Proceedings of the Estonian Academy of Sciences,
2020, **69**, 1, 57–73
https://doi.org/10.3176/proc.2020.1.07
Available online at www.eap.ee/proceedings

CONTROL
SYSTEMS

A brief tutorial overview of disturbance observers for nonlinear systems: application to flatness-based control

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Received 25 March 2019, accepted 19 June 2019, available online 13 February 2020

Abstract. The paper presents a brief overview of the most popular disturbance estimation techniques together with their application to flatness-based control. Two disturbance estimation approaches, the basic disturbance observer and the extended state observer, are described in a tutorial manner. Positive and negative aspects of both approaches are pointed out. Open research questions on disturbance estimation are presented. In the second part of the paper it is demonstrated how to integrate disturbance estimation into flatness-based control. The basic feedback linearization based approach, but also a novel event-based approach for differentially flat systems, are described. It is shown that disturbance estimation can be integrated easily into both of these control approaches. Finally, the results are demonstrated on three models: a heating, ventilation and air-conditioning; an active magnetic bearing; and an underwater vehicle models.

Key words: nonlinear control, disturbance estimation, flatness-based control, event-based control.

1. INTRODUCTION

Disturbances are inevitable part of almost any dynamical system. Since the behaviour of a disturbance in time is usually unknown and often the disturbance is unmeasurable, it causes a lot of trouble when designing controllers for the systems. Historically there exist two approaches to deal with disturbances. First, to design a robust controller, which yields in satisfactory performance even under the influence of the disturbances. Such control approaches are, for example: high-gain feedback; H_∞ control; passivity-based control; and sliding mode control. However, these approaches are robust, because they sacrifice some of the control performance. Second, to avoid the influence of the disturbance, a disturbance decoupling approach (see for instance [21,38]) is used. The goal here is to eliminate the influence of the disturbance from the system output to be controlled. The classical disturbance decoupling approach, unfortunately, is not always applicable. In the simple case of the single output it is important that the relative degree with respect to the disturbance would be strictly larger than the relative degree with respect to the control input. The latter is not necessary to solve the so-called almost disturbance decoupling problem [50], where the influence of the disturbance is not eliminated, but minimized. This, in turn, has similarities with robust control approaches, especially with the high gain feedback approach. However, there exists the third approach, which has become more and more popular in dealing with disturbances: the disturbance-observer-based

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control (DOBC) [32]. The idea of this approach is to estimate the disturbance (and possibly a finite number of time-derivatives of the disturbance) and then integrate this estimate into some control approach. In this paper we address the DOBC approach.

When disturbance decoupling is not possible and robust approaches are not good enough, it is natural to try to estimate disturbance signals. A historic overview of early results on disturbance estimation approaches is described in [22]. An overview of later developments can be seen, for example, in [10]. Over the years the disturbance estimation methods are called by many names: disturbance estimator, perturbation observer, extended state observer, disturbance observer etc. In this paper we refer to all of them as disturbance observers. Usually disturbance observers are studied in combination with DOBC. A positive aspect of DOBC, compared to the robust control approaches, is that when an estimate of the disturbance is used in the inner loop to compensate the effect of disturbance from system outputs, the performance of the outer loop controller is not degraded. That is why disturbance observers are integrated into some of the robust control methods, such as sliding mode control [49,54]. Also, the disturbance decoupling approaches have been studied in the DOBC framework using the disturbance observers [4,18,53]. In this paper the focus is on combination of disturbance observers with the flatness-based control approach.

The flatness or feedback linearization-based control is very natural to combine with disturbance observers. Flatness is a system property which allows to parametrize all system trajectories by the so-called flat output and its time-derivatives. This is true when the system model is exact and no external disturbances are present. If there are disturbances acting on the system equations, these disturbances (and possibly a finite number of their time-derivatives) affect directly the parametrization of the system trajectories. Therefore, by knowing how the disturbances affect the parametrization of the system trajectories, it is very easy to integrate disturbance observers to the basic flatness-based control. While flatness-based control approach can deal well with measurement noises, it is not, in general, robust against external disturbances and model uncertainties. Extending the flatness or feedback linearization-based control by integrating disturbance observers to the controller, one can increase the robustness of the approach. It is common that model uncertainties and external disturbances are integrated into one lumped disturbance vector, which is then estimated and used in the controller design.

The goals of the paper are as follows. First, to give a short overview of some most popular methods to construct the disturbance observers for nonlinear control systems. The paper focuses on two approaches: the basic disturbance observer (BDO), first developed in [9], and the extended state observer (ESO). Both approaches are described and then compared. Second, the paper demonstrates how the disturbance estimation can be integrated into the flatness-based control approach. We address the classical feedback linearization based approach, but also a novel event-based control approach, first introduced in [24], for differentially flat systems. Note that up to the knowledge of the authors it is the first time when disturbance observers are combined with an event-based control approach. Finally, the flatness-based control approaches combined with the ESO and the BDO, are simulated on three examples: on a heating, ventilation and air-conditioning system; an active magnetic bearing system; and on an underwater vehicle.

2. AN OVERVIEW OF DISTURBANCE OBSERVERS

A short overview of some popular methods of finding an estimate of a disturbance is provided. The strengths and weaknesses of various methods are discussed. Note that a fairly good overviews of disturbance observers for linear and nonlinear systems are already given in [8,10,37]. The paper [10] focuses on disturbance-observer-based control methods, but addresses briefly the disturbance estimation methods for linear and nonlinear systems. In [37] a nonlinear disturbance observer is constructed for Euler-Lagrange systems. However, the paper also contains a fairly good overview of disturbance observers is general. The objective of the paper [8] was to provide a historical viewpoint of the development of the so-called basic disturbance observer (see below), the disturbance observer-based control; and to present the link between the disturbance-observer-based control and nonlinear PID for a robotic manipulator under a number of assumptions. Our paper is meant to give a tutorial overview of the most popular approaches for disturbance estimation

Table 1. Overview of papers with different approaches for estimation of disturbance w and assumptions made on disturbances

	Disturbance dynamics known	$w^{(k)} \approx 0$	$w^{(k)}$ bounded	None
BDO	[7,40]	[9,52,55]	[16,45,56]	–
ESO	[13]	[31]	[2,19,41]	–
Sliding mode	[34]	–	[20]	–
Self-learning	–	–	[25,26]	–
Others	[17]	[28]	[3,12,48]	[1]

with a specific goal to apply these methods to improve the flatness-based control approaches. Note that the overview restricts the attention only on the brief exposition of the basic ideas of the approaches. Implementation issues exceed the scope of this paper. However, it is worth pointing out that the paper [10] and the references therein discuss such aspects quite thoroughly.

Disturbance observers are strongly connected to disturbance-observer-based control and in majority of cases, the two are studied together. That is, one does not only find the estimate of a disturbance, but also integrates the estimate to a controller design. Theoretical study of disturbance observers as a separate topic is not much advanced and most of the research is done towards specific applications [10]. For nonlinear disturbance observers there are two popular approaches: (1) the basic nonlinear disturbance observer (BDO) proposed, for example, in [7,9]; (2) the extended state observer (ESO) [2,19,31,41,48]. In the first case only the disturbance is estimated, though in general, the observer equations depend on system states and inputs. So a state observer is necessary unless all the states are measurable. The idea of the ESO is to extend the original state vector by the disturbance vector and possibly, some of its time-derivatives, and then design a state observer for the extended system. There are also other types of disturbance observers, like, sliding-mode-based [20,34], fuzzy [30], self-learning [25,26] and other specific [1,3,12] disturbance observers. In this paper we focus mostly on the BDO proposed in [9] and on the ESO. The reason for such choice is twofold. First, these are most popular approaches in the literature and second, they allow to estimate also the time-derivatives of the disturbances. The latter is especially important in integrating disturbance observers into the flatness-based control approach.

Another way of classifying the nonlinear disturbance observers is by the assumptions made on the disturbance. Some papers [7,13,31,34,40,49] assume that the disturbance dynamics is known, other papers assume that the disturbance and/or some of its time-derivatives are bounded [2,16,41]. It is often assumed that the first [9] or some higher order [28,55] time-derivatives of the disturbance are approximately zero. The assumption that the disturbance dynamics is known or the disturbance is generated by certain, usually linear, dynamics, means in practical terms that the incomplete system model is just improved. When time-derivatives of the disturbance are assumed to be bounded, then most one can prove is that the error dynamics (real minus the estimated disturbance) is also bounded. Obviously, the assumption that the first time-derivative of the disturbance is approximately zero works well for constant or very slowly varying disturbances. This limits applicability of this type of disturbance observers. A much weaker assumption is that the kth order time-derivative of the disturbance is approximately zero. In principle, this assumption allows to estimate all the disturbances, which behave as an analytic function on some (long enough) time-interval, since such functions can be approximated by a polynomial on the given time-interval. An overview of different approaches for disturbance estimation based on the assumptions made on the disturbance w are given in Table 1.

2.1. Basic disturbance observer

Consider a control-affine system of the form

$$\dot{x} = f(x) + g_1(x)u + g_2(x)w, \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state, $u(t) \in \mathbb{R}^m$ is the input and $w(t) \in \mathbb{R}^\rho$ is the disturbance of the system. A disturbance observer was proposed in [9] to estimate the disturbance w in (1) under the assumption that $\dot{w} = 0$. The observer equations are as follows:

$$\begin{aligned} \dot{z} &= -L(x)[f(x)+g_1(x)u+g_2(z+p(x))], \\ \hat{w} &= z+p(x), \end{aligned} \tag{2}$$

where $z(t) \in \mathbb{R}^\rho$ is the observer state, $\hat{w}(t)$ is the estimation of the disturbance $w(t)$, $p(x)$ and $L(x)$ are the observer gains to be chosen, satisfying

$$L(x) = \frac{\partial p(x)}{\partial x}. \tag{3}$$

It has been shown that the error $e = w - \hat{w}$ dynamics is

$$\dot{e} = -L(x)g_2(x)e.$$

Now, the gain $L(x)$ must be chosen such that the error dynamics is stable for every x and $p(x)$ is computed from (3). A systematic way of choosing $L(x)$ is described in [9] for a multilink robotic manipulator. Note that in order to apply the disturbance observer (2), one has to assume that the system state x and input u are known.

The disturbance observer (2) was quickly generalized for the case when the disturbance was generated by a linear dynamics [7], in which case the assumption $\dot{w} = 0$ was not needed anymore. Also an approach to choose the gain values $L(x)$ was given in [7]. The approach has become popular in many applications, see for instance [37,52]. The paper [55] tried to generalize the approach for the case when a higher order time-derivative of the disturbance was assumed to be zero, but is shown to be incorrect [23]. In fact, the correct generalization was already proposed in [16] and later in [6,45,56]. All these results consider the case when the disturbance enters the system dynamics linearly, i.e. the case when $g_2(x)$ is constant or even identity matrix. The paper [16] also assumes the nominal system to be in the Brunovsky canonical form. An important property of the generalized basic disturbance observer is that it also gives estimates of the time-derivatives of the disturbance.

Here we give the generalized basic disturbance observer for a more general class of systems (1). Assume now that the kth time-derivative of w is zero, i.e., $w^{(k)} = 0$. Then, the disturbance observer (2) is generalized to

$$\begin{aligned} \dot{z}_0 &= -L_0(x)[f(x)+g_1(x)u+g_2(z_0+p_0(x))]+z_1+p_1(x), \\ \dot{z}_1 &= -L_1(x)[f(x)+g_1(x)u+g_2(z_0+p_0(x))]+z_2+p_2(x), \\ &\vdots \\ \dot{z}_{k-2} &= -L_{k-2}(x)[f(x)+g_1(x)u+g_2(z_0+p_0(x))]+z_{k-1}+p_{k-1}(x), \\ \dot{z}_{k-1} &= -L_{k-1}(x)[f(x)+g_1(x)u+g_2(z_0+p_0(x))], \\ \hat{w} &= z_0+p_0(x), \\ \hat{\dot{w}} &= z_1+p_1(x), \\ &\vdots \\ \widehat{w^{(k-1)}} &= z_{k-1}+p_{k-1}(x), \end{aligned} \tag{4}$$

where for $i = 0, \ldots, k-1$, $z_i(t) \in \mathbb{R}^\rho$ is the observer state, $\widehat{w^{(i)}}$ is the estimation of $w^{(i)}$, $p_i(x)$ and $L_i(x)$ are observer gains to be chosen, satisfying

$$L_i(x) = \frac{\partial p_i(x)}{\partial x}. \tag{5}$$

Now, let $e_i := w^{(i)} - \widehat{w^{(i)}}$, $i = 0, \dots, k-1$. Then, under the assumption $w^{(k)} = 0$, one has

$$\begin{array}{rcl} \dot{e}_0 & = & e_1 - L_0(x)g_2(x)e_0, \\ \dot{e}_1 & = & e_2 - L_1(x)g_2(x)e_0, \\ & \vdots & \\ \dot{e}_{k-2} & = & e_{k-1} - L_{k-2}(x)g_2(x)e_0, \\ \dot{e}_{k-1} & = & -L_{k-1}(x)g_2(x)e_0. \end{array} \tag{6}$$

Choosing $L_i(x)$, $i = 0, \dots, k-1$, such that dynamical system (6) is stable for all x guarantees that e_i, $i = 0, \dots, k-1$, stabilizes to zero. Then, $p_i(x)$ can be found from (5). To guarantee the stability of (6) for all values of x is not, in general, a simple task. However, if $g_2(x) \in \mathbb{R}^{n \times p}$, then one can also choose $L_i(x) \in \mathbb{R}^{p \times n}$ for $i = 0, \dots, k-1$. Then the error system (6) becomes a linear autonomous system and the gains L_i, $i = 0, \dots, k-1$, can be found such that the roots of the characterizing polynomial of the state transition matrix are on the left-half of the complex plane.

2.2. Extended state observer

The basic disturbance observer (2) and the generalized disturbance observer (4) depend on the knowledge of the system state x and the input u. The extended state observer (ESO) can be used to estimate the disturbance under the same assumptions, i.e., $\dot{w} \approx 0$ or $w^{(k)} \approx 0$. However, the ESO does not need the knowledge of the system state x, but also provides estimates of the state variables.

Consider the system (1) and assume that $w^{(k)} \approx 0$ for some $k \in \mathbb{N}$. Extend the state vector of system (1) to $\bar{x} = (x, w, \dot{w}, \dots, w^{(k-1)})^T$, which yields the extended system equations

$$\dot{\bar{x}} = \bar{f}(\bar{x}) + \bar{g}_1(\bar{x})u. \tag{7}$$

Now, the state x, the disturbance w and its first $k-1$ time-derivatives $w^{(i)}$, $i = 1, \dots, k-1$, for system (1) can be estimated by constructing an observer for the extended system (7). Like in the basic disturbance observer case, one can, instead of $w^{(k)} \approx 0$ assume that the dynamics of the disturbance is known and extends the system accordingly.

Essentially, the state extension based methods rely (sometimes implicitly) on the property that *the extended system* (7) *is observable*. This can at times be a restrictive assumption. For instance, consider the simple linear system

$$\begin{array}{rcl} \dot{x}_1 & = & x_2 + d_1, \\ \dot{x}_2 & = & u + d_2, \\ y & = & x_1 \end{array} \tag{8}$$

and extend it with the dynamics of the disturbance

$$\begin{array}{rcl} \dot{d}_1 & = & 0, \\ \dot{d}_2 & = & 0. \end{array} \tag{9}$$

It is easy to check that the system (8)–(9) is not observable since observability matrix has rank three whereas the number of extended states is four. Thus the disturbance vector to be added to nominal plant equations is limited to the one with dimension 1. Even more, system (8)–(9) with $d_2 = 0$ is still not observable. However, the case with $d_1 = 0$ is observable. This situation corresponds to the so-called matched disturbance case when the disturbance input does not show up earlier on the output than the control input. Therefore, applicability of ESO methods is limited and often scalar disturbances are considered [31]. A bit less restrictive is the case when the disturbance dynamics is assumed to be known. However, the observability assumption is still necessary. If we assume that the disturbances in the system (8) are generated by the model

$$\begin{aligned} \dot{\xi}_1 &= \xi_2, \\ \dot{\xi}_2 &= 0, \\ d_1 &= \xi_1, \\ d_2 &= \xi_2, \end{aligned} \tag{10}$$

then one still cannot observe x_1, x_2, ξ_1 and ξ_2 from the output y and its time-derivatives.

Different approaches have been used to find the state observer for the extended system. In [2,31] the standard Luenberger observer is constructed, while in [19,41,48] more advanced observers are used.

2.3. BDO versus ESO

In this subsection we discuss advantages and disadvantages of both, the basic disturbance observer (BDO) and the ESO. The common feature of both is the assumption made for the disturbance: either a time-derivative of the disturbance is assumed to be zero or the dynamics of the disturbance is assumed to be known. Now, we compare both methods with respect to their applicability.

1. First, to apply the BDO, the system state and input have to be known, whereas it is not required to apply the ESO. However, observability assumption of the extended system is needed in case of the ESO. Under the assumption that the full state vector x is measured, the two methods are both applicable, since then the extended system (7) is always observable. However, if this is not the case, the ESO can still be applied (under the assumption that the extended system (7) is observable), but the BDO, in general, cannot be used.
2. Second, choosing the gains $L_i(x)$ in BDO can be difficult if $g_2(x)$ is highly nonlinear. If the disturbance is added in the system equations linearly, as often assumed in applications, then the gains $L_i(x)$ can be chosen constants and the stability of the error dynamics can be easily guaranteed. The problematic aspect of the ESO may be construction of the observer for the extended system (7). The reason is that the extended system (7) can be highly nonlinear and observer construction for such systems is not a trivial task. Then again, under the assumptions that all the state variables are measurable and the disturbance is added in the system equations linearly, the extended system (7) is in the observer form, for which an observer can be easily constructed.
3. Third, the BDO has been developed for disturbance affine systems, whereas, in principle, the ESO can be used for general nonlinear systems, where the disturbance enters into the system dynamics in non-affine manner.

Summarizing, under the same assumptions (the state vector x is measured and the disturbance is added linearly), the BDO and ESO are approximately equally effective. However, the ESO can be, in general, applicable for non-affine systems, while the BDO does not. It is still worth pointing out that constructing the observer for the extended system in a non-affine case is a difficult problem itself.

Although we have assumed that in both cases – the BDO and the ESO – the kth time-derivative of the disturbance w is zero, both methods succeed also when the assumption is not exactly satisfied. The assumption was made primarily to guarantee the asymptotic stability of the error dynamics. Note that the error dynamics can still be stable in the sense that $||e(t)|| < \varepsilon$, if the kth time-derivative of the disturbance is assumed to be just bounded. Additionally, the value of ε can be lowered by choosing the higher gain values.

2.4. Other approaches

This subsection contains a brief discussion of some other approaches for the design of disturbance observers, developed during the past few years.

The paper [1] relies on the concept of tracking differentiators to define nonlinear disturbance observers. The approach has similarities with the ESO. Instead of assuming that the time-derivative of the disturbance is zero, the time-derivative of the disturbance is assumed to be generated by a tracking differentiator. The authors claim that their disturbance observer can estimate almost all types of disturbances and does not need any prior information about the disturbance. Nevertheless, the applicability of the approach remains questionable, since only scalar systems ($n = 1$) were studied and no hint was given how to generalize the approach for non-scalar case. Also, it seems that the knowledge on the state and input variables is necessary.

A totally different approach is presented in [12]. A *specific matched disturbance*, which affects directly the input, is considered. Then the Hirschorn (left) inverse of the control system is used to compute the estimate $\hat{v}$ of the total input $v := u + w$. Since the inverse depends on the system states, a separate state estimation is necessary. Then, $\hat{v}$ is compared to the input u to receive an estimate of the disturbance w. Low-pass filters are also used to estimate the time-derivatives of the output.

Finally, in [3] a state observer is constructed for systems with bounded exogenous inputs (disturbances and sensor noise). Then an unknown input is estimated based on the observed and measured state variables. However, only systems with specific structure and linear disturbances are considered.

2.5. Future research

A number of future research directions are named in [10] concerning disturbance (and uncertainty) estimation and attenuation. As mentioned in [10], the theoretical research is still well behind the applications in this research area. Many methods for disturbance estimation assume that the system state is measurable (for example BDO). Also, most approaches are developed for control- and disturbance-affine systems or even for systems with linear disturbances. Regarding the above restrictions the ESO is an exception; however, the observer construction in such case may become very difficult if possible at all, since it requires constructing an observer for a general nonlinear control system.

Finally, note that only a few papers [5,27,44] study disturbance observers for discrete-time systems. An ESO was developed in [5] for linear discrete-time systems under the assumption that the disturbance is slowly varying compared to the sampling time. The BDO was generalized in [27] to linear discrete-time systems. Moreover, another observer, similar to the BDO, was given in [27] to relax the assumption that all the states are available for the measurement. These results were further developed in [44].

3. FLATNESS-BASED CONTROL

In this section we describe the basic flatness-based control approach as well as a novel event-based controller for differentially flat systems. The estimates of disturbances and possibly their time-derivatives are incorporated to both controllers. The flatness-based control with disturbance observers has been used before in [17,25,47]. Note that for mismatched disturbances the estimates of some time-derivatives of the disturbance are necessary, whereas for matched disturbances it may not be necessary. Incorporation of disturbance observers into these control schemes is not strictly necessary, but helps to improve the performance while keeping control values smaller, which in many applications corresponds to lower energy usage.

Consider a nonlinear control system of the general form

$$\dot{x} = f(x,u), \tag{11}$$

where $x(t) \in X \subseteq \mathbb{R}^n$ is the system state and $u(t) \in U \subseteq \mathbb{R}^m$ is the system input. It is assumed that the function f is analytic and satisfies on some open and dense subset of $X \times U$ the condition $\operatorname{rank}[\partial f/\partial u] = m$, meaning that there are no redundant inputs.

Recall the flatness property of system (11) as follows [29].

Definition 1. *System* (11) *is said to be flat if there exists an output function*

$$y = h(x, u, \dots, u^{(l)}) \quad l \geq 0 \tag{12}$$

$(y(t) \in \mathbb{R}^m)$*, called flat output, such that*

$$x = \varphi_x(y, \dots, y^{(k)}), \tag{13}$$
$$u = \varphi_u(y, \dots, y^{(k+1)}) \tag{14}$$

for some $k \in \mathbb{N}$ *and functions* φ_x*,* φ_u*.*

A more formal definition of flatness and more thorough discussion can be found, for example, from [14,15,29]. The flatness-based control has attracted a lot of attention throughout last decades, see the books [29,43]. It is known that any differentially flat system is also feedback linearizable by an endogenous state feedback.

However, often disturbances affect the equations (11), i.e., one has

$$\dot{x} = f(x, u, w), \tag{15}$$

for the disturbance $w(t) \in \mathbb{R}^\rho$. In this case the relations (13) and (14) are also affected by the disturbance and some finite number of its time-derivatives, i.e.,

$$x = \tilde{\varphi}_x(y, \dots, y^{(k)}, w, \dots, w^{(\mu)}), \tag{16}$$
$$u = \tilde{\varphi}_u(y, \dots, y^{(k+1)}, w, \dots, w^{(\mu+1)}). \tag{17}$$

When the disturbance and its time-derivatives are not known or estimated, then the nominal model (11) and corresponding relations (13), (14) are used in the control design. If the disturbance and its time-derivatives are estimated, then one can use the more accurate relations (16) and (17) instead.

3.1. Feedback linearization based control

To simplify the situation, consider a single input system (11) with $y = h(x)$ being the flat output. Then, replace y and its first k time-derivatives in (14) (or (17)) by $h(x)$ and its first k time-derivatives and $y^{(k+1)}$ by a new control input v. This gives us the feedback

$$u = \varphi_u(h(x), \dots, h^{(k)}(x), v)$$

or

$$u = \varphi_u(h(x), \dots, h^{(k)}(x), v, w, \dots, w^{(\mu+1)})$$

respectively, which yields a linear closed-loop system $y^{(k+1)} = v$. Now, any linear control approach can be used to control the closed-loop system. For example, one can take

$$v = r^{(k+1)} - \textstyle\sum_{i=0}^{k} q_i (y^{(i)} - r^{(i)}), \tag{18}$$

where $r(t)$ is the reference trajectory of y and $q_i \in \mathbb{R}$, $i = 0, \dots, k$, are chosen such that the error $e = y - r$ dynamics $e^{(k+1)} + q_k e^{(k)} + \cdots + q_0 e = 0$ is stable.

3.2. An event-based approach

A different, an event-based, approach for controlling differentially flat systems is briefly described in this subsection. Assume for simplicity that the flat output $y = h(x)$ is also the output-to-be-controlled of system (11). Here, instead of replacing $y = (y_1, \ldots, y_m)^T$ and its time-derivatives in (14) (or (17)) by $h(x)$ and its time-derivatives, we replace $y = (y_1, \ldots, y_m)^T$ and its time-derivatives by a pre-defined trajectories

$$y_{ir}(t) = p_i(t)\mathrm{e}^{-K_i t} + r_i(t), \qquad i = 1, \ldots, m, \tag{19}$$

which converge to the desired trajectories $r_i(t)$ of y_i, $i = 1, \ldots, m$. The polynomial $p_i(t) \in \mathbb{R}[t]$ is used to match the actual initial conditions of the system states and desired initial states, i.e., to guarantee that

$$\begin{aligned} x(0) &= \varphi_x(y_r(0), \ldots, y_r^{(k)}(0)), \\ u(0) &= \varphi_u(y_r(0), \ldots, y_r^{(k+1)}(0)), \end{aligned} \tag{20}$$

where $y_r = (y_{1r}, \ldots, y_{mr})^T$. Finally, the constant parameters K_i, $i = 1, \ldots, m$, can be freely chosen and affect the speed at which the output y_{ir} converges to the desired reference trajectory $r_i(t)$.

Then one gets a feedforward controller

$$u = \varphi_u(y_r, \ldots, y_r^{(k+1)})$$

or

$$u = \varphi_u(y_r, \ldots, y_r^{(k+1)}, w, \ldots, w^{(\mu+1)}),$$

which directs the system output y to follow the trajectory y_r. Because uncertainties and disturbances affect the system, the actual output trajectory starts to deviate from the desired one y_r. If this happens, an event is generated and a new desired trajectory y_r is computed based on the actual measurements of the system states. Since y_r always converges to $r = (r_1, \ldots, r_m)$, then, if K_i, $i = 1, \ldots, m$, are appropriately chosen, y also converges to r.

4. EXAMPLES

In this section we demonstrate on three examples from different areas how the flatness-based control together with disturbance estimation improves the system performance compared to the case when no disturbance estimation is used. Also, up to the authors knowledge, this is the first time when disturbance observers are integrated into an event-based control approach. Doing so we not only improve the performance of the closed-loop system, but also reduce the number of events necessary to achieve such closed-loop performance. The number of events, in turn, corresponds to communication load between the sensors and the controller.

First, an ESO is constructed to estimate a *slowly varying* unmeasured thermal load acting on a heating, ventilation and air-conditioning (HVAC) model. Second, a *very fast disturbance* and its time-derivative are estimated to control an active magnetic bearing system. We show that although the assumptions of the generalized BDO (4) are not satisfied (second derivative is not approximately zero) the observer (4) can still be used to estimate the disturbance and its derivative. Third, a BDO is constructed to estimate disturbances acting on an underwater vehicle and integrated to the event-based controller, described in Subsection 3.2.

4.1. Heating, ventilation and air-conditioning system

A model of heating, ventilation and air-conditioning system (HVAC) was given in [35] for one thermal zone as follows:

$$\begin{aligned} \dot{x}_1 &= \tfrac{c_p}{C_1}(T_s - x_1)u + \tfrac{1}{C_1 R}(x_2 - x_1) + \tfrac{1}{C_1 R_o}(T_o - x_1) + w, \\ \dot{x}_2 &= \tfrac{1}{C_2 R}(x_1 - x_2), \\ y &= x_1, \end{aligned} \tag{21}$$

where the system variables and parameters are described in Table 2.

Table 2. Descriptions of different variables and parameters in model (21)

Symbol	Value	Physical description
x_1	State variable	Air temperature of the thermal zone
x_2	State variable	Temperature of floors, walls, furniture etc.
u	Input variable	Mass flow rate of supply air
w	Disturbance variable	Unmeasured thermal load
c_p	0.000281, kWh/kg*K	Heat capacity of thermal zone air
C_1	0.00275, kWh/K	Thermal capacitance of air
C_2	1.87733, kWh/K	Thermal capacitance of floors, walls, furniture etc.
T_s	17, °C	Temperature of supply air
R	2.08, K/kW	Thermal resistance between C_1 and C_2
R_o	11.849, K/kW	Thermal resistance between the thermal zone and outside air
T_o	27, °C	Outside air temperature

We want to estimate w. Assuming that $\dot{w} = 0$, we extend the equations (21) with $x_3 = w$, which yields an extended system

$$\begin{aligned} \dot{x} &= Ax + g(y,u), \\ y &= x_1, \end{aligned} \tag{22}$$

where

$$A = \begin{pmatrix} -\frac{1}{C_1R} - \frac{1}{C_1R_o} & \frac{1}{C_1R} & 1 \\ \frac{1}{C_2R} & -\frac{1}{C_2R} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

and

$$g(y,u) = \begin{pmatrix} \frac{c_p}{C_1}(T_s - y)u + \frac{T_o}{C_1R_o}. \\ 0 \\ 0 \end{pmatrix}.$$

System (22) is observable and moreover, in the observer form, thus one can construct an observer

$$\dot{\hat{x}} = A\hat{x} + g(y,u) + L(y - \hat{x}_1). \tag{23}$$

Let the error be $e = x - \hat{x}$, then the error dynamics becomes

$$\dot{e} = (A - LC)e,$$

where $C = (1 \ \ 0 \ \ 0)$. The matrix $L = (l_1 \ \ l_2 \ \ l_3)^T$ can be chosen such that $A - LC$ is an asymptotically stable matrix. The estimate $\hat{x}_3$ of (23) gives the estimate of w.

In model (21) we want to control the room temperature x_1. Note that, x_1 is not the flat output of the system. Instead, $y = x_2$ can be chosen as the flat output. Therefore, using the feedback linearization based control approach we can only control directly the variable x_2. Nevertheless, from the second equation of (21), by driving x_2 to a constant value, the variable x_1 will achieve the same constant value. The goal is to change room temperature x_1 from 27 °C to 20 °C when outside temperature is 27 °C. The simulations in Fig. 1 show that the observer (23) tracks the disturbance and that the feedback linearization control approach is much improved compared to the case when no disturbance estimation is added to the controller.

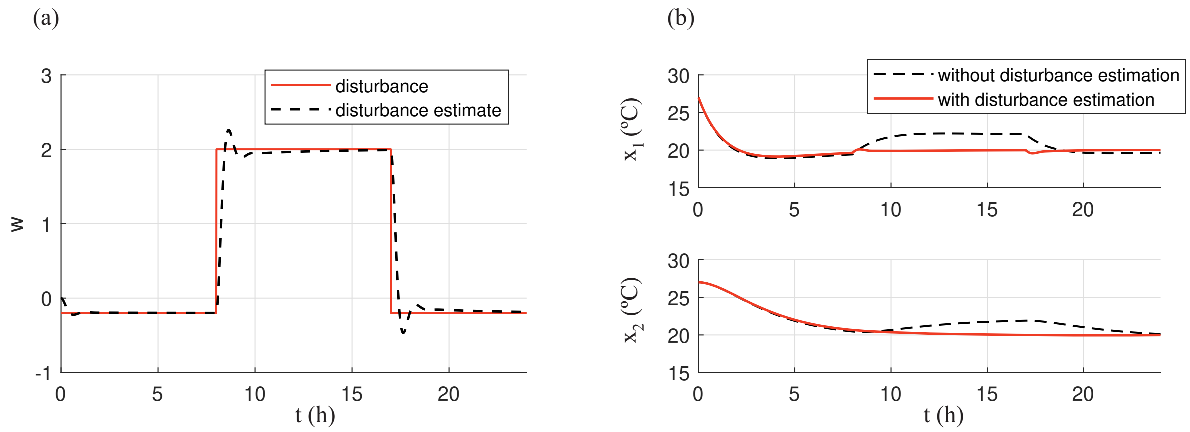

Fig. 1. The disturbance (a) and state variables (b) for system (21).

4.2. Active magnetic bearing system

A *voltage-controlled* model of an active magnetic bearing (AMB) system is given in [36] and described by the dynamics

$$\begin{array}{rcl} \dot{x}_1 & = & x_2, \\ \dot{x}_2 & = & \vartheta_1 x_3 x_4 + \frac{1}{m} w, \\ \dot{x}_3 & = & \frac{1}{N} u_1 + \vartheta_2 x_3 + \vartheta_3 x_1 x_4, \\ \dot{x}_4 & = & \frac{1}{N} u_2 + \vartheta_2 x_4 + \vartheta_3 x_1 x_3, \\ y & = & x_1, \end{array} \tag{24}$$

where x_1 is the rotor position [m], x_2 is the rotor speed [m/s], $x_3 = \phi_b$, $x_4 = \phi_c$, for $\phi_b = \phi_1 + \phi_2$, $\phi_c = \phi_1 - \phi_2$, where ϕ_1 and ϕ_2 are the electromagnetic fluxes [Wb] of two opposite electromagnets, respectively. Moreover, $\vartheta_1 = \frac{1}{\mu_0 m A}$, $\vartheta_2 = \frac{-2s_0 R}{\mu_0 N^2 A}$ and $\vartheta_3 \frac{2R}{\mu_0 N^2 A}$ are known constants, with system parameters described in Table 3.

The flat output of system (24) can be chosen as $y_1 = x_1$ and $y_2 = x_4$. According to the feedback linearization approach, we design the controller

$$\begin{array}{rcl} u_1 & = & N(\frac{v_1}{\vartheta_1 x_4} - \vartheta_2 x_3 - \vartheta_3 x_1 x_4 - \frac{x_3 v_2}{x_4} - \frac{\dot{w}}{m\vartheta_1 x_4}), \\ u_2 & = & N(v_2 - \vartheta_2 x_4 - \vartheta_3 x_1 x_3), \end{array} \tag{25}$$

where

$$\begin{array}{rcl} v_1 & = & -q_{12}\ddot{y}_1 - q_{11}\dot{y}_1 - q_{10}y_1 \\ & = & -q_{12}(\vartheta_1 x_3 x_4 + \frac{1}{m}w) - q_{11}x_2 - q_{10}x_1, \\ v_2 & = & -q_{20}(y_2 - y_2^{ref}) = -q_{20}(x_4 - 0.001). \end{array} \tag{26}$$

The parameters q_{1j}, $j = 0,1,2$, and q_{20} are chosen such that the linearized system equations are stable. In the current simulations we want the convergence to be fast, so we choose $q_{10} = 64000$, $q_{11} = 4800$, $q_{12} = 120$ and $q_{20} = 40$. Note that the controller (25), (26) depends on the disturbance w and its first time-derivative $\dot{w}$. To estimate them, we use the BDO approach, described in Subsection 2.1.

The disturbance observer (4) is constructed under the assumption that $\ddot{w} = 0$. We choose $L_0 = (0,500,0,0)$ and $L_1 = (0,16000,0,0)$, which yield stable error dynamics (6) and fast enough convergence of the disturbance estimation. Thus, from (5) one has $p_0(x) = 500x_2$, $p_1(x) = 16000x_2$, and the disturbance observer

$$\begin{aligned}
\dot{z}_0 &= -500[\vartheta_1 x_3 x_4 + \tfrac{1}{m}(z_0 + 500x_2)] + z_1 + 16000x_2, \\
\dot{z}_1 &= -16000[\vartheta_1 x_3 x_4 + \tfrac{1}{m}(z_0 + 500x_2)], \\
\hat{w} &= z_0 + 500x_2, \\
\hat{\dot{w}} &= z_1 + 16000x_2.
\end{aligned} \tag{27}$$

Now, the estimates $\hat{w}$ and $\hat{\dot{w}}$ are integrated to controller (25), (26). The actual disturbance is simulated as a sinusoid with constant frequency and amplitude. Figure 2 shows the simulation results. In Fig. 2a the disturbance and its time-derivative together with their estimates is presented. Note that since the assumption ($\ddot{w} = 0$) made in the disturbance observer construction is not actually satisfied, the error does not converge to zero, but is bounded around zero. Figure 2b presents the state variables of the closed loop system in two cases: when the feedback linearization based control is combined with the disturbance observer (27) and when it is not. Clearly, the addition of disturbance observer to the controller improves the performance of the closed-loop system significantly.

Note that there are many papers (for example, [11,17,33,39,40,46,51]) that present results on controlling the AMB system where a disturbance observer is integrated into the control scheme. Some of the papers ([11,33,39,40,46]) consider linear voltage-controlled models, others [17,51] nonlinear current-controlled models. Also, different methods for disturbance estimation are being used. For example, the BDO [40, 46], the ESO [33,51] or inversion-based method with low-pass filters [39]. Moreover, different control approaches are implemented, such as flatness-based-control [17], a linear state feedback [40,46], an output

Table 3. Values and descriptions of AMB system parameters

Symbol	Value	Physical description
s_0, m	0.0004	Air gap
m, kg	2.5	Mass of the rotor
L_o, H	0.0025	Coil inductance
L_s, H	0.0005	Coil inductance losses
R, Ω	0.5	Coil resistance
N	108	Number of turns of wire in the coil
$\mu_0, H/m$	1.25×10^{-6}	Permeability of free space
A, m^2	0.0014	Cross sectional area of air gap
$k_i, N/A$	15.625	Current stiffness
$k_s, N/m$	97656.25	Displacement stiffness

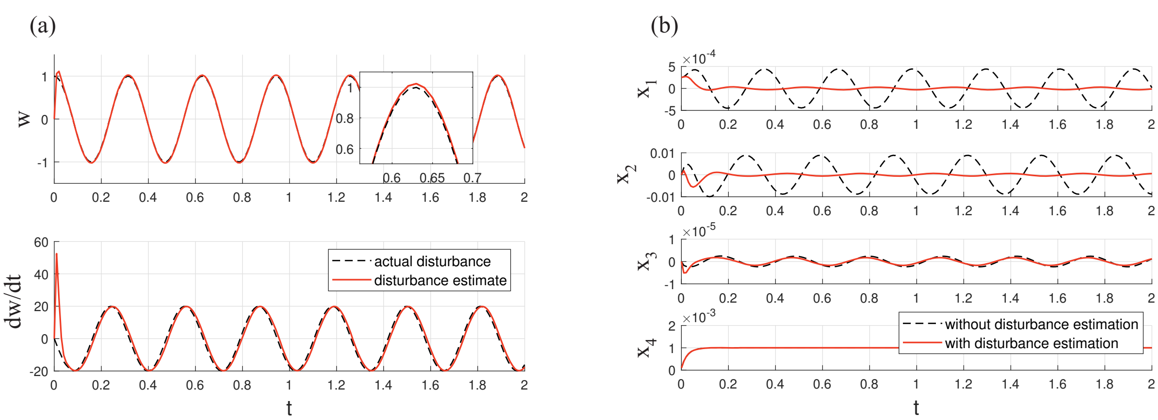

Fig. 2. (a) Disturbance w, its time-derivative $\dot{w}$ and their estimate; (b) The state trajectories compared to the case when there is no disturbance estimation added to the control design.

feedback control law [51], H_∞ control [39] etc. However, up to the authors knowledge, there are no papers, where a disturbance-observer-based control method is applied to a nonlinear voltage-controlled model, such as (24).

4.3. Underwater vehicle

A model of an underwater vehicle, called U-CAT, is proposed in [42]. It has two motion modes, denoted by SLOW and FAST, which depend on how the four fins of the vehicle are configured. The state space model for SLOW mode is as follows:

$$\begin{array}{rcl} \dot{x}_1 &=& x_2\cos(x_5) - x_4\sin(x_5), \\ \dot{x}_2 &=& -\frac{C_1}{C_2}x_4x_6 - \frac{X_{uu}}{C_2}x_2|x_2| + \frac{u_1}{C_1} + w_1, \\ \dot{x}_3 &=& x_2\sin(x_5) + x_4\cos(x_5), \\ \dot{x}_4 &=& -\frac{C_3}{C_4}x_2x_6 - \frac{Y_{vv}}{C_4}x_4|x_4| + \frac{u_2}{C_4} + w_2, \\ \dot{x}_5 &=& x_6, \\ \dot{x}_6 &=& -\frac{C_5}{C_6}x_2x_4 - \frac{N_{rr}}{C_6}x_6|x_6| + \frac{u_3}{C_6} + w_3, \end{array} \quad (28)$$

where $w = [w_1, w_2, w_3]^T$ represents the unknown disturbance vector. The parameter values of the model (28) are displayed in Table 4.

The flat outputs of the system (28) are chosen as $y_1 = x_1$, $y_2 = x_3$, and $y_3 = x_5$, which we also want to control. The relations (16) do not depend on the disturbance. The relation (17) depends on the disturbance w, but not on the time-derivatives of the disturbance. Thus, only the estimates of the disturbances are necessary.

The BDO (4) is constructed to estimate the disturbances w_i, $i = 1,2,3$, in (28). Note that these disturbances are simulated as noisy signals and the disturbance observer estimates only their mean value. However, this is fine for us, since the event-based control approach, described in Subsection 3.2, can deal with the noise in the disturbances.

The situation, where the underwater vehicle starts from the point $(-4;4)$ on a (x_1,x_3)-plane with $x_5 = x_2 = x_4 = x_6 = 0$ and does circles around 0 with radius 2, is simulated. At the same time, the angle x_5 will go from 0 to the new set point $\pi/2$. The parameters K_i, $i = 1,2,3$, are all taken equal to 2. The error threshold is chosen as $\varepsilon = 0.05$. Two situations are considered: first, when the disturbance estimate is continuously sent to the controller. Second, when the disturbance estimate is sent to the controller only at the event times.

The results of the case when disturbance estimate is continuously sent to the controller are displayed in Fig. 3. The disturbance and its estimate are presented (a) together with the output trajectories compared to their reference trajectories (b).

Second, the case when the disturbance estimate information is sent to the controller only on the event times, was simulated. The disturbance estimate is as before (Fig. 3a), however, the output trajectories are displayed in Fig. 4a.

Finally, if no disturbance estimate is used in the controller design, then the corresponding output trajectories are displayed in Fig. 4b. The number of event times in all three cases are given in Table 5. As one can expect, addition of disturbance observer reduces the number of events significantly.

Table 4. Values of the parameters for U-CAT model

Parameter	Value	Parameter	Value
C_1	–40	C_2	59
C_3	59	C_4	40
C_5	–19	C_6	2.8179
X_{uu}	56	Y_{vv}	551
N_{rr}	0.7226		

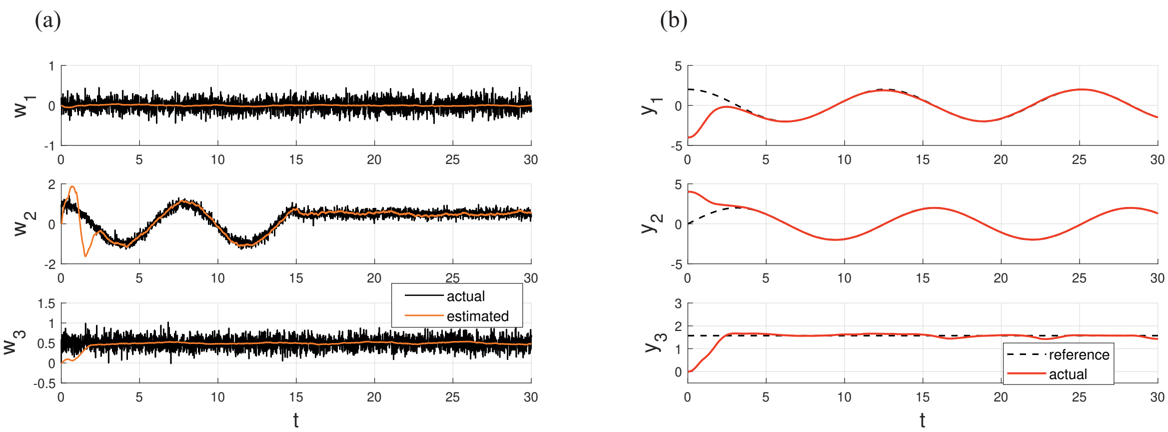

Fig. 3. (a) The disturbance estimation results for the U-Cat model; (b) The output trajectories of the U-Cat model in the case when continuous disturbance estimate signal is sent to the controller.

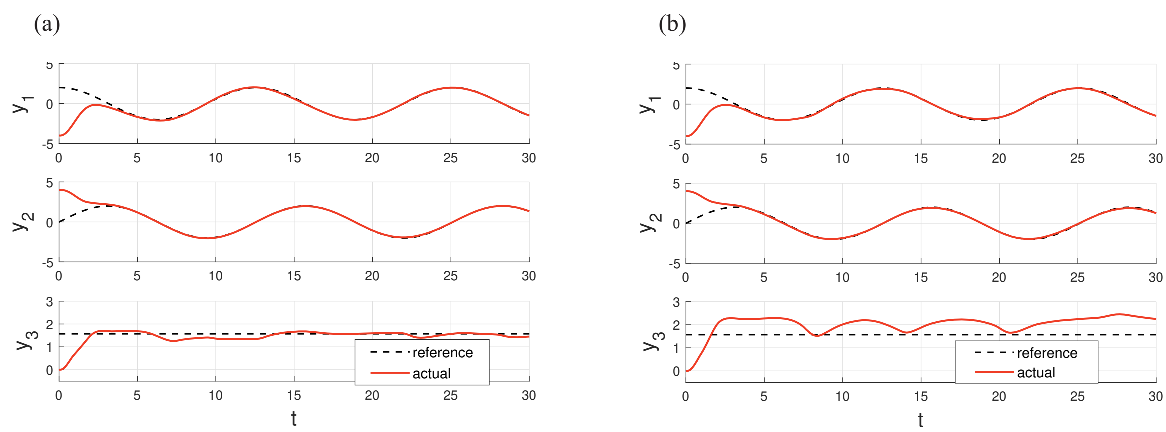

Fig. 4. (a) The output trajectories of the U-Cat model in the case when disturbance estimate signal is sent to the controller only at event times; (b) The output trajectories of the U-Cat model in the case when no disturbance estimate is sent to the controller.

Table 5. Number of event times in different scenarios

Scenario	Number of events
No disturbance estimate added to the controller	64
Constant disturbance estimate sent to the controller	15
Disturbance estimate sent to the controller at event times	20

5. CONCLUSIONS

A brief overview of disturbance observer approaches was given with focus on two of the most popular ones: the basic disturbance observer and the extended state observer. These methods were described in detail and compared with respect to their applicability. Then the disturbance estimates were integrated into the flatness-based control (feedback linearization based and event-based approaches) and the effectiveness of the combination was demonstrated on three practical examples: the HVAC, the AMB and the underwater vehicle models.

Some future research directions for disturbance observers were mentioned in Subsection 2.5. As for the event-based control together with the integrated disturbance observer, it remains to prove the stability of the closed-loop system.

ACKNOWLEDGEMENTS

The work of A. Kaldmäe and Ü. Kotta were supported by the Estonian Center of Excellence in IT (EXCITE), funded by the European Regional Development Fund. The publication costs of this article were covered by the Estonian Academy of Sciences.

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Lühiülevaade häiringu vaatlejatest mittelineaarsetes süsteemides: rakendus lameduse omadusel põhinevale juhtimisele

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On antud lühiülevaade populaarsetest häiringu hindamise meetoditest ja näidatud, kuidas seda hinnangut on võimalik kasutada süsteemi lameduse omadusel põhineva juhtimismeetodi täpsemaks muutmisel. Algul on antud üldine ülevaade häiringu hindamise deterministlikest meetoditest, seejärel on kirjeldatud lähemalt kaht kõige populaarsemat meetodit eeldusel, et leidub häiringu lõplikku järku tuletis, mis on null. Esimene neist on nn tavaline häiringu vaatleja. Lihtsustatult öeldes põhineb antud vaatleja süsteemi olekute mõõdetud väärtuste ja mudeli poolt ennustatud väärtuste võrdlemisel. Seega eeldab antud meetod, et kõik süsteemi olekud oleksid mõõdetavad. Teine meetod häiringu hindamiseks on nn laiendatud olekutaastaja konstrueerimine. Antud juhul laiendatakse süsteemi olekuruumi häiringute ja mingi lõpliku arvu häiringute tuletistega. Laiendatud süsteem eeldatakse olevat häiringuvaba ja sellise laiendatud süsteemi jaoks konstrueeritakse olekutaastaja, mis muuhulgas hindab ka esialgse süsteemi häiringuid ning selle tuletisi. Artiklis on põhjalikumalt võrreldud neid kaht meetodit nende rakendamise võimalikkuse vaatepunktist. Töö teises pooles on näidatud, kuidas häiringute hinnanguid saab kasutada lamedate süsteemide juhtimiseks. Häiringu vaatleja kombineeritakse tagasisidega lineariseerimisel põhineva juhtimismeetodi ja uudse sündmuspõhise juhtimismeetodiga, mille tulemusel saavutatakse täpsem tulemus. Töö viimases osas on näidatud kolmel praktilisel näitel (kütte- ja ventilatsioonisüsteemil, aktiivsel magnetlaagersüsteemil ning veealusel robotil) eelkirjeldatud häiringu hindamise kui ka täiustatud juhtimismeetodite tulemuslikkust.