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OPTIMAL LOAD DISPATCH IN POWER PLANT UNDER PROBABILISTIC INFORMATION

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The optimal operation of power plants, electrical networks and power systems is a very important problem also in the electricity market conditions. There are many unsolved problems in the field of optimal operation of power systems and power plants. Usually these problems are tackled as deterministic ones. Actually, the initial information is never complete. The information may occur in several forms: probabilistic, uncertain and fuzzy. This paper presents the principles of optimal dispatch of thermal and electrical power between boilers and power units in a power plant under incomplete information.

Introduction

This paper presents the principles of optimal dispatch of thermal power between boilers and electrical power between the condensing units in a power plant under incomplete information. Computation problem of optimum active power generation schedules in a power plant for a certain time period (day, week) is one of relevant optimization assignments in power plant control. Usually this problem is tackled as a deterministic one where the objective function, constraints and uncontrollable factors are single-valued [1].

At the same time, it is assumed that planned generation schedules would be exactly realized. Actually, the initial information is never complete, and the planned generation schedules would be corrected several times. Inconsideration of these circumstances decreases the efficiency of optimization. Therefore it is necessary to elaborate methods of power plant state optimization, which take into account really existing incompleteness of the initial information and deviations of factual states of power plant from the planned ones. This problem is very actual in these power systems where there are lots of wind power, engine and gas turbine cogeneration plants [2–4].

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Initial Mathematical Model

The input-output characteristic of a condensing unit can be presented as a composite function:

$$C = cB(Q_T(P)) = C(P) \tag{1}$$

where c – price of the fuel;

C – fuel cost of the unit;

P – power output of the unit;

$B(Q_B)$ – input-output characteristic of the boiler, assuming that $Q_B = Q_T$;

Q_T – heat input of the turbine;

$Q_T(P)$ – input-output characteristic of the condensing turbine;

$C(P)$ – input-output characteristic of the condensing unit.

Condensing power units have usually continuous, piecewise smooth and strictly convex cost functions.

The optimal dispatch problem in a condensing power plant is to determine the loads of the units so that the production cost is minimized and all constraints are met. The initial deterministic optimization model is

$$\min_P C = \sum_{i=1}^{n} C_i \tag{2}$$

subject to the following constraints:

1) power balance equation:

$$G = P_P + \sum_{i=1}^{n} P_{ai}(P_i) - \sum_{i=1}^{n} P_i = 0 \tag{3}$$

2) power limitation constraints of power units (i = 1…n)

$$P_i^- \le P_i \le P_i^+ \tag{4}$$

where n – number of units;

P_P – active power demand of the power plant;

C_i – fuel cost of i-th power unit;

P_i – active power of i-th power unit.

P_i^-, P_i^+ – lower and upper limits of i-th power unit, respectively.

$P_{ai}(P_i)$ – auxiliary power characteristic of i-th power unit.

The problem (2)–(4) gives an opportunity to determine optimal active power generation schedules of power units in a condensing power plant in the ideal case, when all the initial data are known exactly, and optimal power schedules would also be realized exactly.

Conditions of Optimality

The Langrangian function of this problem is

$$\Phi = C + \lambda G \tag{5}$$

where λ is Lagrangian multiplier.

By the use of the Lagrangian function, the problem of power economic dispatch can be written as follows:

$$\min_{P} \max_{\lambda} \Phi \tag{6}$$

The conditions for the optimum are:

$$\frac{\partial \Phi}{\partial P_i} = \frac{\partial C_i}{\partial P_i} + \lambda \cdot \left(\frac{\partial P_{ai}}{\partial P_i} - 1 \right) \begin{cases} = 0, when: P_i^- \le P_i^o \le P_i^+ \\ \ge 0, when: P_i^o = P_i^- \\ \le 0, when: P_i^o = P_i^+ \end{cases} \tag{7}$$

The superscript "o" of the symbols means the optimal value of the parameter. The optimal solution of the problem (2)–(4) must satisfy the conditions (3) and (7).

Sub-problem: If the power plant consists of double-units there are two boilers and one turbine in a unit. In Estonia all power units fired with oil shale are double-units.

There is an optimization problem of power unit control: the optimal dispatch of thermal power of turbine input between boilers of a double-unit. The optimal load dispatch problem between boilers in a power unit is to determine the loads of boilers so that the production cost is minimized and all constraints are met. The initial deterministic optimization model is

$$\min_{Q_B} C = \sum_{i=1}^{2} C_i \tag{8}$$

subject to the following constraints:

1) equation of thermal power balance:

$$G = Q - \sum_{i=1}^{2} Q_{Bi} = 0 \tag{9}$$

2) thermal power limitation constraints of boilers (i = 1,…,2)

$$Q_{Bi}^- \le Q_{Bi} \le Q_{Bi}^+ \tag{10}$$

where Q – thermal input of the turbine;

C_i – fuel cost of i-th boiler;

Q_{Bi} – thermal power of i-th boiler;

Q_{Bi}^-, Q_{Bi}^+ – lower and upper limits of i-th boiler, respectively.

The problem (8)–(10) gives an opportunity to determine optimal thermal power generation schedules of boilers in double-units in ideal case, when all initial data are known exactly and optimal power schedules would also be realized exactly. The input-output characteristics of boilers and power units depend on lots of parameters which have random deviations from their nominal or planned values.

Some Results of Statistical Analysis of State Parameters of Power Units and Boilers

During many years the probabilistic characteristics of boiler state parameters, loads of boiler and power units are analyzed at Estonian power plants. The fuel of Estonian condensing power plants is oil shale. The calorific value of oil shale varies within great limits.

The results of statistical analyses of deviations of state parameters of the power unit and boiler are shown in the Table.

The Results of Statistical Analysis of Deviations of State Parameters of the Power Unit and Boiler

Parameter	Mathematical expectation, %	Root-mean-square, %	Coefficient of asymmetry
Load of power units	–2.5…4.5	5…10	1…3
Load of boilers	–15…16	5…18	–2…1.5
Flue gas temperature	–8…16	5…10	–3…–2
Feedwater temperature	–1…15	4…6	–2…–1
Calorific value of oil shale	–1.8…1.2	3…6	–1…2
Moisture of oil shale	0.1…0.2	7…10	–1…–2

Optimization Under Probabilistic Information

Let us assume that all the initial functions (characteristics of power units, boilers, and auxiliary power) and uncontrollable parameters in the problem (2)–(4) are random functions and variables, the initial information on which is available in probabilistic form. The input-output characteristics of boilers depend on flue gas temperature after boiler, fuel parameters, etc. Turbine characteristics depend on the following parameters: vacuum in condenser, pressure and temperature of steam at turbine inlet, etc. All these parameters are random. On the basis of probabilistic information it is possible to calculate new characteristics of boilers and turbine by the formulas:

- boiler:

$$C_B(Q_B) = C_B(Q_B, m_{X1}, \dots, m_{Xn}) + \frac{1}{2}\sum \frac{\partial^2 C_{Bj}(X)}{\partial X_j^2}\sigma_{Xj} + \frac{1}{2}\sum_{k=1}^{n}\sum_{j=1}^{n}\frac{\partial^2 C_B(X)}{\partial X_k \partial X_j}\cdot k_{Xk} k_{Xj}$$

- turbine:

$$Q_T(P_T) = Q_T(P_T, m_{X1}, \dots, m_{Xn}),$$

where m – mathematical expectations of the parameter;

σ – root-mean-square of the parameter.

These characteristics are the first-degree planned characteristics of units.

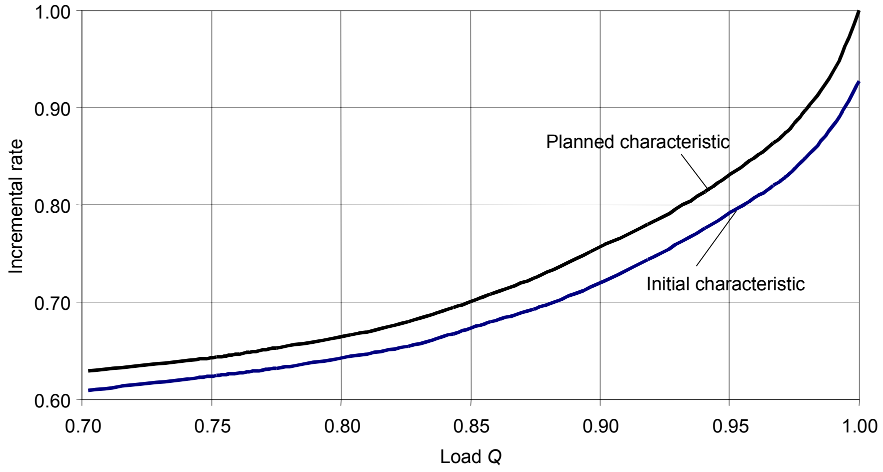

Fig. 1. Initial and planned characteristics of boiler

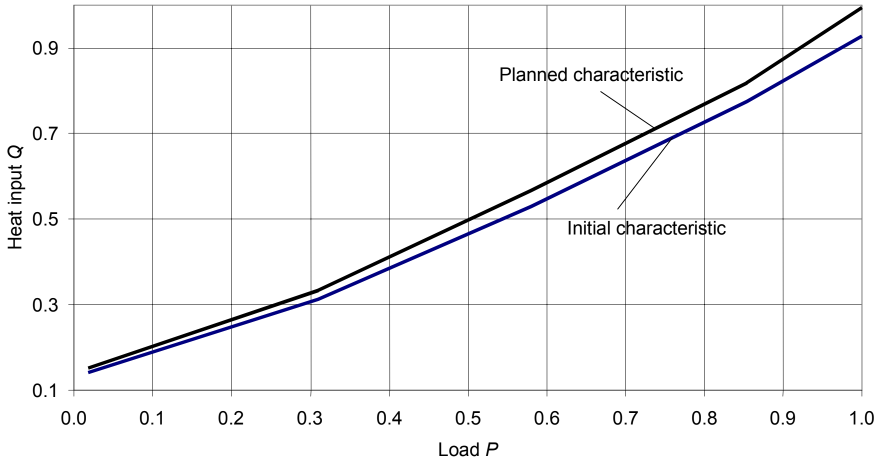

Fig. 2. Initial and planned characteristics of turbine

Moreover, random deviation of factual values of electricity generation of power units and boilers from their planned values will be considered. For that purpose the factual values of power units and boilers (double-units) are presented as sums of two summations:

$$\widetilde{P}_i = \overline{P}_i + \Delta\widetilde{P}_i \tag{11}$$

where $\overline{P}$ – planned values (mathematical expectations) of power or heat generations;

$\Delta\widetilde{P}$ – random deviations of generations that may be presented as random functions of planned values;

~ denotes the random character of parameter.

Deviations of power generation of units have different origins. The processes of deviations of controllable parameters are very complicated stochastic processes, which depend upon scheduling, errors of in forecasting initial data, methods of correction, effect of automatic regulators, influences of various distributed generation sources (wind- or solar power, engine- and gas turbine cogeneration plants), and several other factors. The deviations of boiler thermal power may be caused by deviations of fuel parameters (oil shale) and by the state of boiler steam-releasing surfaces.

In these conditions the actual minimum value of total fuel cost in power plant cannot be guaranteed, as factual values of electrical and heat powers and objective function are the random ones.

Incompleteness of information causes a possible fuel over-consumption in power plant.

Instead of (2), the mathematical expectation of total fuel cost can be minimized:

$$\min_{\overline{P}} \sum m\widetilde{C}_i(\overline{P}_i + \Delta\widetilde{P}_i) \qquad (12)$$

subject to corresponding constraints.

The constraints must guarantee the realization of optimal active generation schedules of power units on an average. For this purpose, it is necessary to substitute the equality constraints by inequalities of their mathematical expectations or by corresponding probabilistic equations.

Solution Method

The solution of this probabilistic optimization problem can be reduced to:

1. Determination of planned characteristics of boilers, auxiliary power and power units.
2. Solution of its deterministic equivalent.

The determination of planned characteristics is particularly complicated in general case. However, the functions in the problem of optimal scheduling are usually the additive ones. Therefore the calculation of planned characteristics considerably simplifies. The results of statistical analysis made in Estonian power plants pointed out that probabilistic information on random factors is sufficiently stable in time. These planned characteristics must be calculated relatively seldom. These planned characteristics are called the second-degree planned characteristics.

General expression for calculation of the second-degree planned characteristics is as follows:

$$\overline{C}(\overline{P}) = \int_{\Delta P^-}^{\Delta P^+} C(\overline{P}+\Delta P) f_{\Delta\tilde{P}}(\Delta P)\, d\Delta P \tag{13}$$

where $f_{\Delta\tilde{P}}$ – density of simultaneous distribution function of vector $\Delta\tilde{P}$.

The Fortran program has been worked out for calculation of the second-degree planned input-output characteristics of power units and boilers.

The second-degree planned characteristics of units and boilers take into account the random deviation of the loads of boilers and power unit from the planned optimal value.

If the input-output characteristics are continuous and strictly convex cost functions, we can simplify the expression (13). One approximate expression is:

$$\overline{C}(\overline{P}) = C(\overline{P}) + \frac{1}{2}\frac{\partial^2 C_B(\overline{P})}{\partial \overline{P}^2}\cdot\sigma^2_{\Delta\tilde{P}} \tag{14}$$

where $\sigma_{\Delta\tilde{P}}$ – a quadratic mean value of random deviation of load;

$a_{\Delta\tilde{P}}$ – a coefficient of asymmetry of random deviation of load.

The expression (14) may be used to calculate the planned input-output characteristics of boilers of double-units (two boilers and one turbine in unit) and power units which have continuous and strictly convex input-output characteristics. Double-units are used in Estonia and in countries, where low-calorific fuel (for example: oil shale, brown coal, etc.) is used.

Solution of the deterministic equivalent may be realized with ordinary computer programs and methods, which are elaborated for solution of deterministic optimal scheduling problems in thermal power plants.

In Figure 3 the planned incremental cost rate characteristic of power unit is presented.

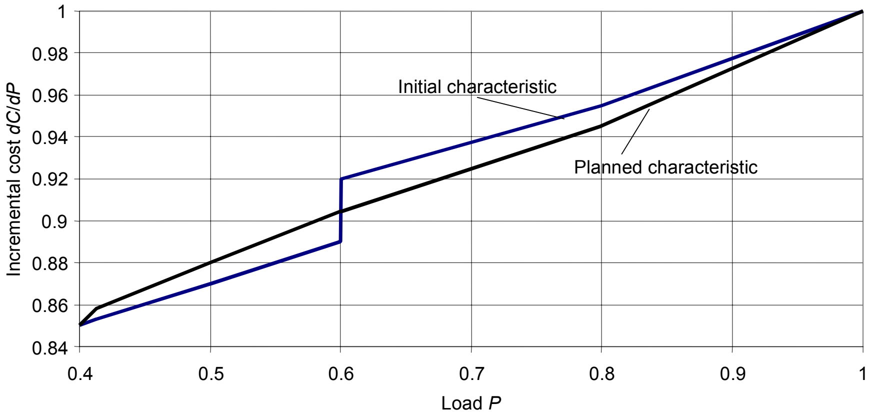

Fig. 3. The initial and the planned incremental cost rate characteristics (in relative units) of power unit

In case of need, the planned cost rate and planned input-output characteristics of units are calculated.

Computer Programs

The methodology described above is realized in a program complex at Tallinn University of Technology. Modules for state optimization enable to compute planned input-output characteristics of power units under probabilistic information and solve the optimization problem in power plants. That method may be realized by uncomplicated supplements for existing programs.

Conclusion

The methodology described here enables to consider the information in probabilistic, uncertain or fuzzy forms rather simply and to decrease the maximum possible economic loss caused by the incompleteness of information. The method of planned characteristics is used also in the programs for optimal scheduling of power generation in power systems [3].

The method of optimal dispatch in a power plant which takes into account the probabilistic information about random factors enables to economize the fuel by up to 0.9%.

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