complete and relatively simple presentation of the input-output characteristics of units than the deterministic form of characteristics used commonly. After changing the form of presenting the input-output characteristics, it is necessary to change also the problems of operation, planning and analyzing. The models considered here, can be used also for Monte Carlo simulation of input-output characteristics of power generating units.

Acknowledgements

Authors thank the Estonian Science Foundation (Grants No. 3389, No. 5176 and No. 5181) and State target financed research projects (0140223s98 and 0142512s03) for financial support of this study.

REFERENCES

1. *Steinberg, M. J., Smith, T. H.* Economy Loading of Power Plants and Electric Systems. – New York : John Wiley and Sons, 1943.
2. *Gornstein, V. M.* Economical Load Distribution between in Parallel Operating Power Plants. – Moscow : *Gosenergoizdat*, 1949 [in Russian].
3. *Valdma, M., Viilup, L., Mölle, K., Tammoja, H.* About estimation of input-output characteristics of thermal plant's units // Transactions of Tallinn Technical University, 1976. 403. P. 17–27 [in Russian].
4. *Valdma, M., Tammoja, H.* A method of optimal load planning between units and calculation of input-output characteristics of thermal plant // Transactions of the

Tallinn Technical University. 1976. 403. P. 29–44 [in Russian].

5. *Valdma, M.* Initial propositions of power systems state optimization under incomplete information // Transactions of Tallinn Technical university. 1983. 549. P. 3–18 [in Russian].
6. *Valdma, M*. Optimal control of power systems under incomplete information // Proc. Second International Conference on Automation, Robotics and Computer Vision (ICARCV '92). Singapore. 1992. P. CO-11.6.1.
7. *El-Hawary, M. E*. Electric Power Applications of Fuzzy Systems. – New York : IEEE Press, 1998.
8. *Zadeh, L. A.* Fuzzy Sets // Information and Control. 1965. Vol. 8. P. 338–353.

Received November 11, 2004